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Quality-Preserving Algorithms for Grasp Synthesis from Prototypes

by

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Figure Captions

- Figure 1: An example grasp prototype, represented by forces applied at the numbered contacts. In the coordinate system shown, with origin at the object center of mass, force / torque vectors for the five contact points are (f_x, f_y, τ): (-.56, .83, .71), (-.21, -.98, -.70), (-.53, .85, .08), (.46, .89, -.59), (.41, -.91, .81).

- Figure 2: The grasp wrench space of the example grasp. The coordinate frame at the top of each figure illustrates the directions of the force and torque vectors, but note that the actual coordinate frame origin is in the center of the wire frame cylinder (inside the shaded polyhedron).

- Figure 3: (L) Contact wrenches w_i of a grasp prototype with wrench space origin O and convex hull bounding GS for this grasp prototype. (R) Wrench space regions $w_i^{'}$ that make up $GGP(TS_{ball}, Q)$. A grasp having a grasp quality measure greater than or equal to Q can be formed by selecting one contact wrench from each of the four regions. The convex hull formed from these contact wrenches is guaranteed to contain the wrench space ball of magnitude Q.

- Figure 4: Independent wrench space regions for the five contacts of the example grasp, such that any grasp formed by selecting one contact wrench within each of the five regions shown is guaranteed to have a grasp quality measure of 75% of the original grasp quality measure ($Q \geq 0.26$). These regions of normalized contact wrenches are presented in a space of force orientation vs. weighted torque values. The normalized wrenches corresponding to the five contacts of the original grasp are shown.

- Figure 5: Gradient plot of independent wrench space regions for the example prototype grasp. The regions are striped at contact quality intervals of 0.1, with the outer, black stripe representing the contact quality interval [0.0, 0.1).

- Figure 6: Independent wrench space regions superimposed on the object boundary curve of the example polygon. The object boundary curve indicates what contact wrenches are possible for this object. The position of the object boundary curve along the force orientation axis of this plot depends on the relative orientation of the object with respect to the fixed coordinate frame of the grasp prototype.

- Figure 7: Independent contact regions that guarantee a grasp quality measure of 75% of the grasp quality measure of the original grasp. Only point-edge contacts are shown. The region corresponding to contact three overflows onto the edge containing contact one.

- Figure 8: The best match of this new target object to the example grasp has a grasp quality measure of 0.33. In the plot, the object boundary curve of the new object has been superimposed on the 0.0875 quality regions. This object boundary curve has not been shifted with respect to the independent contact regions of the grasp prototype, indicating that the orientation shown is the optimal one. The contact wrenches of the grasp are indicated in the plot, and independent contact region sizes for the quality threshold of 0.0875 can be estimated by examining the plot.

- Figure 9: The best match of this new target object to the example grasp has a grasp quality measure of 0.27. In the plot, the object boundary curve of this new object has been superimposed on the 0.0875 quality regions. This object boundary curve has been shifted by π with respect to the independent contact regions of the grasp prototype, indicating that this object best matches the grasp prototype if it is rotated by 180 degrees (try it). The contact wrenches of the grasp are indicated in the plot, and independent contact region sizes for the quality threshold of 0.0875 can be estimated by examining the plot.

- Figure 10: Two views of the Salisbury hand grasping the toy plane.

- Figure 11: Two views of the Salisbury hand grasping the bubble gun.

Abstract

This paper introduces a grasp synthesis algorithm that can use any grasp prototype as the starting point in a search for a good grasp. The algorithm makes the given prototype more effective by generalizing it for a specific task. This generalization step expands the range of application of the prototype to a wide variety of target object geometries, while ensuring that the resulting grasps are appropriate for the intended task. The generalization step can be performed offline, and the resulting space of grasps is designed so that synthesis of a grasp of a new target object can be easily parallelized.

Features of the algorithm include a rigorous definition of grasp quality and applicability to three-dimensional objects of arbitrary (including curved) geometry. The algorithm does not limit the number of contacts that can make up a grasp, although this number is determined by the grasp prototype used. Although the bulk of the paper presents two-dimensional examples to illustrate the concepts involved, the algorithm has been used to synthesize whole-hand grasps of three-dimensional objects. An example grasp synthesized for the Salisbury hand is shown at the end of the paper.

1 Introduction

Robotic hands are very versatile mechanisms. It has been very difficult, however, to successfully automate the processes of acquiring and manipulating objects using today's robotic hands. One tempting solution is to develop prototype grasps, such as a generic cylinder grasp, to be applied to common objects. Prototype grasps solve part of the problem, and they may require very little computation, but they do not seem sufficiently robust to work in complex situations. At the opposite extreme, a thorough analysis of each grasping or manipulation task would allow more robust grasps to be designed, but this type of analysis is too computationally complex to be feasible. This paper presents a novel, hybrid solution to the problem of grasp synthesis. It shows how a grasp prototype can be analyzed offline to determine how it can be adapted to a wide variety of situations, while maintaining the effectiveness of the grasp for the intended task. This is achieved by focusing on the wrench spaces of the grasp and the task rather than on the appearance of the grasp.

1.1 Previous Work: Finding Good Sets of Contacts

The problem addressed in this paper is selection of a set of contacts between a robot hand and a target object that are optimal for performing a specified task. An important part of finding good points of contact with the target object is to decide how prospective contact sets will be evaluated. The first requirement is that a grasp be stable. Work on the analysis of stable grasps can be found in Yoshikawa [29], where a framework for defining passive and active closure for grasps is clearly specified, Montana [21], where spatial and contact grasp stability are defined, Cutkosky [6] where stable grasps involving curved and soft contacts are studied, Kerr and Roth [15] where linear programming is used to find internal grasp forces, Salisbury [27], and Salisbury and Craig [28], where a grasp matrix is used to map fingertip forces to object forces.

Stable grasps can be evaluated according to various optimality criteria or grasp quality measures. Representative of work on grasp quality measures, are Li, Hsu, and Sastry [17], where the optimality criterion is the ability to manipulate a grasped object; Li and Sastry [18], Kirkpatrick, Mishra, and Yap [16], and Ferrari and Canny [9], where the ability to apply task forces and torques to a grasped object is measured; and Ponce et al. [25] and Faverjon and Ponce [8], where the optimality criterion is robustness of a grasp to errors in contact placement.

Once an optimization function has been specified, optimal sets of contacts can be found on a target object. Computationally efficient grasp optimization techniques for two-dimensional objects have been widely studied. Markenscoff and Papadimitriou [19] illustrates a technique for synthesizing three-contact grasps which minimize the sum of forces required to lift a two-dimensional polygon. Chen and Burdick [4] shows an algorithm for synthesizing two-contact grasps which minimize the size of the friction coefficient required to grasp a two-dimensional curved object. Baker et al. [1] shows that a stable grasp of a polygon can be formed by placing contacts at points of intersection of the polygon with the maximum radius circle that can fit inside it. Hanafusa and Asada [11] shows that stable grasps of two-dimensional objects can be found by minimizing the energy stored in springs at the contacts. The efficiency of these techniques is a result of constraints on the dimensionality of the object, the task to be performed, and the number of contacts of the grasp.

In addition to the work mentioned above, Mishra et al. [20] shows that 12-contact force closure grasps of three-dimensional objects can be found in $O(n)$ time, although these grasps are in general not optimal, and Nguyen [23] describes how to identify a variety of robust contact configurations, including three-contact grasps of polyhedral objects.

A detailed survey of objective functions, tasks, and constraints assumed for various grasp optimization techniques that have been proposed in the literature can be found in Pollard [24]. One goal of this paper is to introduce a computationally tractable grasp optimization algorithm that does not require many of the assumptions used to make these techniques tractable. In particular, the algorithm described in this paper can be applied to three-dimensional objects and any given task, and the number of contacts of the grasp is limited only by the number of contacts in the grasp prototype.

1.2 Previous Work: Standard Grasps and Tailored Grasping Strategies

Work on developing prototype grasps and grasping strategies for articulated hands is motivated by studies of human grasping. Napier [22] divides grasps into the categories of power and precision. Jeannerod [13] studies the effect of sensor feedback on hand shapes and hand trajectories during prehension tasks. Iberall and MacKensie [12] contains a good overview of work in this area.

Representative of work in the category of developing prototype grasps for robot hands is Bard and Troccaz [2], where the target object is decomposed into ellipsoids and a grasping strategy for ellipsoidal objects is explored. Cutkosky and Howe [7] outlines a taxonomy of manufacturing grasps and develops an expert system to choose between them. Brock [3] develops tools for the construction and analysis of sensor-based grasping strategies that eliminate the need to know an object's exact configuration in the workspace. Kang [14] illustrates a technique for mapping a human grasp directly into a manipulator grasp of the same object.

One problem with many of these techniques is that it is not always easy to determine a priori when they will work. Brock [3] addresses this problem for uncertainty in object location, given a grasp that has been designed for the object that is being grasped. This paper addresses the complementary problem of dealing rigorously with variations in object geometry.

1.3 Overview

This paper presents a technique for optimal contact point selection that brings together ideas from both bodies of work described above. The main contribution of the system described in this paper is a novel technique for generalizing any given grasp prototype for any given task and then using the resulting generalized prototype to synthesize grasps of new target objects.

Prototype grasps are used because they can drastically reduce the complexity of the search for a good grasp, but they can be very inflexible with respect to geometric variations in the object to be grasped. The grasp generalization technique described in this paper addresses the problem of inflexibility by showing how prototype grasps can be generalized, while ensuring that the resulting grasps are appropriate for the intended task. Wrench space descriptions of the grasp and task are used to perform this generalization step and to apply the resulting set of grasps to a new target object.

Section 2 of this paper continues with a statement of the problem addressed here, and Section 3 gives a description of notation. Section 4 describes the grasp quality measure used and introduces a simple example task. Section 5 describes how a grasp prototype can be generalized so that it applies to a wide range of target objects, and Section 6 shows how optimal grasps can be synthesized using this generalized grasp prototype. Section 7 discusses the problem of synthesizing optimal grasps of three-dimensional objects and briefly describes how robot hand kinematics and geometry can be integrated into the search for an optimal grasp. Sections 8 and 9 present a discussion and conclusions.

