A Framework for Efficient and Composable Oblivious Transfer

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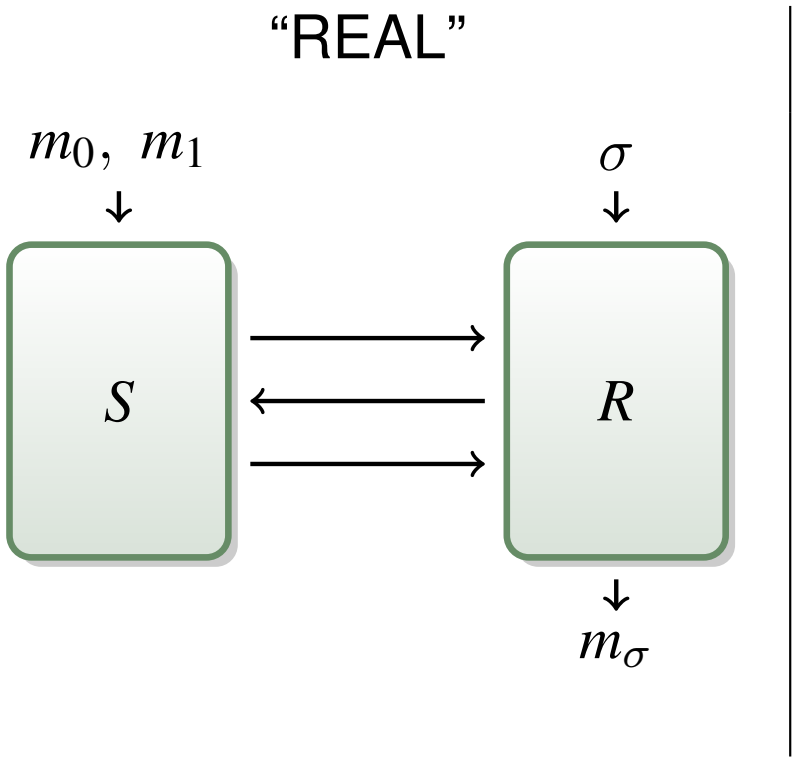
CRYPTO 2008

Oblivious Transfer

[R81,EGL85,BCR86,C87,K88,CK88,C89,CS91,CGT95,DCP95,FMR96,BC97,. . .]



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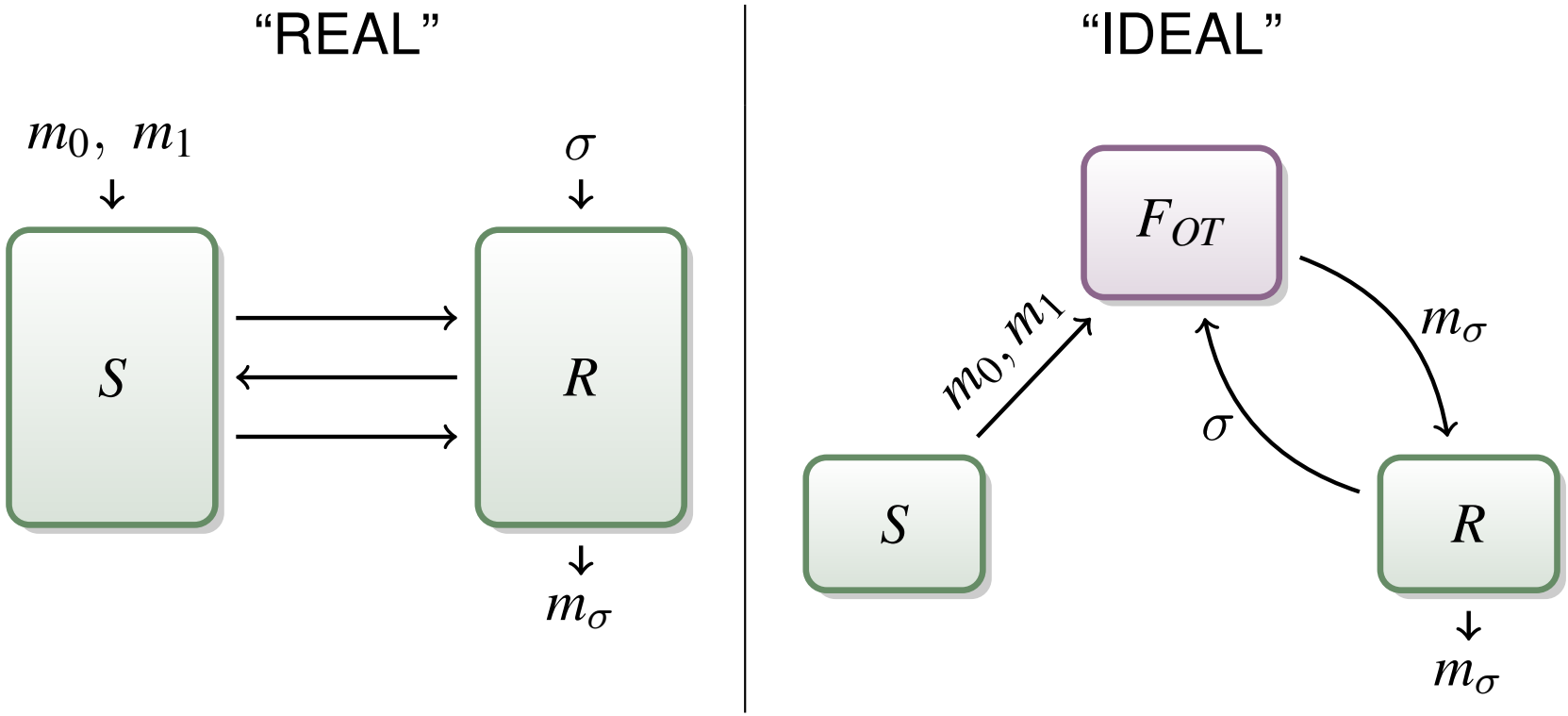