2 Problem Statement

This section outlines the specifications of the problem addressed in this paper. A few comments on these specifications can help to clarify them. First, this paper describes a grasp as a set of contact points on a target object, and no mention is made of how the robot hand will achieve this set of contacts. Incorporation of robot hand kinematic constraints into the search for a good grasp is deferred to Section 7. Second, this paper assumes grasps are composed of frictionless point contacts. This is in fact the most difficult case for grasps that are designed to counteract specified task forces. If a good grasp can be found without relying on friction, then the existence of friction will only improve the effectiveness of the grasp. Third, these specifications refer to the notion of grasp quality. The quality measure in use is defined in Section 4.

Definitions:

- **A grasp:** a set of contacts on a target object. Figure 1 is an example of a grasp of a two-dimensional object.
- **A task:** a set of wrenches (force/torque vectors) that must be applied to the target object to achieve the desired goal. For example, the task of lifting an object requires applying a resultant force to the object that counteracts the force of gravity.
- **A grasp prototype:** an example object and a high-quality grasp of that object. For example, a grasp formed by frictionless contact at the points illustrated in Figure 1 would be capable of counteracting task wrenches in any direction. This grasp can be used as a prototype for the task of resisting arbitrary disturbance forces.

Inputs:

- A geometric model of a target object
- A grasp prototype
- A task

Assumptions:

- Non-singular, frictionless point contacts. Frictionless point contacts are assumed, and the algorithm will not attempt to make use of contacts that require exact placement of the robot with respect to the target object in order to bring two features into contact.

For two-dimensional objects, allowable contacts are point contact on a straight or curved edge of the target object and line contact at a vertex of the target object. For three-dimensional objects, allowable contacts are point contact on a face or curved surface of the target object, line contact on an edge of the target object, and face contact on a vertex of the target object.

Problem:

- Find the grasp of the target object that is most suitable for achieving the specified task.

3 Notation

A number of terms are used in this paper:

$$
\begin{aligned}
\underline{w}_i &= \text{wrench due to applied force at contact } i = \begin{bmatrix} \underline{f}_i \\ \underline{\tau}_i \end{bmatrix} \\
\underline{f}_i &= \text{applied force at contact } i \\
\underline{\hat{n}}_i &= \text{unit normal at contact } i \text{, pointing outward from object surface} \\
\underline{\tau}_i &= \text{scaled torque due to applied force at contact } i, \quad = \lambda\,(\underline{f}_i \times \underline{d}_i) \\
\underline{d}_i &= \text{vector from contact point to object center of mass} \\
\lambda &= \text{multiplier to convert torque values to units of force} \\
c &= \text{number of contacts}
\end{aligned}
$$

Note that torque multiplier λ is used to relate torque magnitudes to force magnitudes. In this paper, torque multiplier λ is defined for a given object as the reciprocal of the maximum magnitude torque that could possibly be applied to that target object using unit applied force, and it has units of $\frac{1}{length}$. This means that the capabilities of a particular grasp will be independent of target object scale.

4 Measuring Grasp Quality

Before the grasp synthesis algorithm that is the main topic of this paper can be introduced, a grasp evaluation function, or grasp quality measure, must be established. A grasp quality measure is an estimate of the suitability of a grasp for the task to be performed. In order to make this evaluation, a method must be defined for describing grasps and tasks, and for measuring how well a given grasp fits a specified task. This section draws heavily from similar work in [16], [9], and [18]. In particular, Kirkpatrick, Mishra, and Yap [16] propose to specify a task as a wrench space ball centered at the wrench space origin, and to minimize the sum magnitude contact forces required to counter the worst case wrench within this task space. Ferrari and Canny [9] develop this measure and propose a second alternative that minimizes the maximum force required to achieve the same task. Li and Sastry [18] propose the use of task wrench space ellipsoids, and they develop a grasp quality measure that minimizes the sum magnitude applied forces required to counter the worst case task wrench within a given task wrench space ellipsoid. This paper uses similar measures, but extends the definition of a grasp quality measure to apply to any task that can be described as a subset of wrench space.

4.1 The Grasp Wrench Space Region (GS)

In this paper, the common domain within which grasps and tasks are described is wrench space. The grasp wrench space region (GS) is the set of wrenches that can be applied to the target object from the contacts of a grasp, given certain limitations on applied forces. The contact wrenches that are possible can be determined as follows. Legal values for applied forces $\underline{f}_i$ can be computed from the contacts of a grasp and the assumption of frictionless point contacts. Torques $\underline{\tau}_i$ can be computed from contact points and applied forces. Contact wrenches $\underline{w}_i$ can be formed from these forces and torques.

In this paper, GS is defined by limiting the sum of magnitudes of contact forces to 1, resulting in the following expression:

$$GS = \{\underline{w} \mid \underline{w} = \sum_{i=1}^{c} \alpha_i \underline{w}_i, \;\; \alpha_i \geq 0, \;\; \sum_{i=1}^{c} \alpha_i \leq 1, \;\; |\underline{f}_i| = 1\}. \tag{1}$$

where α_i is a scalar coefficient representing the range of magnitudes that may be assumed for contact force $\underline{f}_i$ at contact i for grasps with resultant wrenches in GS.

In other words:

- GS is bounded by the convex hull of the contact wrenches formed from unit applied forces at the contacts of the grasp.

The definition of grasp quality will involve scaling GS. GS can be scaled by κ as follows:

$$\kappa\, GS = \{\underline{w} \ \mid \underline{w} = \sum_{i=1}^{c} \alpha_i \underline{w}_i, \ \ \alpha_i \geq 0, \ \ \sum_{i=1}^{c} \alpha_i \leq \kappa, \ \ |\underline{f}_i| = 1\}. \tag{2}$$

It is interesting to pursue the question of how this definition for GS relates to the physical constraints of the robot hand. In particular, the limits placed on GS in Equations 1 and 2 can be related to torque limits on the motors that drive the hand. This works as follows. Given the geometry of the hand, the maximum torque required to counteract a unit applied force can be estimated. From this worst case torque and from the motor limitations, an upper bound can be obtained on the magnitude of any single force that the hand can be guaranteed to resist. If there are multiple contacts on the same finger of the hand, then their effect on the joint torques required at that finger is additive. This results in an upper bound on the sum magnitude forces applied to that finger. The effects of forces on independent fingers should be independent, so limiting the sum magnitude of all applied forces results in a conservative estimate of the capabilities of the hand.

4.2 The Task Wrench Space Region (TS)

A task can be described in terms of the wrenches that must be applied to the object by the robot in order to complete the task. This set of wrenches is called the task wrench space region (TS). A very simple task is TS_{ball}, defined as a unit wrench space ball, centered about the wrench space origin. In other words:

$$TS_{ball} = \{\underline{w} \ \mid \ |\underline{w}| \leq 1\} \tag{3}$$

Optimizing a grasp for task TS_{ball} ensures that the robot will be able to counter at least some magnitude wrench applied in any direction. Although TS_{ball} is used for illustration throughout this paper, the algorithms described can be applied to any task definition. This is illustrated in Sections 5.2 and 5.4.

4.3 A Grasp Quality Measure (Q)

Intuitively, the grasp quality measure is the amount the robot has to squeeze the object in order to be capable of resisting all task forces while maintaining the grasp. The grasp quality measure used in this paper (Q) can be defined as follows:

$$Q = \max\left(\frac{1}{\kappa}\right) \mid TS \in \kappa GS \tag{4}$$

In other words:

- Grasp quality is the reciprocal of the amount by which GS must be scaled so that it just contains TS.

A grasp quality measure will exist whenever GS spans the full range of directions spanned by TS. A special case exists when GS completely contains the wrench space origin. When GS contains the origin, then if the robot hand squeezes hard enough, a grasp can be formed that can successfully counter all wrenches within *any* definition of TS. The applied force magnitudes may be unnecessarily large, however, if the shape of GS bears little resemblance to that of TS, and this is reflected in a small grasp quality measure. Grasps can be compared based on the grasp quality measure for a given task. The relative effectiveness of a grasp for executing different tasks can also be evaluated by comparing the grasp quality measures of the grasp for the different tasks.

The grasp quality measure for the simple example task TS_{ball} can be computed as follows:

1. Compute the convex hull of the grasp:

$$H = CH\{\underline{w}_i : i = 1, \ldots, n\}. \tag{5}$$

2. Define the following vectors associated with each facet j of H:

$$\begin{aligned} \underline{\hat{n}}_j &= \text{unit outward pointing normal of facet } j, \\ \underline{p}_j &= \text{vector from wrench space origin to an arbitrary point on facet } j. \end{aligned}$$

3. Then quality measure Q_{ball}, which is the minimum perpendicular distance from a facet to the wrench space origin (for this task), can be computed as follows:

$$Q_{ball} = \min_{j} \left(\underline{p}_j \cdot \underline{\hat{n}}_j \right). \tag{6}$$

Equation 6 can be used as the objective function in a search for an optimal grasp of a target object. This is a relatively complex objective function, however, because it involves constructing a convex hull in wrench space (which is six-dimensional for three-dimensional objects). The objective function becomes even more complex when TS is not as simple as TS_{ball}. This paper shows how offline generalization of a grasp prototype for a given task can minimize the amount of run-time computation required to generate a good lower bound for quality measure Q and any given TS.

5 Generalizing a Grasp Prototype (Offline)

One problem with applying a single prototype grasp to a set of objects with varying geometries is that the prototype may not apply equally well to all objects within this set. This section describes how a grasp prototype can be generalized to represent a set of grasps that are appropriate for a specified task. This more general representation of the grasp prototype will allow the prototype to be customized to form grasps of objects with different object geometries, and it does not require prior knowledge of what these geometries may be. The generalization step can be performed offline and the results stored in a grasp library for access at run-time. Section 6 describes how the resulting generalized grasp is applied to a new target object.

The generalized grasp prototype $GGP(TS, Q)$ is defined for a given task TS and for a given grasp quality threshold Q. It is a set of grasps designed to meet the following conditions:

1. All grasps within $GGP(TS, Q)$ have grasp quality measure greater than or equal to Q for task TS.

2. If q is less than or equal to the quality measure of the grasp prototype for task TS, then $GGP(TS, q)$ contains the grasp prototype.

3. $GGP(TS, Q)$ is described in terms of independent constraint sets for each contact c of a grasp. A proposed grasp of a new target object will fall within the set of grasps $GGP(TS, Q)$ iff each contact of the proposed grasp meets the constraints given within $GGP(TS, Q)$ for that contact. This means that the contacts of a grasp can be selected independently.

The first condition states that $GGP(TS, Q)$ is defined based on a lower bound grasp quality measure. The second condition states that the generalized grasp prototype is defined based on the given grasp prototype. The third condition states that constraints on each contact of the grasp are independent. This condition is designed to allow the search for a grasp within $GGP(TS, Q)$ to be easily parallelized.

5.1 Generalizing the Grasp Prototype for An Example Task

An example grasp prototype is shown in Figure 1. Unit contact wrenches can be computed from the object geometry, the contact points shown, and the assumption of frictionless point contact. The grasp wrench space region for this grasp GS_{proto} is bounded by the convex hull of the unit contact wrenches. Figure 2 shows four views of GS_{proto}. Although a coordinate frame attached to the top of each view is provided for reference, the actual wrench space origin is at the center of each view. The cylindrical wire frame boundary represents the constraints on GS for any grasp: the plane perpendicular to the cylinder major axis represents forces, which are constrained by the definition of GS to have magnitude less than or equal to one in any direction; the cylinder major axis represents torques, which are constrained by the definition of GS to have magnitude less than or equal to one.

The quality measure of the prototype grasp Q_{proto} can be computed from GS_{proto} and a task. For the simple task TS_{ball}, Q_{proto} is 0.35. The convex hull shown in Figure 2 must be scaled by $\frac{1}{0.35}$ (or 2.86) before it will contain the unit wrench space ball.

The generalized grasp prototype $GGP(TS_{ball}, Q)$ is defined based on GS_{proto} as follows:

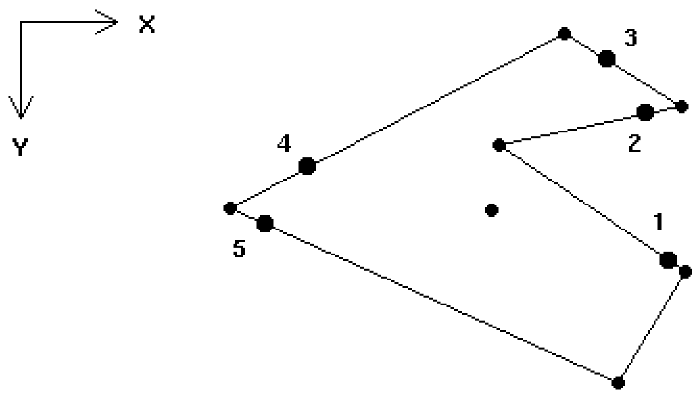

Figure 1: An example grasp prototype, represented by forces applied at the numbered contacts. In the coordinate system shown, with origin at the object center of mass, force / torque vectors for the five contact points are (f_x, f_y, τ): (-.56, .83, .71), (-.21, -.98, -.70), (-.53, .85, .08), (.46, .89, -.59), (.41, -.91, .81).

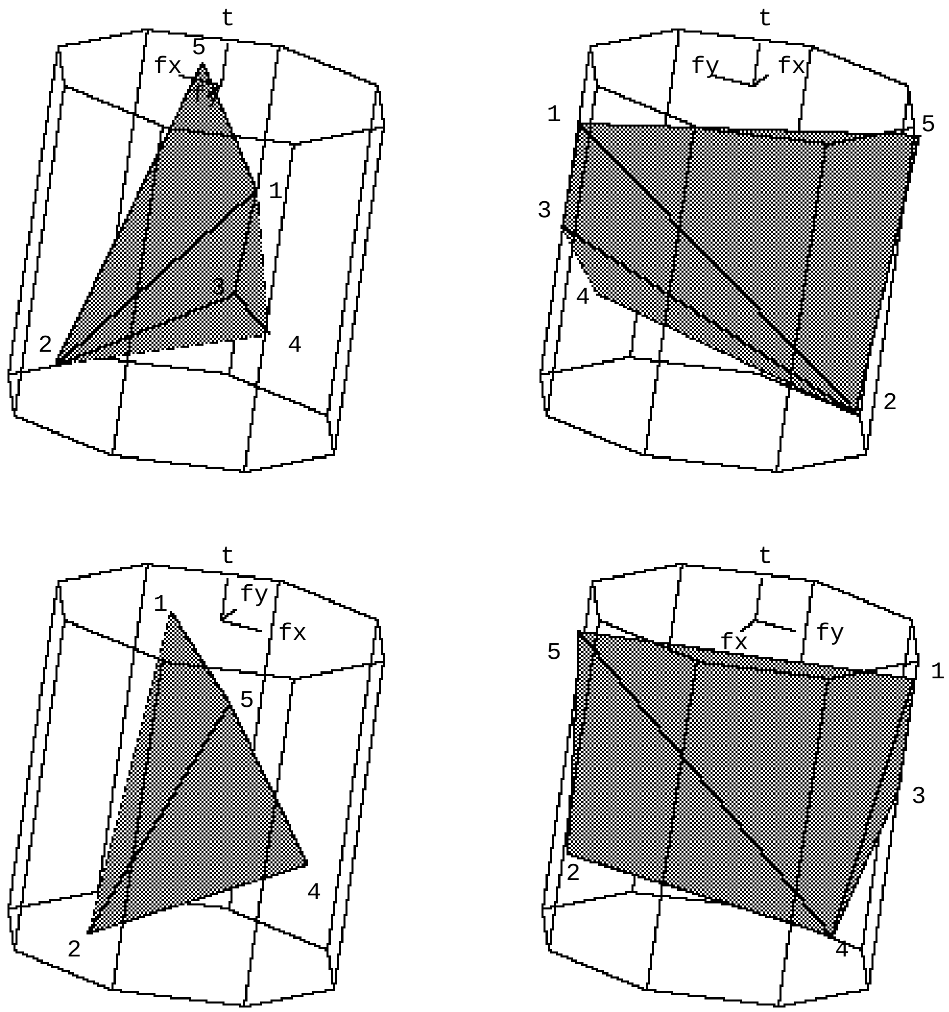

Figure 2: The grasp wrench space of the example grasp. The coordinate frame at the top of each figure illustrates the directions of the force and torque vectors, but note that the actual coordinate frame origin is in the center of the wire frame cylinder (inside the shaded polyhedron).

$$\begin{aligned} \underline{w}_i &= \text{contact wrench } i \text{ of the grasp prototype,} \\ g_j &= \text{facet } j \text{ of the convex hull bounding } GS_{proto}, \\ \underline{\hat{n}}_j &= \text{unit outward pointing normal of facet } g_j. \end{aligned}$$

Let G_i represent the set of facets containing $\underline{w}_i$ as follows:

$$G_i \quad = \quad \{j : g_j \text{ contains } \underline{w}_i\}.$$

A grasp falls within $GGP(TS_{ball}, Q)$ iff each wrench of the new grasp $\underline{w}_i^{'}$ is placed so that it meets the following conditions:

$$\forall j \in G_i, \quad \left[(\underline{w}_i^{'} \cdot \underline{\hat{n}}_j) \geq Q \right]. \tag{7}$$

Regions $\underline{w}_i^{'}$ for a hypothetical two-dimensional wrench space are shown in Figure 3.

We can test whether this definition satisfies the conditions posed for $GGP(TS, Q)$ at the beginning of this section:

1. *All grasps within $GGP(TS, Q)$ have grasp quality measure greater than or equal to Q for task TS.* Here TS is TS_{ball}, and Figure 3 shows that this condition holds. A proof is sketched below.

2. *If q is less than or equal to the grasp quality measure of the grasp prototype for task TS, then $GGP(TS, q)$ contains the grasp prototype.* By definition of the grasp quality measure for TS_{ball} (see Equation 6):

$$\forall j \in G_i, \quad \left[(\underline{w}_i \cdot \underline{\hat{n}}_j) \geq Q_{ball} \right]. \tag{8}$$

   This means that the grasp prototype falls within $GGP(TS_{ball}, Q)$ for any $Q \leq Q_{ball}$.

3. *$GGP(TS, Q)$ is described in terms of independent constraint sets for each contact c of a grasp.* Equation 7 deals with each contact i of a grasp independently. Contacts of a grasp can be chosen in parallel.

To visualize what is happening in Equation 7, imagine dropping the lines containing the facets of the convex hull shown in Figure 3 perpendicular to their normals until they are

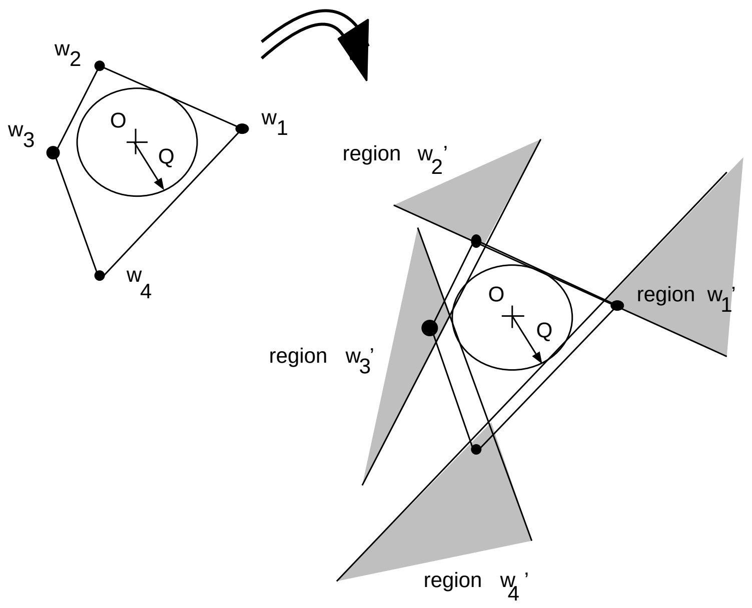

Figure 3: (L) Contact wrenches w_i of a grasp prototype with wrench space origin O and convex hull bounding GS for this grasp prototype. (R) Wrench space regions w_i' that make up $GGP(TS_{ball}, Q)$. A grasp having a grasp quality measure greater than or equal to Q can be formed by selecting one contact wrench from each of the four regions. The convex hull formed from these contact wrenches is guaranteed to contain the wrench space ball of magnitude Q.

tangent to the wrench space ball of radius Q. Form the convex hull defined by these planes. Call the convex hull $H_{tangent}$, the region bounded by this convex hull $GS_{tangent}$, and the facets of the convex hull $g_j^{'}$. Because every facet $g_j^{'}$ is distance Q from the wrench space origin,

$$(Q)\,TS_{ball} \in GS_{tangent}. \tag{9}$$

Equation 7 can be visualized based on $GS_{tangent}$. The geometric interpretation of this equation is that the set of wrenches $\underline{w}_i^{'}$ is the intersection of the outer halfspace regions defined by the facets $g_j^{'}$ indexed by G_i (the sets $\underline{w}_i^{'}$ are the regions shown in Figure 3).