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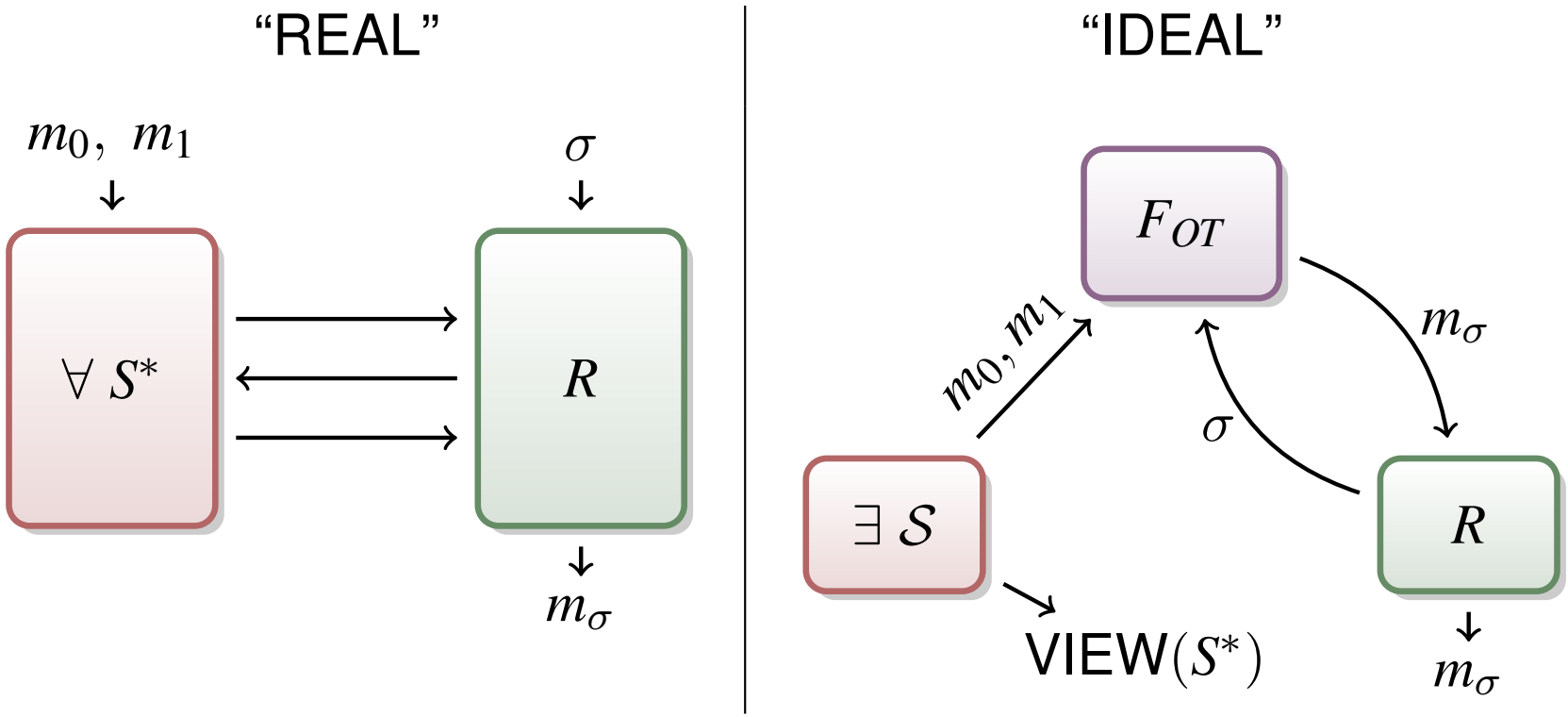

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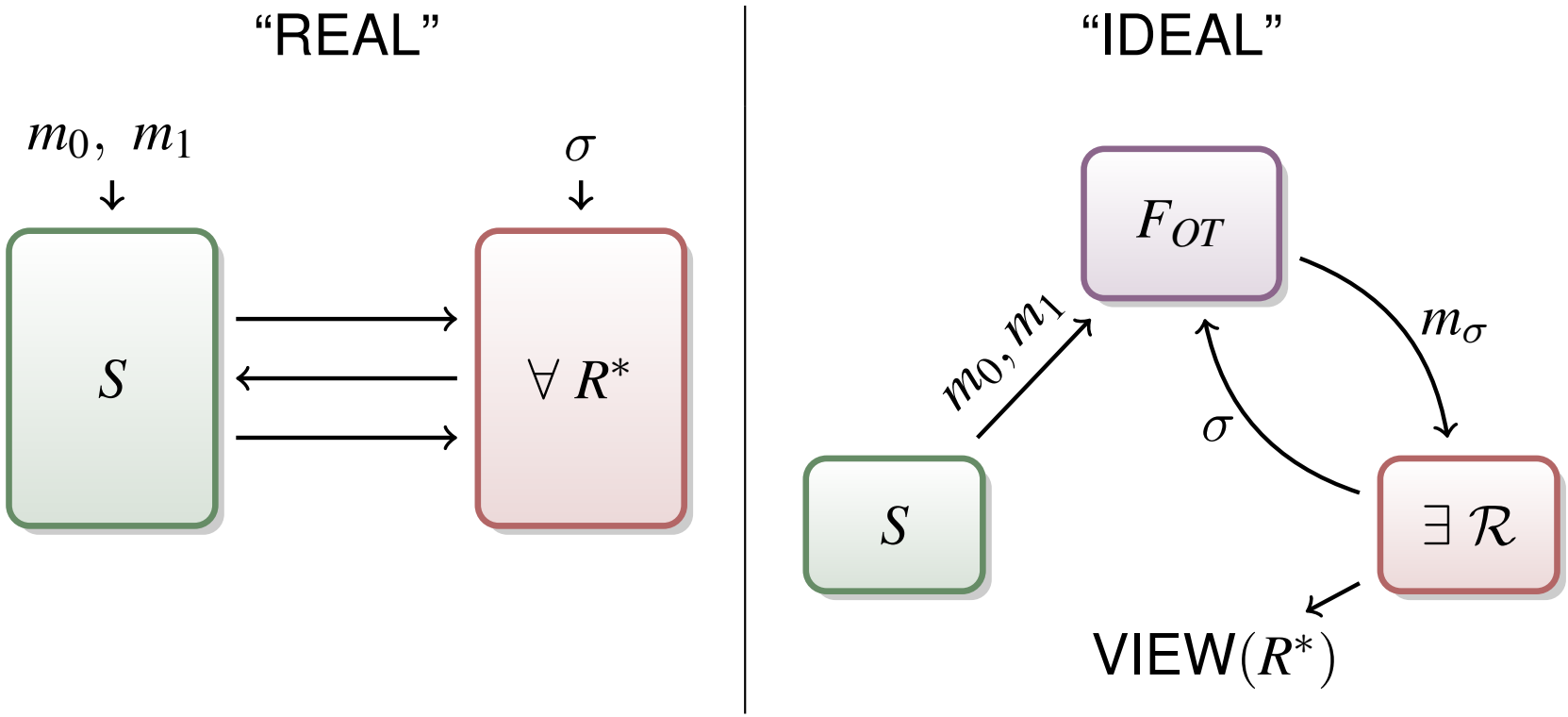