A new grasp with grasp wrench space region GS_{new} can be formed from any set of contact wrenches $\underline{w}_i^{'}$ that satisfy Equation 7. Let H_{new} be the convex hull bounding GS_{new}. We can show that the new grasp has a grasp quality measure of at least Q by showing that GS_{new} contains $GS_{tangent}$, which contains $(Q)\,TS_{ball}$. This sketch of a proof can easily be adapted for the arbitrary task wrench spaces ($TS \neq TS_{ball}$) that will be described in the next section.

1. *No facet of H_{new} intersects the interior of $H_{tangent}$.* Suppose that there is some such facet. Construct the hyperplane containing this facet and call it P. Hyperplane P divides space into inner and outer halfspaces, and by definition of a convex hull, all points of H_{new} will either lie within hyperplane P or in the inner halfspace of P. Since P intersects the interior of $H_{tangent}$, by definition of a convex hull, there must be at least one vertex of $H_{tangent}$ in the outer halfspace of P. By definition of the contact constraint sets in Equation 7, a vertex of $H_{tangent}$ that is perpendicularly furthest to the outside of P will have its entire contact wrench region contained in the outer halfspace of P. By construction of the new grasp, there is a contact wrench within this region. This contact wrench is in the outer halfspace of P. □

2. *H_{new} contains the interior of $H_{tangent}$.* Choose any point in the interior of $H_{tangent}$ and call it p. Suppose that H_{new} does not contain p. Then there exists some hyperplane P that divides space into inner and outer halfspaces and passes through p, such that all wrenches of H_{new} either lie within hyperplane P or in the inner halfspace of P. $H_{tangent}$ by definition *does* contain p. P must then intersect the interior of $H_{tangent}$. By

definition of a convex hull, the outer halfspace of P contains some vertex of $H_{tangent}$. By the argument in 1 above, there is at least one point of H_{new} within the outer halfspace of P. □

3. *H_{new} contains the interior of the task wrench space.* We know that $H_{tangent}$ contains the task wrench space. By 2 above, H_{new} also contains this task wrench space.

For two-dimensional objects, there is another way to describe and visualize the contact constraint sets of a generalized grasp prototype. The examples in Section 5.3 should make these concepts more clear, but first, Section 5.2 introduces the minor variation required to generalize a grasp prototype for a task that is not TS_{ball}.

5.2 Generalizing the Grasp Prototype for any Given Task

The previous section described a method for generalizing a grasp based on a ball-shaped task wrench space. The obvious shortcoming of that generalization is that it is not applicable to task wrench spaces that differ significantly from a ball. This excludes many interesting tasks that may require applying task wrenches predominantly in certain directions (e.g. using a hammer), as well as tasks that may not require coverage of the entire wrench space (e.g. drinking from a glass). This section describes an extension to the previous algorithm that allows generalization of grasp prototypes for a task wrench space of any shape.

The extension required to the algorithm of the previous section is very simple: every facet of the prototype convex hull is associated with a scale factor that is the factor by which GS must be scaled so that the entire unit task wrench space is just on the inside of the hyperplane containing that facet. A revised contact constraint set is then constructed as follows. Let the scale factor be γ_j for facet g_j. Replace Equation 7 with the following:

$$\forall j \in G_i, \quad \left[(\underline{w}_i' \cdot \underline{\hat{n}}_j) \geq \gamma_j \, Q \right]. \tag{10}$$

It is interesting to note that this technique can be applied to generalize a grasp prototype even when TS is not known. If we set $TS_{unknown}$ to be GS_{proto}, so that $Q_{proto} = 1$, the set of grasps $GGP(TS_{unknown}, 1)$ will represent grasps at least as good as the grasp prototype for *any* task.

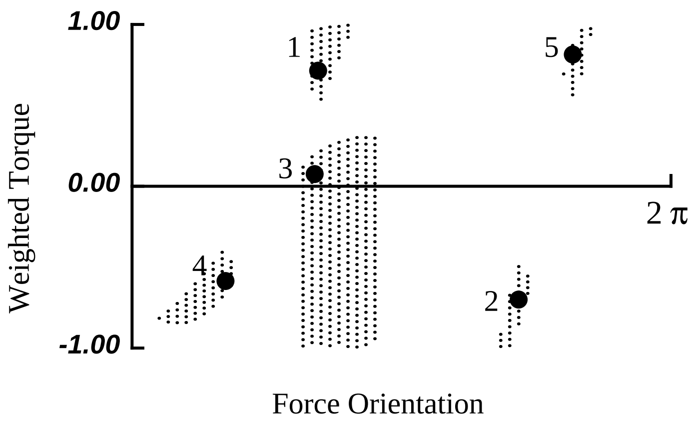

Figure 4: Independent wrench space regions for the five contacts of the example grasp, such that any grasp formed by selecting one contact wrench within each of the five regions shown is guaranteed to have a grasp quality measure of 75% of the original grasp quality measure ($Q \geq 0.26$). These regions of normalized contact wrenches are presented in a space of force orientation vs. weighted torque values. The normalized wrenches corresponding to the five contacts of the original grasp are shown.

5.3 Visualizing Contact Constraint Sets

The contact constraint sets for a given grasp prototype, task TS, and grasp quality threshold Q are the set of contact wrenches $\underline{w}_i^{'}$ that satisfy Equation 10. Because the forces of these contact wrenches are normalized, the contact wrenches can be plotted on a two-dimensional graph.

Figure 4 shows the contact constraint sets for the grasp prototype of Figure 1, task TS_{ball}, and $Q \geq 0.75Q_{proto}$. The x-axis represents orientation of the force vector in radians, and the y-axis represents the scaled torque vector. The shaded regions illustrate wrenches having contact quality measures of at least 75% of the grasp quality measure of the grasp prototype. The numbered points within these regions represent the contact wrenches of the grasp prototype. Each contact wrench falls within its corresponding contact constraint set.

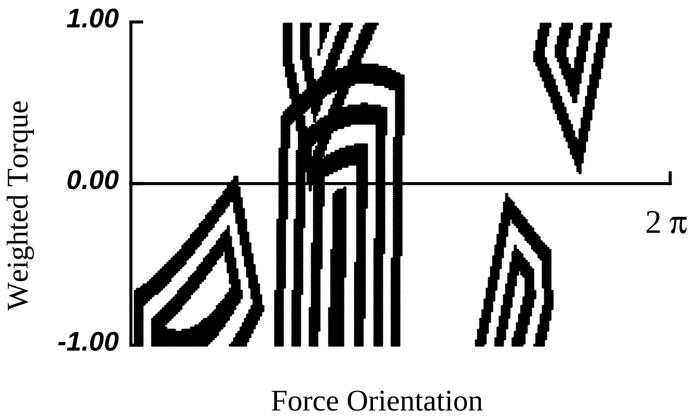

Figure 5: Gradient plot of independent wrench space regions for the example prototype grasp. The regions are striped at contact quality intervals of 0.1, with the outer, black stripe representing the contact quality interval [0.0, 0.1).

5.4 Optimizing Contact Wrenches Independently

Each contact wrench $\underline{w}_i^{'}$ within a contact constraint set i of Figure 4 will satisfy Equation 10 for a range of grasp quality thresholds. The *contact quality measure* for a contact wrench $\underline{w}_i^{'}$ is defined as the maximum grasp quality threshold for which Equation 10 is satisfied. Contact quality measures can be represented graphically by using gradient curves as in Figure 5.

To optimize the contact quality measure of each contact for task TS_{ball}, simply place each new contact wrench $\underline{w}_i^{'}$ such that

$$\min_{j \in G_i} (\underline{w}_i^{'} \cdot \hat{\underline{n}}_j) \tag{11}$$

is maximized.

A lower bound on the quality of the grasp can then be stated as:

$$Q_{min} = \min_i \left[\min_{j \in G_i} (\underline{w}_i^{'} \cdot \hat{\underline{n}}_j) \right]. \tag{12}$$

The grasp falls within the set of grasps described by generalized grasp prototype $GGP(TS_{ball}, Q_{min})$.

To maximize the contact quality measure for a more complex task wrench space, a weighting factor γ_j is needed for each facet g_j of the grasp wrench space convex hull. Each new contact wrench $\underline{w}_i'$ should be placed to maximize

$$\min_{j \in G_i} \left[\frac{(\underline{w}_i' \cdot \hat{\underline{n}}_j)}{\gamma_j} \right], \tag{13}$$

and a lower bound on grasp quality for this task shape is

$$Q_{min} = \min_i \left(\min_{j \in G_i} \left[\frac{(\underline{w}_i' \cdot \hat{\underline{n}}_j)}{\gamma_j} \right] \right). \tag{14}$$

The technique just described optimizes the quality of a grasp by independently optimizing the quality of each contact wrench. This is a very powerful approach, and lends itself to the implementation of parallel algorithms for grasp synthesis.

6 Synthesizing Optimal Grasps (Run-time)

The previous section showed how a grasp prototype could be generalized to create sets of grasps that are guaranteed to have grasp quality measures greater than a given grasp quality threshold. It also showed how the contacts of a grasp could be independently optimized, and how a lower bound grasp quality measure could be obtained from any selected set of contact wrenches. This section pulls these ideas together to show how a generalized grasp prototype, $GGP(TS, Q)$, computed offline, can be used to generate high quality grasps of new target objects at run-time. The section begins by introducing the object boundary curve, which can be used to visualize the space of possible grasps. It then introduces two techniques for finding high quality grasps within this space.