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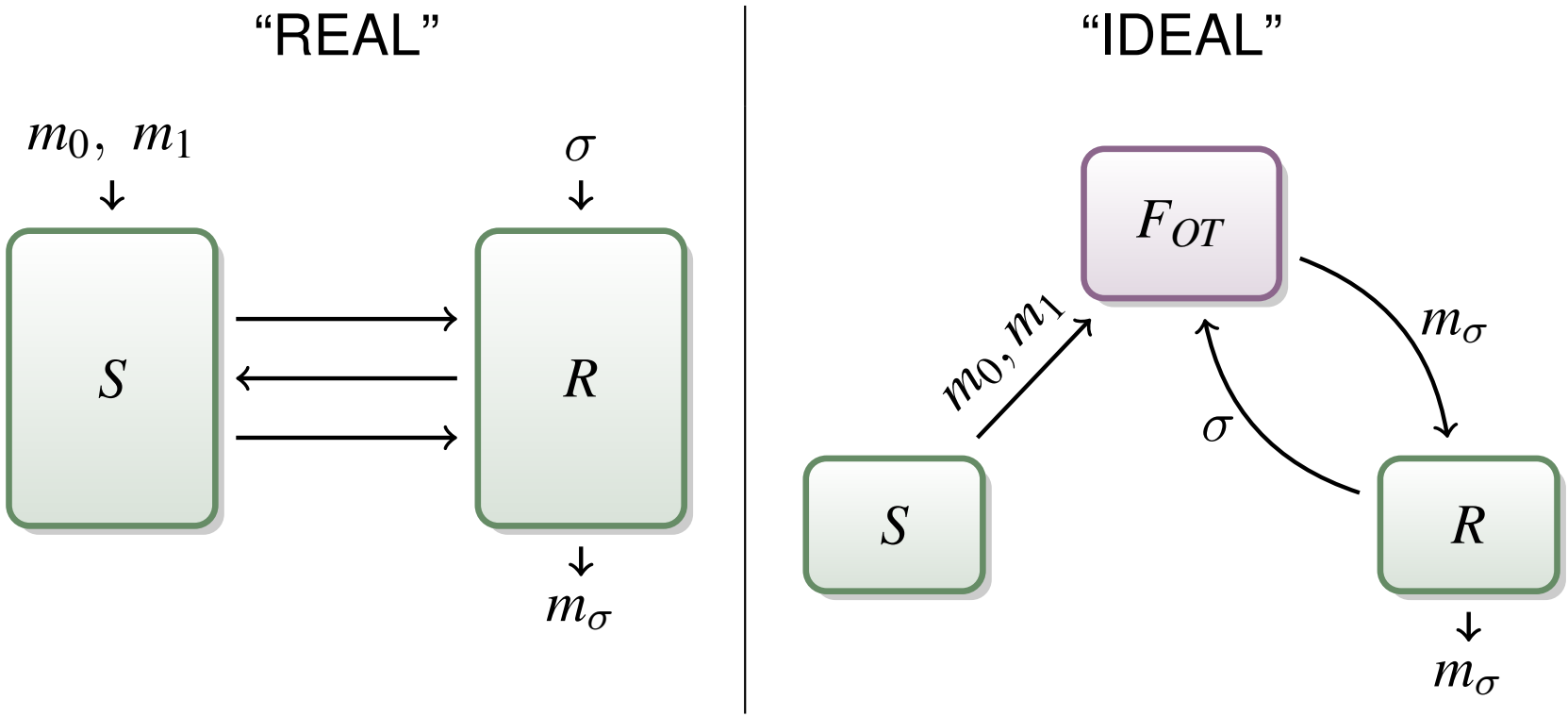

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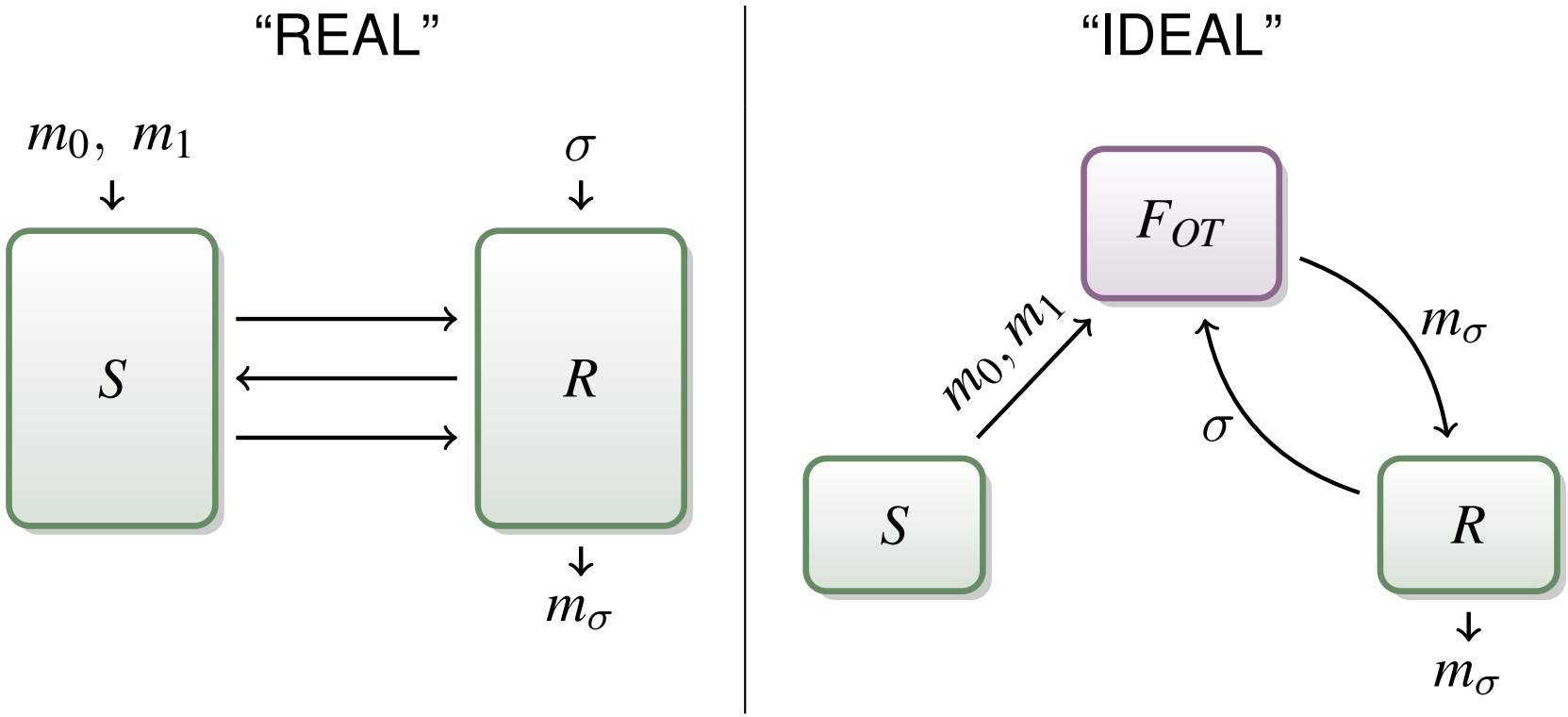

- 'Complete' for secure computation [Yao82,GMW87,Kil88]
- Feasible: (enhanced) TDPs + zero knowledge [EGL85,GMW86]

Prior Efficient Protocols

- Two messages: [NP01,AIR01,Tau05,CS02]

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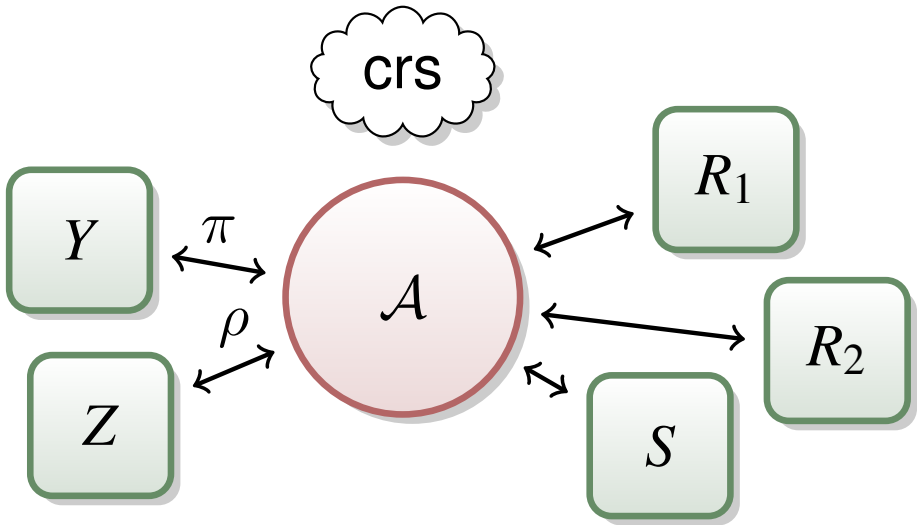
UC framework [Can01]

Requires *setup* [CF01]



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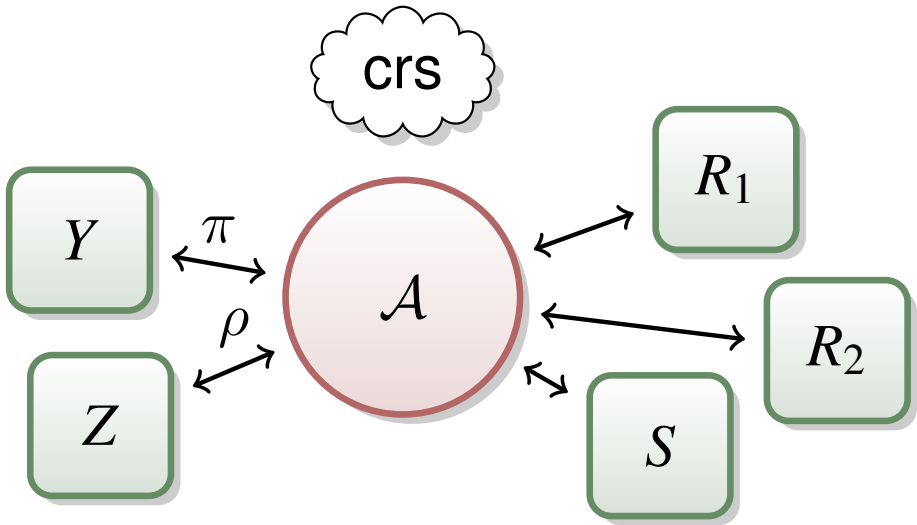