6.1 The Object Boundary Curve

The set of normalized wrenches that represent frictionless point contact with a given object can be plotted in the same space as the contact constraint sets of Figures 4 and 5. Figure 6 shows the *object boundary curve* for the prototype object of Figure 1 overlaid on the plot of 75% contact constraint sets. This object boundary curve indicates the contact wrenches that can be achieved through frictionless point contact with the prototype object. Contact on an edge of this object results in a force normal to that edge and a torque dependent on

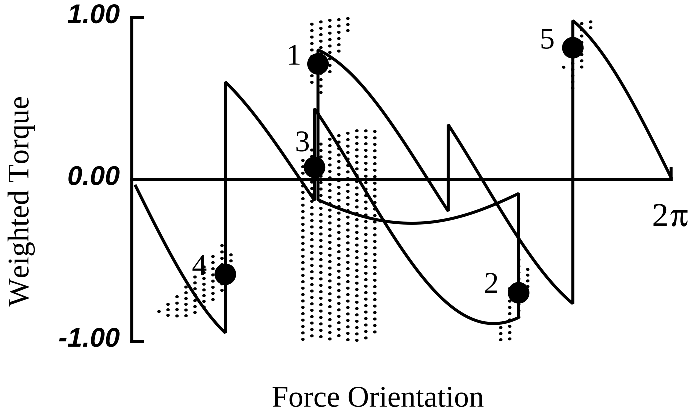

Figure 6: Independent wrench space regions superimposed on the object boundary curve of the example polygon. The object boundary curve indicates what contact wrenches are possible for this object. The position of the object boundary curve along the force orientation axis of this plot depends on the relative orientation of the object with respect to the fixed coordinate frame of the grasp prototype.

the direction of this normal as well as on the position of the contact relative to the object center of mass. The vertical lines in the plot represent contact with edges of the target object. Contact on a vertex of the object results in a force dependent on the angle of contact and torque dependent on the force direction and the position of the contact relative to the object center of mass. Curved lines represent contact with vertices of the target object. The contacts of the prototype grasp are on the edges of the object, and so they fall on the vertical lines of the object boundary curve. Note that the object boundary curve of a smoothly curved object would look somewhat different. In particular, it would have no vertical lines.

The combined plot of object boundary curve and contact constraint sets tells us a great deal about the robustness of a grasp to errors in contact placement. Consider Figure 7, for example. From the plot in Figure 6 we see that as long as point contacts are placed within the highlighted regions of Figure 7, the resulting grasp will have a grasp quality measure of at least 75% of the example grasp. Note that the contact region corresponding to contact 3 is

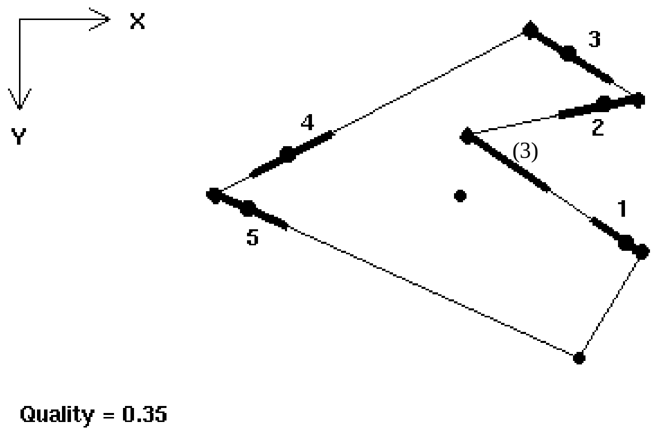

Figure 7: Independent contact regions that guarantee a grasp quality measure of 75% of the grasp quality measure of the original grasp. Only point-edge contacts are shown. The region corresponding to contact three overflows onto the edge containing contact one.

discontinuous; this region overflows onto the edge containing contact 1. This can be seen in Figure 6 as well: there are two nearly overlapping vertical lines intersecting region 3, which corresponds to contact regions on two separate edges of the target object. Also note that contact regions may involve not only a point of the robot in contact with an edge of the object, but also an edge of the robot in contact with a vertex of the object. For example, Figure 6 shows that the contact constraint set of contact 4 can be satisfied by one of two discontinuous contact regions: the region on the edge of the target object shown in Figure 7, or a separate region of edge contact on the nearby target object vertex.

The effects arising from a variety of changes in the grasp can be read from the plot in Figure 6. For example, uncertainty in contact placement displaces contact points on the object boundary curve. In addition, uncertainty in object orientation causes the curve to slide on the orientation axis, and changes in object geometry cause the object boundary curve to warp. As edge lengths shrink, the vertical lines shrink. As edge orientations change, these lines move horizontally. We can get an idea of how robust a grasp might be by noting how easily such changes can cause the contact points to slip out of the good contact regions. This figure provides a way of visualizing the problems of designing an appropriate grasp for a given object or designing an object that can be easily grasped.

Figure 6 also provides some insight as to why a prototype grasp can be mapped to a grasp of a new object. A successful grasp of this new object will be found when the object boundary curve of the new object contains features that overlap the regions shown in the figure. Before the object boundary curve of the new target object can be plotted on the same graph as Figure 6, however, the orientation of the target object with respect to the grasp prototype must be specified. As this orientation is changed, the object boundary curve of the target object will move along the force orientation axis. An orientation can be selected to optimize grasp quality or robustness (see Section 6.2).

If the regions that represent the contact constraint sets of the generalized grasp prototype are small, it may be very difficult to find an orientation of an arbitrary object where the object boundary curve overlaps all of the regions. As the quality requirements are lowered, however, these regions grow in size. If we require only that the guaranteed grasp quality measure be greater than zero, in fact, these regions fill much of the two-dimensional space (see Figure 5). This implies that as objects become less like the prototype, we can still expect to find a grasp, although we may have to reduce the quality requirements for that grasp. The actual geometry of the target object becomes more important, however, when we move to higher dimensional wrench spaces. Intuitively, there is increasing sensitivity to object geometry in higher dimensional wrench spaces simply because the wrench space is much larger, and so the contact constraint sets may fill a much smaller portion of this wrench space. This topic will be discussed in more detail in Section 8.

6.2 Optimizing Grasp Robustness and Grasp Quality

The object boundary curve provides one way of projecting a generalized grasp prototype onto a target object. The object boundary curve can be intersected with contact constraint sets as in Figure 6 to obtain regions of good contact on the target object, as shown in Figure 7. As long as a grasp contains one contact within each of the regions shown in Figure 7, the grasp is guaranteed to have quality greater than or equal to 75% of the quality of the grasp prototype.

Because the results of intersecting the object boundary curve with the contact constraint sets of Figure 6 are dependent on the orientation of the target object, a method for finding an optimal grasp must consider the space of possible orientations. If contact region size is optimized over this space of orientations, the following objective function can be used:

$$R_{max} = \max_{\theta}\left(\min_{i} R(i,\theta)\right). \tag{15}$$

The robustness measure for a given contact is written $R(i,\theta)$, a function of the contact (i) and the relative orientation of the grasp prototype and target object (θ). It represents the size of the valid region of contact for contact i and a given grasp quality threshold, and is measured by taking the length of the largest continuous portion of the object boundary curve passing through the independent wrench space region corresponding to contact i (see Figure 6). This implies a conversion factor between distance along an edge of an object and change in contact angle at a vertex of that object.

To find an optimal grasp using this measure requires minimization over the contacts of a grasp (i) and maximization over the relative orientation between the grasp prototype and the target object (θ). The best grasp will be found at the value of θ producing R_{max}. This may be different for different grasp quality thresholds.

Contact quality spaces such as that shown in Figure 5 can also be used to find optimal quality grasps. Contact quality can be expressed as $CQ(i,\theta,s)$, a function of contact (i), relative orientation of grasp prototype and target object (θ), and position on the object boundary curve (s). Then the optimal guaranteed grasp quality measure is:

$$Q_{max} = \max_{\theta}\left(\min_{i}\left(\max_{s} CQ(i,\theta,s)\right)\right). \tag{16}$$

The best grasp will be found at the value of θ producing Q_{max}, and the contacts will be placed at the corresponding optimal values of s. The actual quality of the resulting grasp will in general be greater than Q_{max}, but it is guaranteed never to be less than Q_{max}.

A brute force process for finding an optimal grasp using either Equation 15 or Equation 16 can be summarized as follows:

1. Given a grasp prototype, extract the contact wrenches $\underline{w}_i$ of that grasp prototype. *(Offline)*

2. Given a task TS, compute the generalized grasp prototype $GGP(TS, Q)$ for that task as a function of grasp quality threshold Q. This corresponds to finding values for G_i, $\underline{\hat{n}}_j$, and γ_j in Equation 13 so that the contact quality of any given wrench $\underline{w}_i^{'}$ can be tested. Note that this requires only a minimization over a few dot products. *(Offline)*

3. Given a new target object, find an optimal set of contacts on that target object as follows: *(Run-time)*

   (a) Sample all orientations of the target object. (In a less brute force approach, this would not be required.)

   (b) For each orientation, compute the objective function at that orientation as follows:

      i. Compute $R(i, \theta)$ (Equation 15) or $[\max_s CQ(i, \theta, s)]$ (Equation 16) for each contact i.

      ii. Minimize the results over the contacts i to obtain the value of the objective function at this orientation.

   (c) Find the maximum value of the objective function over all orientations.

   (d) Place each contact at the center of the largest region found for that orientation (Equation 15) or at the point of maximum contact quality for that orientation (Equation 16). This is the optimal set of contacts for the given object based on the generalized grasp prototype $GGP(TS, Q)$.

Figures 8 and 9 show the optimal grasps found for two test objects using Equation 15. $R(i, \theta)$ was estimated for these examples by using a parallel machine to sample the boundary curve and test each sampled point, but it can also be found numerically by identifying the points at which the object boundary curve moves into and out of the contact constraint regions whose boundaries are specified by Equation 10. The grasp quality threshold used in measuring the sizes of the valid contact regions was set at 25% of the quality of the grasp prototype, but note that the qualities of the grasps are much better than the guaranteed lower bound of 0.0875. They are very near the quality of the grasp prototype, which is 0.35.