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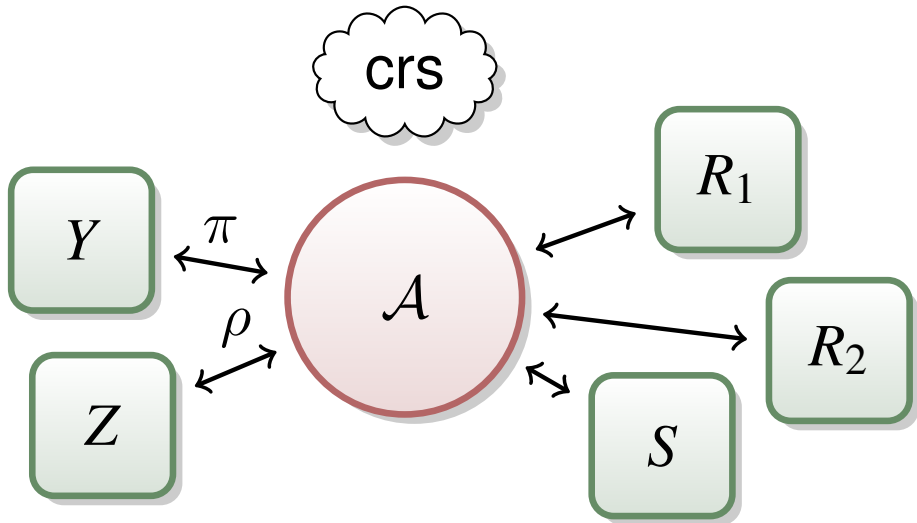

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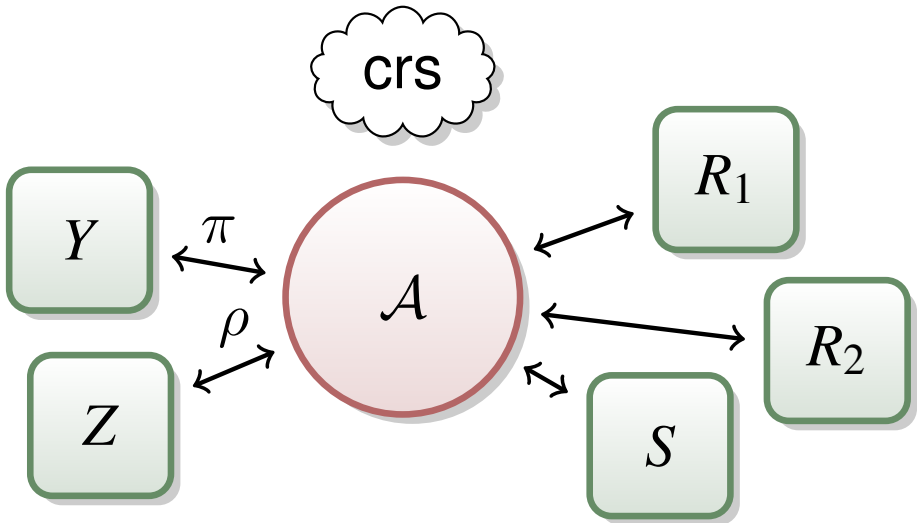

COMPOSABILITY aids *EFFICIENCY*

- Stronger OT variants, specific assumptions, 4+ messages [JS07,GMY04,DN03,GH08]

A New OT Framework

Main Attractions

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- *Statistical* security for either party
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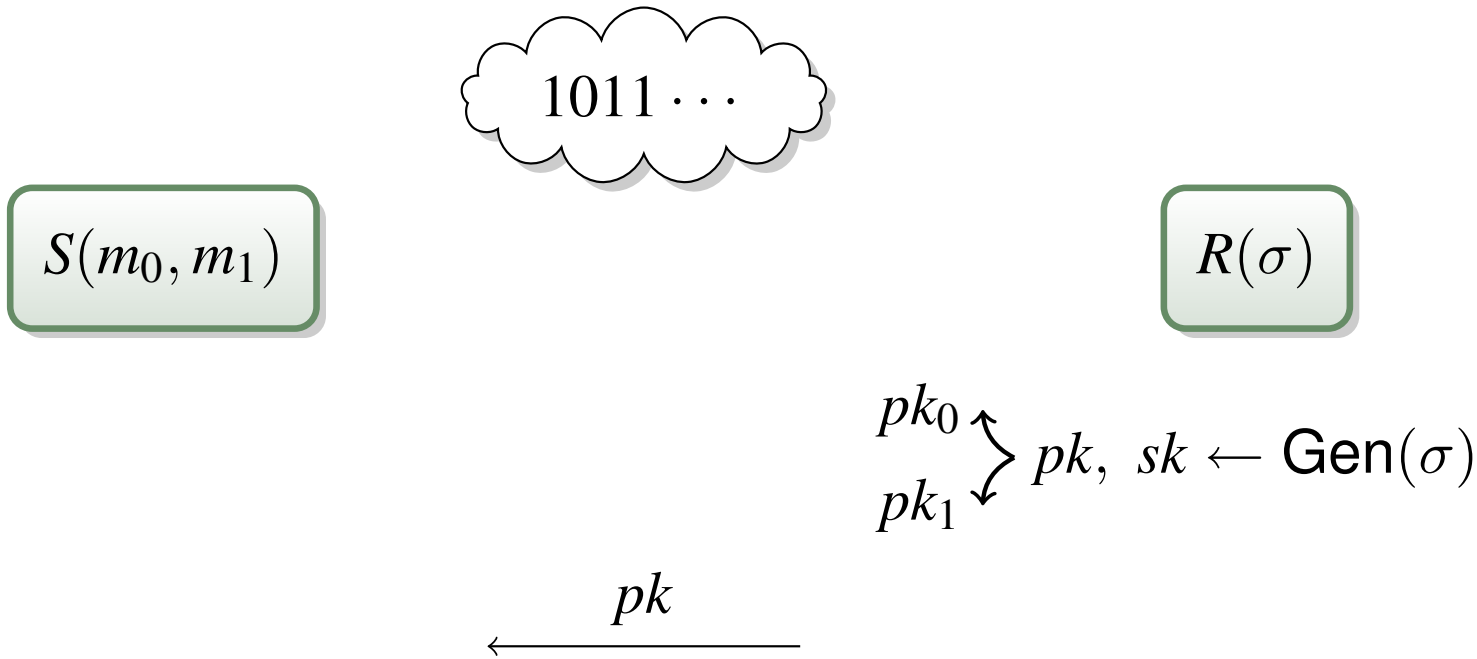
Conceptual Tools