Although in this experiment the orientation of the target object with respect to the grasp prototype was exhaustively sampled, it is not necessary to approach the problem in this way. For example, if we compare the object in Figure 8 to the prototype object in Figure 7, we may be able to guess at an appropriate orientation for the new object based on similarities in object features. This orientation could be used as a starting point in a local search for the orientation that produced the best grasp. This type of geometric fitting is more important for three-dimensional objects, which have a three-dimensional orientation space.

7 Synthesizing Grasps of 3D Objects

When we move from two-dimensional to three-dimensional objects, the techniques used to construct contact constraint sets and contact quality measures and to synthesize optimal grasps from these constructs do not change. The contact types and the required intermediate constructs do become more complex, however. Important changes in dimensionality are listed below:

- The wrench space is six-dimensional.
- There is a two-dimensional object boundary surface rather than a one-dimensional object boundary curve.
- The target object orientation can vary in a three-dimensional orientation space, rather than a one-dimensional orientation space.

The grasp synthesis techniques described in this paper were tested for a seven contact cylindrical grasp prototype and a model of the Salisbury three-fingered hand [27]. The grasp prototype has contacts on each of the two distal links of each finger and on the palm of the hand. It was designed for a task that is a four-dimensional task wrench space ball. Frictional properties of the grasp were introduced to accommodate the other two dimensions: forces along the cylinder major axis and torques about this major axis. This was done by projecting

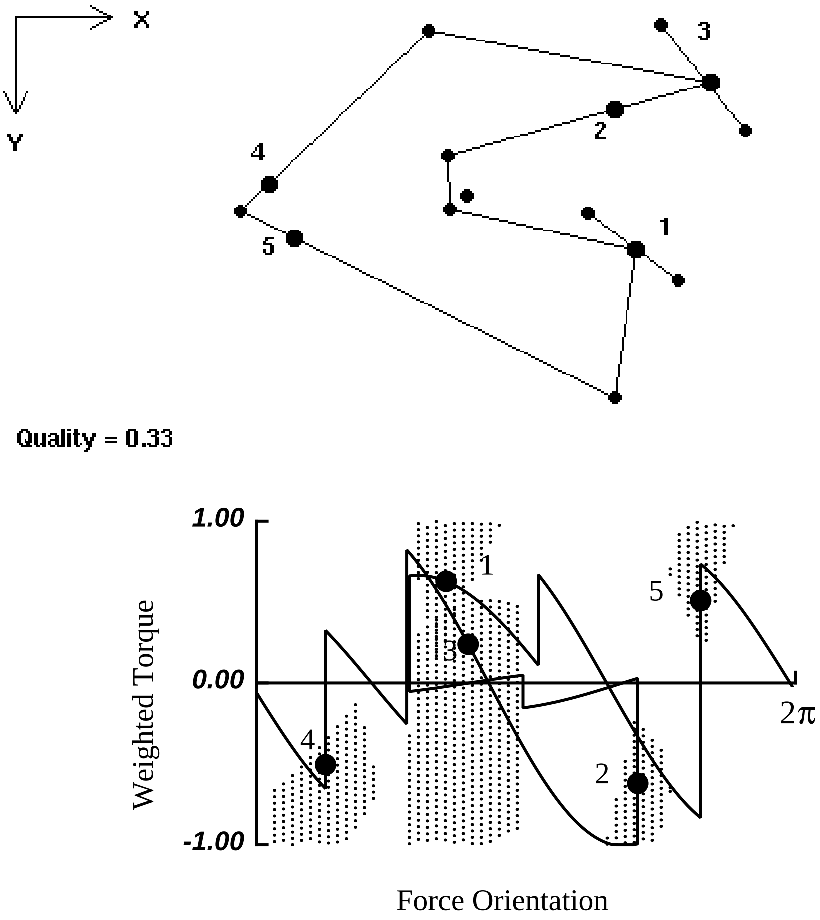

Figure 8: The best match of this new target object to the example grasp has a grasp quality measure of 0.33. In the plot, the object boundary curve of the new object has been superimposed on the 0.0875 quality regions. This object boundary curve has not been shifted with respect to the independent contact regions of the grasp prototype, indicating that the orientation shown is the optimal one. The contact wrenches of the grasp are indicated in the plot, and independent contact region sizes for the quality threshold of 0.0875 can be estimated by examining the plot.

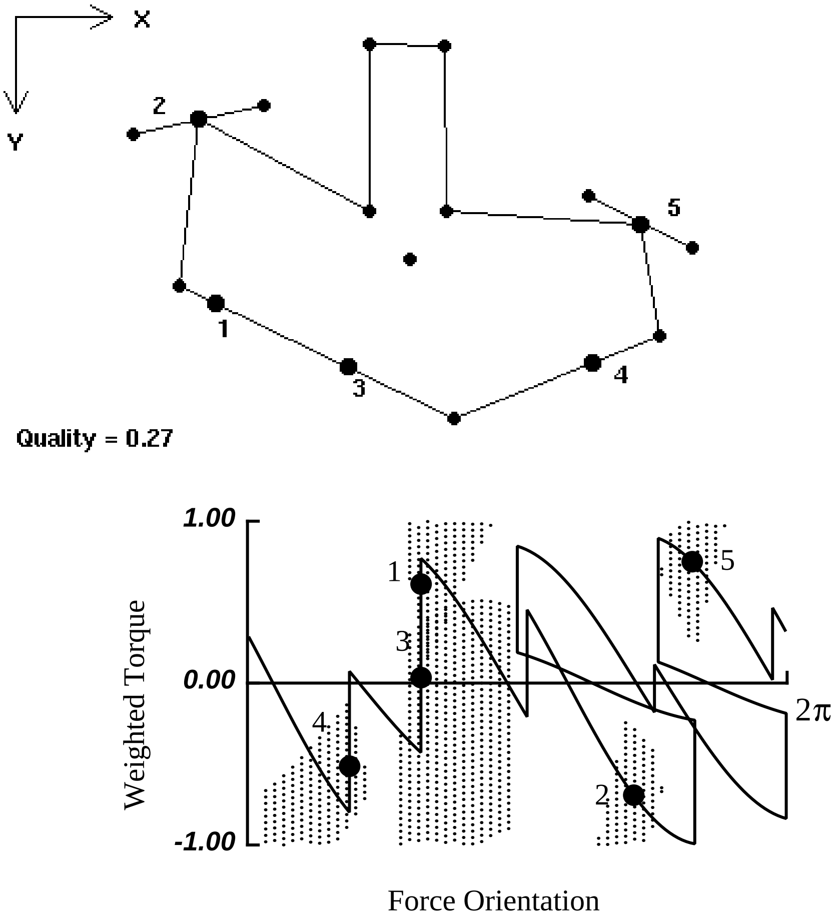

Figure 9: The best match of this new target object to the example grasp has a grasp quality measure of 0.27. In the plot, the object boundary curve of this new object has been superimposed on the 0.0875 quality regions. This object boundary curve has been shifted by π with respect to the independent contact regions of the grasp prototype, indicating that this object best matches the grasp prototype if it is rotated by 180 degrees (try it). The contact wrenches of the grasp are indicated in the plot, and independent contact region sizes for the quality threshold of 0.0875 can be estimated by examining the plot.

frictionless contact wrenches into the four-dimensional wrench space of interest and ruling out any contact wrenches where the projection process caused the contact wrench to fall outside the friction cone at the point of contact.

Optimal quality grasps were synthesized using this prototype for a variety of approximately cylindrical objects, including a toy airplane and a bubble gun. Optimal grasps for these two objects were tested with the Salisbury three-fingered robot hand, and the grasps were found to be easily achievable and very robust. Figures 10 and 11 show two of these grasps. Due to the approximate geometric models used for the Salisbury hand and for these two objects, actual points of contact with the objects varied by as much as $1cm$ to $2cm$ from the contact points chosen by the model. If the hand were equipped with sensors that allowed forces and torques to be measured, the grasps could be adjusted at run-time to more closely match the modeled grasp.

In the algorithm described in Section 6.2, the orientation space of the target object and the object boundary curve of that object were exhaustively sampled as part of the algorithm for finding an optimal grasp. This is not a problem for two-dimensional objects, but exhaustive sampling of the orientation space and object boundary surface of a three-dimensional object is a prohibitively expensive operation. Geometric information can be used to generate a good starting orientation for a search, however. One possible approach is to find an alignment between certain geometric features of the target object and prototype object. In the case of a cylindrical prototype object, for example, the main axis of a new target object can be found and aligned with the main axis of the cylinder. This leaves only a single degree of orientation freedom to be searched: orientation around the cylinder axis.

A second approach is to identify parts of the object that are important for the current task. This might involve finding the head of a hammer or the slot of a screwdriver, for example. Because these portions of the object are used in a very specific way for the task, they can drastically constrain the set of feasible orientations of the target object with respect to the task coordinate frame, which is also the coordinate frame of the grasp prototype.

Once a set of promising orientations are chosen, numerical routines can be used to optimize contact quality measures over the object surface. For example, in one experiment, optimal grasps of the toy plane in Figure 10 were found using the following procedure:

1. The seven contact wrenches were extracted from the cylindrical grasp prototype.

2. This grasp prototype was generalized for task TS_{ball}.

3. The contact quality measures were optimized for the seven contacts as follows:

   (a) The plane was decomposed into seven ellipsoids and one cylinder. (Ellipsoids were: front portion of the main airplane body, rear portion of the main airplane body, left wing, right wing, main tail section, left tail fin, right tail fin. The cylinder was used for the central portion of the main airplane body.)

   (b) The orientation space of the plane was sampled for orientations where the plane major axis was approximately (± 0.5 radians) aligned with the prototype cylinder major axis.

   (c) For each orientation, an optimal quality grasp was chosen as follows:

      i. Contact quality measures were optimized for each contact on the surface of each ellipsoid or cylinder using the Powell numerical optimization routine [26] and Equation 16 (adapted for three-dimensional objects).

      ii. Contacts inside other ellipsoids or cylinders (i.e. not on the airplane surface) were rejected and the best grasp was composed from the remaining contacts.

This process resulted in high quality sets of contact points found for the majority of the sampled orientations (127 of 200 orientations had quality greater than 0.2). The process required several seconds per orientation on a Pentium PC. Although the space over which the Powell algorithm was used did contain some local minima, these local minima were few, and simple techniques such as picking a small number of random starting points for each search served to virtually eliminate this problem.

It is not sufficient to find an optimal set of contact points using this type of algorithm, however, because the constraints of the robot hand kinematics and geometry must also be satisfied. In Pollard [24], this was done using a parallel algorithm that first found good configurations for links of the robot hand independently and then chained these links together in kinematically feasible ways to form high-quality collision-free grasps of the given target object. The ability to optimize contacts independently and the existence of regions of good contacts were crucial to the success of this algorithm.

8 Discussion

This paper has illustrated an algorithm that is useful for synthesis of whole-hand grasps of three-dimensional objects. There are a few issues that must be addressed, however, to make this a complete and practical system for real-time grasp synthesis:

- Computational issues: The complete grasp synthesis algorithm cannot currently be run in real time.
- Construction and use of a grasp library: The algorithm assumes the existence of a grasp library and selection of a suitable grasp prototype.
- Incorporation of kinematic constraints into the search for a good grasp

The first two of these issues are discussed below. The third is addressed in detail in Pollard [24].

8.1 Computational Issues

Section 7 illustrated that geometric fitting and numerical methods can be used to find a single set of good contact points very quickly (within a minute for the airplane example). The complete grasp synthesis algorithm, however, requires regions of good contact points to be found, so that hand kinematic constraints can be incorporated into the solution. The current complete implementation (see Pollard [24]) requires approximately one hour to compute a set of high quality, kinematically feasible grasps of a new target object, using a Thinking Machines CM2, a parallel, single instruction, multiple data (SIMD) computer. This is actually quite good, given the global nature of the solution, the size of the solution space (15-dimensional in this example), and the variety of constraints that must be taken into account (the grasp must

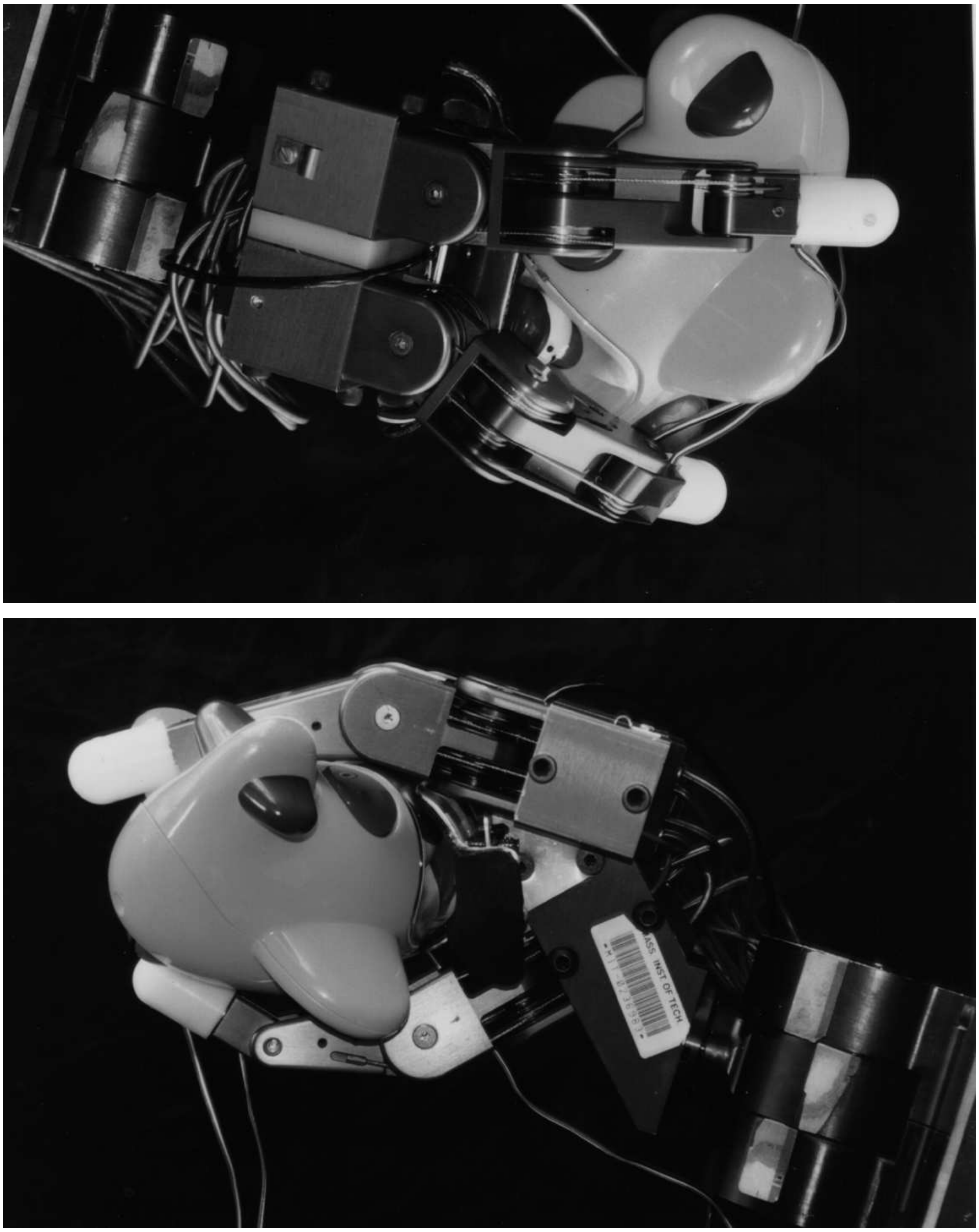

Figure 10: Two views of the Salisbury hand grasping the toy plane.

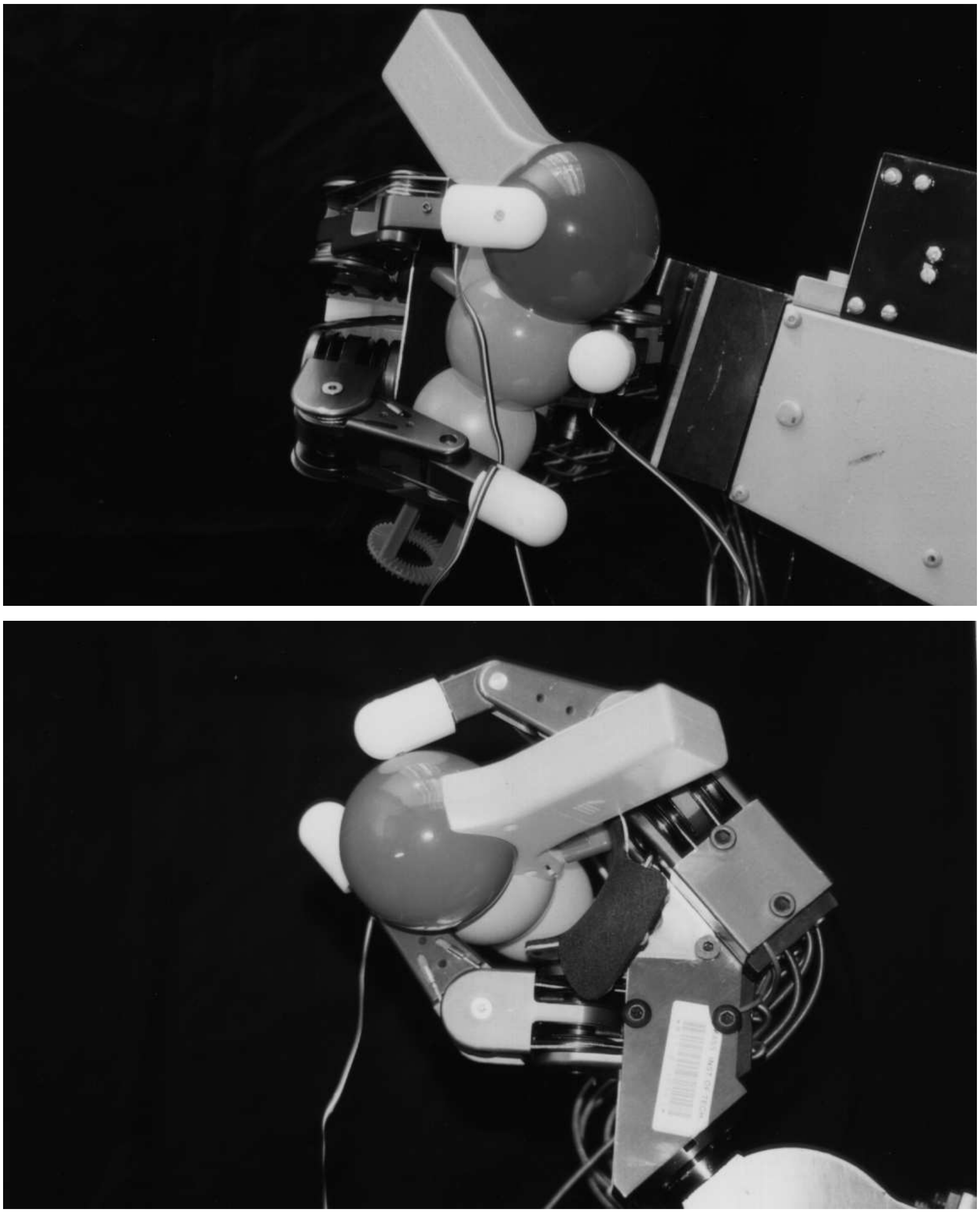

Figure 11: Two views of the Salisbury hand grasping the bubble gun.

be stable, kinematically feasible, and free of collisions). It is not, however, a real-time solution to the problem of grasp synthesis.

The current implementation is based around a large amount of global search, however. As discussed in Section 7, object geometry and task constraints may drastically reduce the search space. In addition, a global description of the space of high quality grasps may often not be the real objective of the grasp synthesis process. It may be sufficient to find a single high quality, kinematically feasible solution, and it may often be possible to find such a solution by optimizing locally around the grasp prototype. This type of approach would bring the algorithm much closer to a real-time solution. It is worth noting, however, that surprising adaptations of a prototype grasp to a new object will not be found using this type of local search. An example of this would be using a hole or a handle to generate grasp wrenches when the prototype object does not have such features. Searching locally around a given hand configuration is unlikely to result in serendipitous use of new object features, which may require a very different hand configuration to produce a successful grasp.