- *Messy* public keys (‘*me*ssage-lo*ssy*’) aka ‘meaningless’ [KN08]
- New abstraction: *Dual-mode* cryptosystem

Our Protocol



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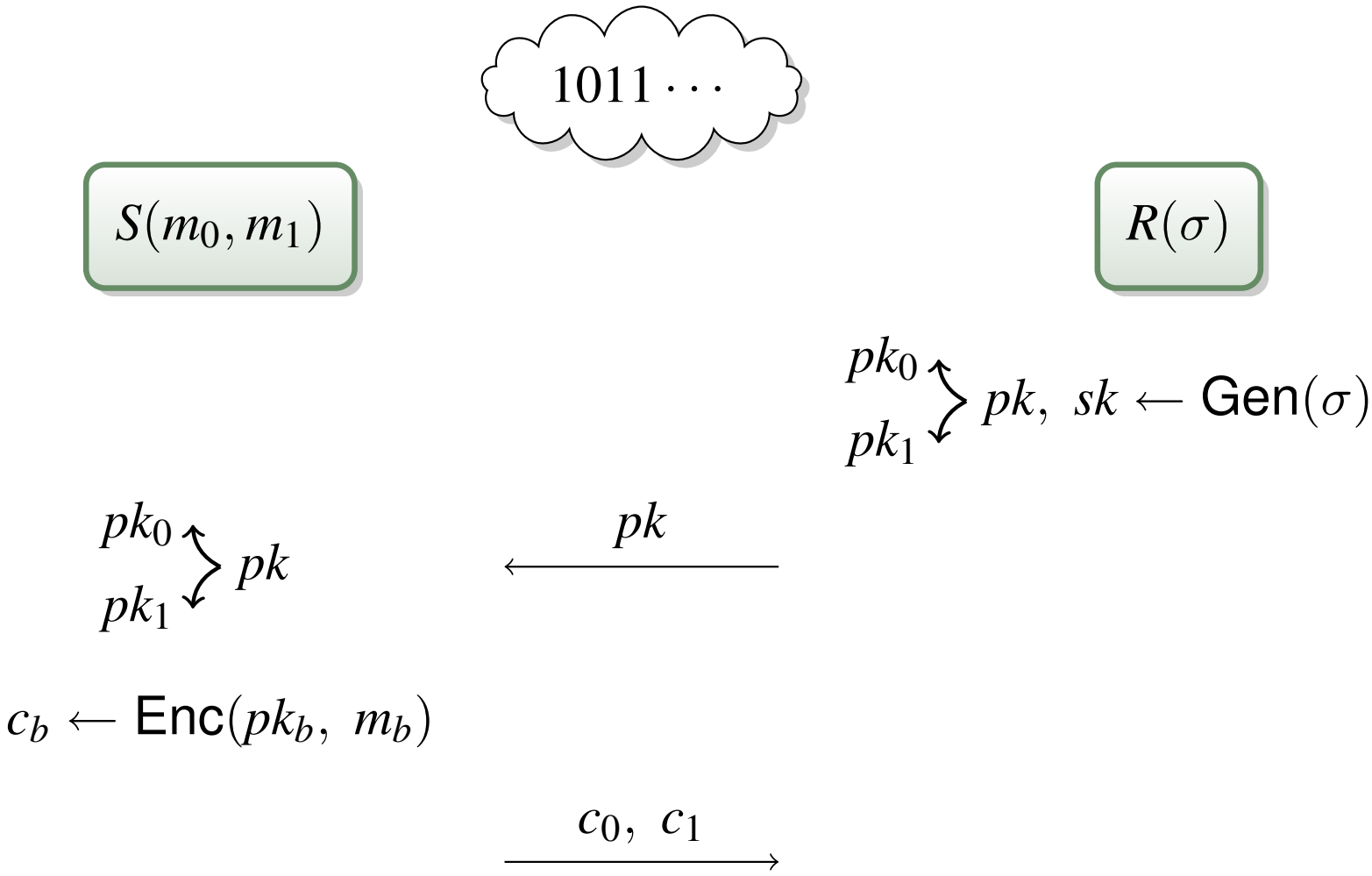

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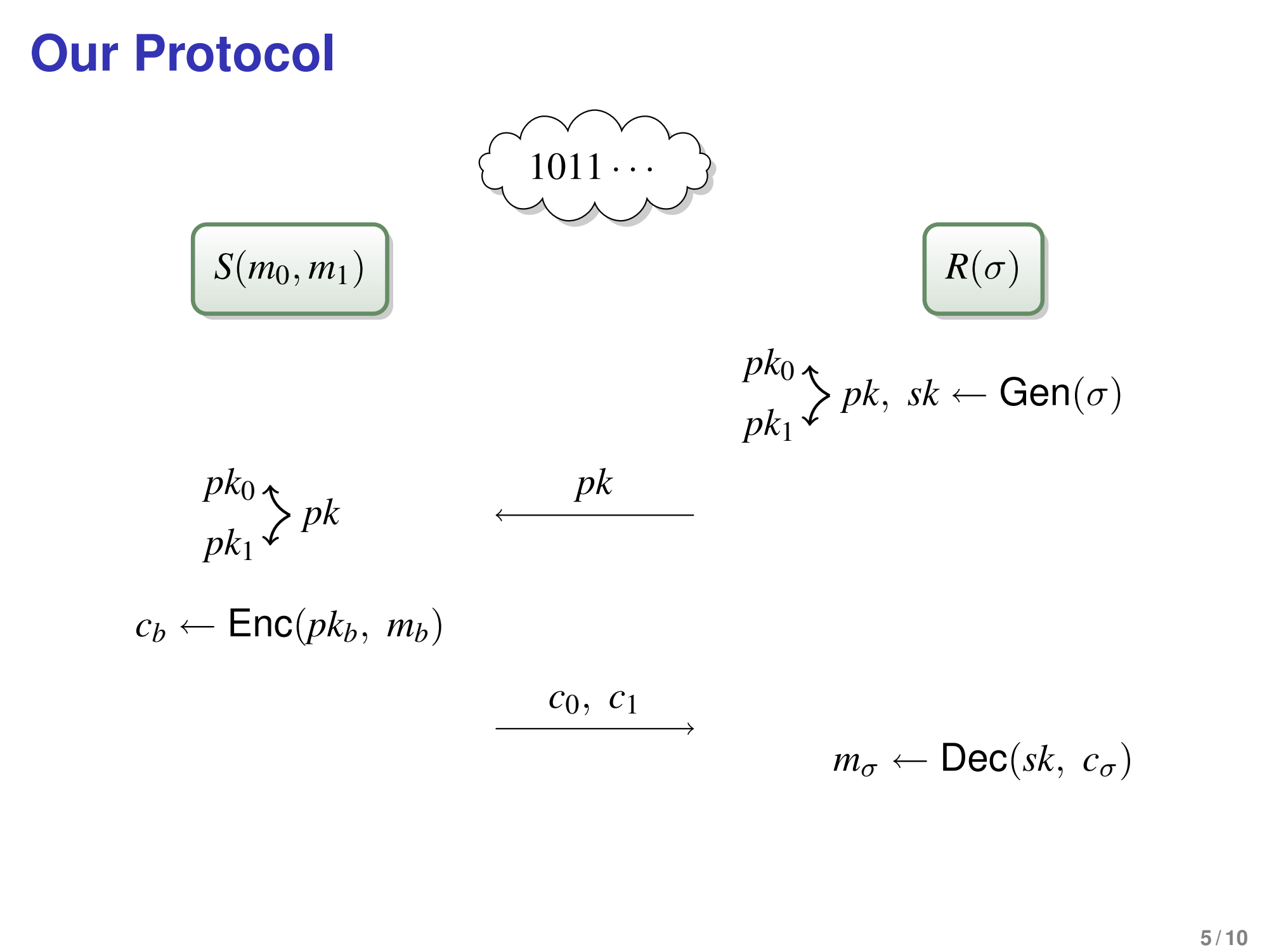

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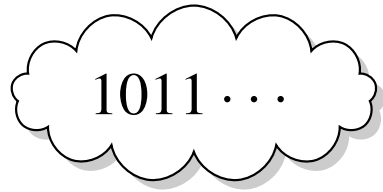

$S(m_0, m_1)$

$R(\sigma)$

pk_0, pk_1 $\leftrightsquigarrow$ $pk,\ sk \leftarrow \mathsf{Gen}(\sigma)$

pk_0, pk_1 $\leftrightsquigarrow$ pk

$\xleftarrow{\quad pk \quad}$

$c_b \leftarrow \mathsf{Enc}(pk_b,\ m_b)$

$\xrightarrow{\quad c_0,\ c_1 \quad}$

$m_\sigma \leftarrow \mathsf{Dec}(sk,\ c_\sigma)$

Needed: *Dual-mode cryptosystem*

Messy Encryption

Decryptable Public Keys

$$\mathsf{Enc}(pk,\ m_0) \quad \overset{c}{\approx} \quad \mathsf{Enc}(pk,\ m_1)$$

- Decrypt with sk.

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Cryptosystems with Messy Keys

- Cocks ID-based [Coc01]
- Lattice-based [AD97, Reg03, Reg05]
- ElGamal, Paillier variants [ElG84,Pai99]

Dual-Mode Cryptosystem

$$\{\text{dec}, \text{mes}\} \ni \text{mode} \longrightarrow \boxed{\text{Setup}} \longrightarrow (\text{crs},\ \text{trap})$$

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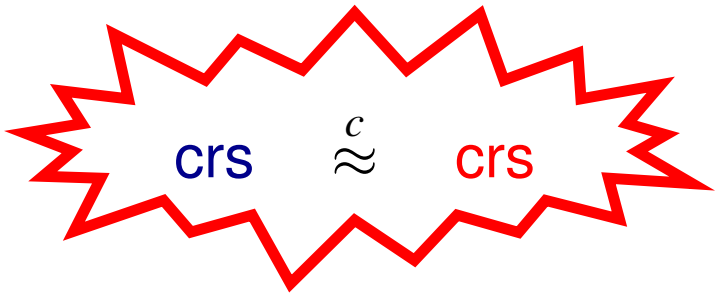

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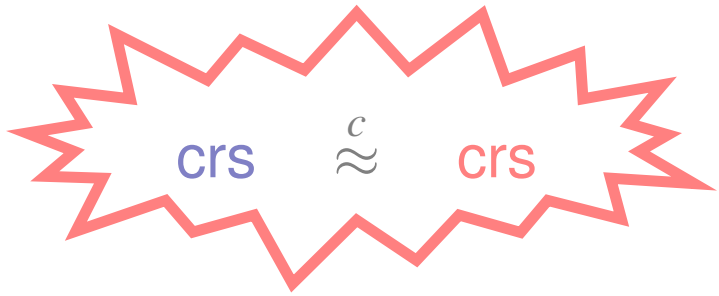

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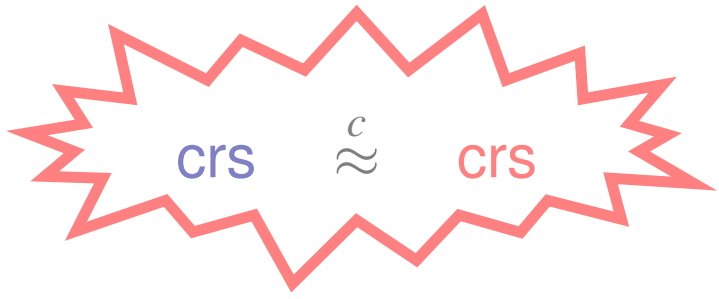

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(pk_σ ↓ Enc, sk_σ ↓ Dec)