Local search can be used to produce a grasp interactively at run-time, with very little computational overhead, even if the exact geometry of the object to be grasped is not known. If the contact wrenches of a grasp can be measured, then the contact quality of those wrenches can be computed. This is a very fast process, involving only minimization of a few dot products at each contact. This information could be used to test or refine a grasp at run-time. This would allow local grasp synthesis techniques such as those found in Coelho and Grupen [5] and Grupen and Weiss [10] to be used with a more sophisticated grasp quality measure.

Some geometric information is needed, however, as the algorithm is sensitive to the position of the coordinate frame. Consider the object boundary curve in Figure 6 for the prototype object in Figure 1. If the origin of the coordinate frame of this object is translated in the positive X direction, the torques associated with contacts 1 and 2 move in toward the origin, and the torques associated with contacts 3, 4, and 5 move away from the origin. As the coordinate frame is translated further in the X direction, contacts 1 and 2 will cross the origin and move to the opposite torque extreme, resulting in a very polarized object boundary curve. In general, the object center of mass is a good point to use for the origin of the coordinate frame, because it results in a well balanced object boundary surface. If the object boundary surface has better coverage of the wrench space, it will be more likely to result in good matches to an arbitrary prototype grasp. Very coarse approximations to object geometry are sufficient to estimate the center of mass of an object, however, and so this should not be much of a limitation.

8.2 Construction and Use of a Grasp Library

This paper assumes the existence of a grasp library and selection of an appropriate grasp prototype. One set of questions that must be addressed to make this a complete algorithm for grasp synthesis involves the construction of prototypes to populate the grasp library, the selection of a prototype matching a given target object, and the identification of the range of grasps for which such a library will be useful. This is an area of future work, but some ideas are sketched below.

8.2.1 Populating the Grasp Library

Because of the way in which grasp prototypes are used in this paper, they need to be constructed around a particular task or set of tasks. We cannot assume that a prototype constructed for one task will be appropriate for another, very different task. Because tasks are described in wrench space, one possible approach for constructing an optimal grasp prototype for a given task might be to select an arbitrary number of contacts and optimize the wrenches associated with those contacts in order to maximize the quality of the grasp (using the quality measure of Equation 4). The best grasp of a task wrench space ball, for example, would have an even distribution of contact wrenches about the surface of the wrench space ball.

This will not in general be a good idea, however, because the approach does not take object geometry into account. Object geometry does matter, and even constructing a single grasp prototype for a single task will probably not work. Consider the very similar tasks of drinking from a wine glass, a water glass, and a coffee mug. Because of variations in object geometry and other characteristics, such as weight, the usual human grasps of these three objects are very different. This example implies that prototype grasps may need to be designed with families of objects in mind.

An additional question involves how to determine the appropriate number of contacts for a grasp prototype, since any grasp synthesized from that prototype will have the same number of contacts. To make the prototype more flexible in this respect, it may be useful to create a family of prototypes having different numbers of contacts. The consequences of applying each of the prototypes in this family to a new target object can be evaluated when synthesizing a grasp of that object.

8.2.2 Selecting a Prototype

Because the match between the target object and grasp prototype is performed in wrench space, the target object does not have to be similar in appearance to the grasp prototype. It may not even have the same topology. The important features that this object must have are characterized as features in wrench space.

These wrench space features can be easily mapped into geometric features, however. Consider a contact constraint set consisting of a single wrench in the wrench space of a two-dimensional object. For each contact type (point on edge, line on vertex), there is a one-dimensional family of contact configurations from which this contact constraint set could be achieved. Determining how this contact constraint set could be met for a given target object would simply require finding a feature and orientation of that object for which a contact configuration within one of these one-dimensional sets could be achieved.

A complete generalized prototype has more than one contact constraint set, and the full contact constraint set for each contact will in general result in a much broader set of allowable contact configurations, but it may still be possible to use a high level characterization of a prototype grasp in terms of necessary geometric features when choosing the prototype that best fits a given target object. The contact constraint sets of any given generalized grasp prototype would be converted into very simple sets of geometric features for which the target object would be tested. Ideally, this testing process would also serve to identify good orientations of target object with respect to the grasp prototype.

8.2.3 Scope of the Grasp Library - Friction

One of the assumptions made in specifying the problem to be addressed by the grasp synthesis algorithm in this paper was that only frictionless grasps would be synthesized. When the task is to achieve coverage of a certain portion of wrench space, this assumption is not prohibitive. A solution that works for frictionless grasps will work even better for grasps with friction.

Prototypes that use friction to counter important task wrenches will not be effectively represented in the methodology described in this paper, however, as it focuses on the portion of the task space achieved by frictionless contact with the prototype object. This does not entirely rule out grasps that rely somewhat on friction, however. The cylinder grasp prototype used in Section 7, for example, characterized the main task wrench space as a four-dimensional space including all wrenches except those that involved force along the main cylinder axis and torque about that axis. The algorithm for synthesizing new grasps using this prototype worked within this four-dimensional task wrench space, but ensured that the contact forces chosen for any grasp would fall within the friction cones at the points of contact with the target object. It was assumed that frictional properties of the grasp would be sufficient to accommodate any minor task wrenches along the two dimensions not covered by the task wrench space (e.g. lifting the cylinder, if it is standing on end). If frictional properties of the grasp would not be sufficient for such tasks, the prototype grasp should be redesigned, and the main task redefined to include more of the complete six-dimensional wrench space.

8.2.4 Scope of the Grasp Library - Number of Contacts

In general, the algorithm described in this paper is appropriate for grasps of any number of contacts. Even three-contact fingertip grasps can be found using this technique (if a three-contact grasp prototype is available). The contact constraint sets that result from generalizing a grasp with a small number of contacts will cover only a small portion of wrench space, however, and so it may be difficult to map such a grasp to a grasp of a new object.

In general, the success of the mapping from a prototype grasp to a grasp of a new object depends on the number of contacts in the prototype grasp, the variation in object geometry between the prototype object and the target object, and the wrench space coverage required by the task. As the number of contacts in the prototype grasp increases, the fraction of wrench space covered by the contact constraint sets of the generalized prototype increases. This is true not only because there is now a larger number of contact constraint sets, but also because each of these contact constraint sets will be larger. Intuitively, the size of each contact constraint set increases because adding additional contacts means that any given contact i has responsibility for coverage of a smaller portion of wrench space. This means that there is more flexibility in placing each contact wrench i, which makes it easier to map the generalized grasp to a grasp of a new object.

Objects that are less similar require contact constraint sets that cover a larger portion of wrench space, because geometric features are less likely to exist on the new object that allow the robot to apply contact wrenches that are very similar to the contact wrenches of the grasp prototype. Conversely, if the set of objects to be grasped using a given grasp prototype is very similar, then the prototype grasp does not need to have a large number of contacts.

Similarly, tasks requiring coverage of a larger portion of wrench space require more contacts, because there is more wrench space to cover. A larger number of contact constraint sets are required to obtain adequate coverage of this space.

If the problem to be solved is simply to find large regions of high quality contact points on an object, it is always better to begin with a grasp prototype having a large number of contacts. To a certain extent, this is true for finding kinematically feasible solutions as well. As the number of contacts in a prototype grasp increases, it will be easier to find large regions of high quality contact points on a target object, and the existence of these large regions of high quality contacts provide the flexibility to meet hand kinematic constraints.

When the number of contacts becomes very large, however, it will become more difficult to find a configuration of the hand where all of the necessary contacts can be achieved. The number of contacts in a grasp prototype must be large enough to cover the task wrench space and provide flexibility in mapping the prototype grasp to grasps of new objects, but it must not be so large as to overconstrain the problem of finding feasible kinematic configurations of the hand that can achieve all of the required contacts.

The algorithm described in this paper has been extensively tested for five contact grasps where the task was a three-dimensional task wrench space ball, and seven contact grasps where the task was a four-dimensional task wrench space ball. In both cases, the number of contacts was adequate to form high quality grasps of the objects tested, but not so large as to make it difficult to find kinematically feasible solutions. An area for future work is to better characterize the relationship between the size and dimensionality of the task wrench space, the similarity of the objects to be grasped, and the number of contacts required to produce a useful grasp prototype.

9 Conclusion

This paper presented a novel technique for synthesizing grasps by generalizing a single example grasp, or grasp prototype, and applying this generalized prototype to a new target object. The technique can be summarized as follows:

1. Given a grasp prototype, extract the contact wrenches $\underline{w}_i$ of that grasp prototype. *(Offline)*

2. Given a task TS, compute the generalized grasp prototype $GGP(TS, Q)$ for that task as a function of grasp quality threshold Q. *(Offline)*

3. Given a new target object, find an optimal set of contacts on that target object. *(Runtime)*

The method used for generalizing the prototype was shown to decrease the complexity of the search for a good grasp of the target object by allowing the contacts of a proposed grasp to be evaluated and even optimized independently. Although the space over which contact quality measures are optimized does contain local minima, simple numerical optimization routines can be used with success. If independent contact quality measures are all above some quality threshold, then the quality of the resulting grasp is guaranteed to also be above that threshold.

The major strengths of this algorithm are listed below.

Global Region Information: The technique of finding grasps that match a generalized grasp prototype provides lower bound quality values over a space of possible grasps. This region information is important, because it can be used to determine the robustness of a grasp to errors in contact placement. It also provides the flexibility in contact placement that is needed to adapt to the constraints of the robot hand kinematics.

Offline Computation: This technique allows a grasp to be optimized for a particular task offline. The generalized grasp prototype for that given task can then be created and stored in a grasp library for later access. This means that any given grasp design and task description can incorporate knowledge gained from extensive analysis and experimentation.

Wide Applicability: This approach does not require many of the assumptions used by other grasp optimization techniques to reduce the complexity of the grasp synthesis problem. In particular, there is no limit to the number of contacts in a grasp, and this approach can be applied to three-dimensional, curved objects and any given task wrench space.

Speed: No matter how complex and time-consuming the offline grasp design and analysis process may be, the run-time process of testing the quality of any potential grasp is very fast. It is reduced from the problem of constructing a grasp wrench space convex hull and finding the scale of the largest task wrench space that fits within that convex hull to the problem of minimizing over a few dot products for each contact of the grasp. The quality measure for each contact can be independently calculated, thus allowing the placement of each contact to be independently optimized, resulting in an algorithm that is easily parallelizable.

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