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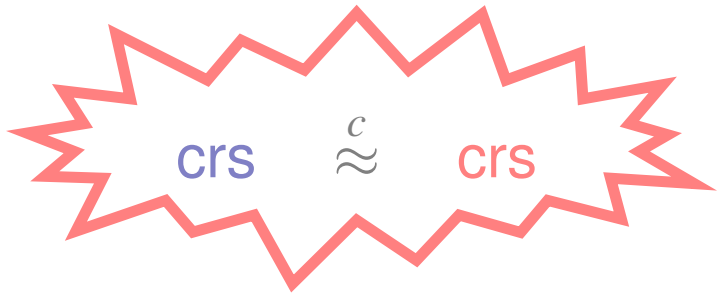

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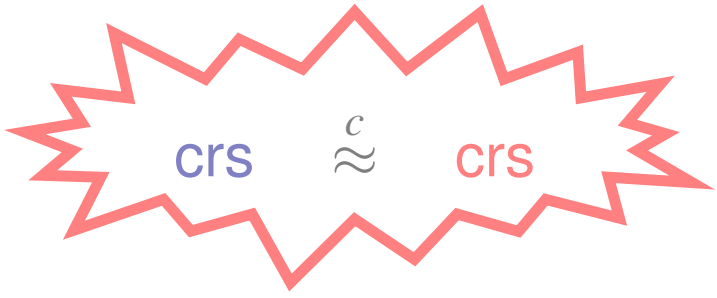

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$pk^* \longrightarrow$ FindMessy $\longrightarrow pk^*_\mu$

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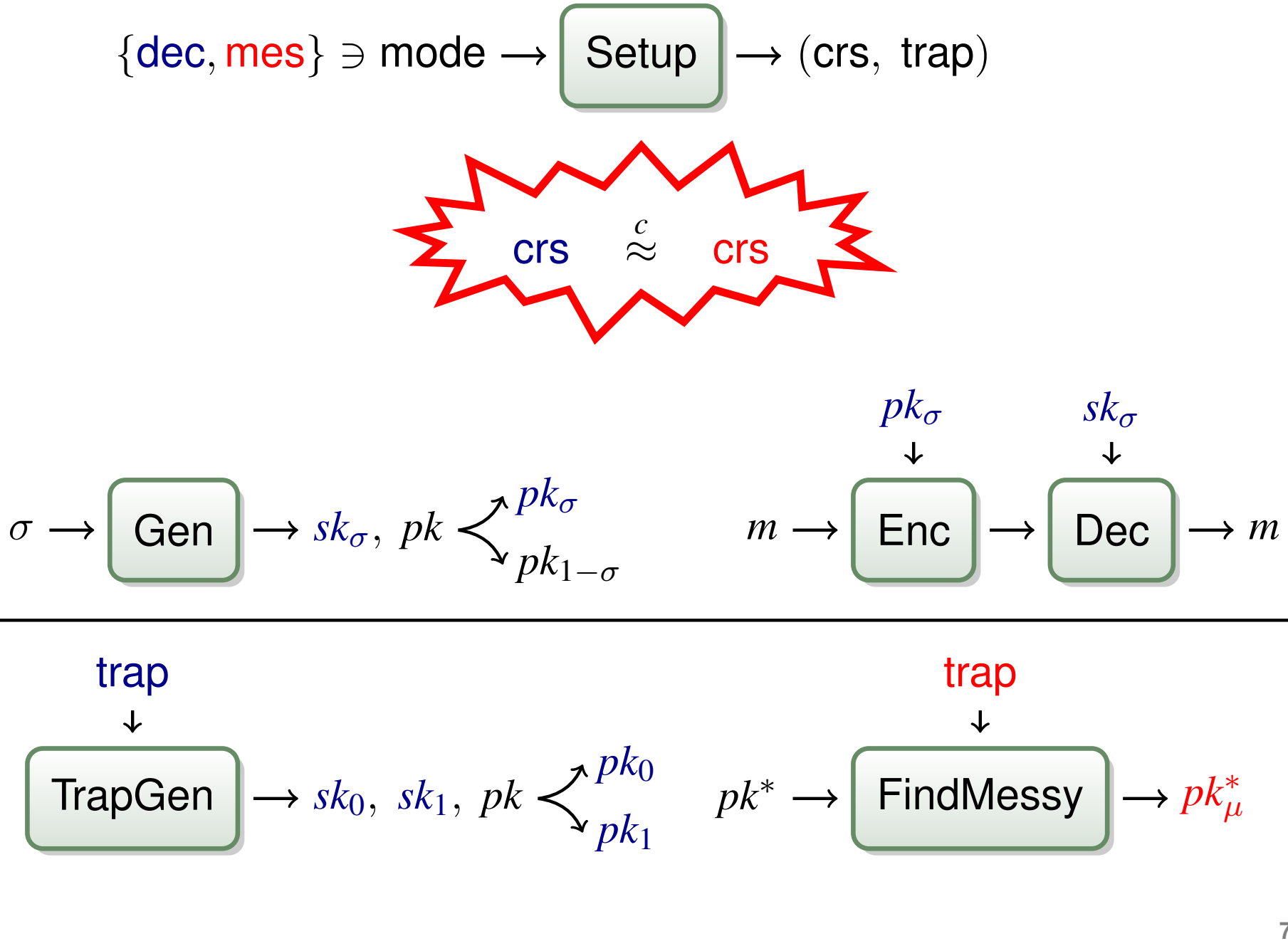

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Main Theorem

Dual-mode cryptosystem $\Longrightarrow$ 2-message UC-secure OT

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1. $\forall$ real $\mathcal{S}^*$, $\exists$ ideal $\mathcal{S}$ (TrapGen)

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Security in decryption mode (cf. [GOS06]):

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Quadratic Residuosity Construction

Cocks Encryption [Coc01]

- Global: $N = pq$
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Decryption mode

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TrapGen:	FindMessy:
$sk_0 = \sqrt{y},\ sk_1 = \sqrt{y} \cdot \sqrt{z}$	$y \notin \mathbb{QR}_N \quad \text{or} \quad y \cdot z \notin \mathbb{QR}_N$

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