The Energy-Limited Capacity of Wireless Networks

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***Abstract*— The performance of large-scale wireless ad hoc networks is often limited by the broadcasting nature of the wireless medium and the inherent node energy constraints. While the impact of the former on network capacity has been studied extensively in the literature, the impact of energy constraints has not received as much attention. In this paper, we study the capacity limitations resulting from the energy supplies in wireless nodes. We define the energy-limited capacity of a wireless network as the maximum amount of data the network can deliver before the nodes run out of energy. This energy-limited capacity is an important parameter in networks where operating lifetime is critical, such as ad hoc networks deployed in hazardous environments and sensor networks. We study two types of static networks, networks without any infrastructure support and networks where base stations with unlimited energy are deployed to support data forwarding. We consider two kinds of traffic models motivated by ad hoc networks and sensor networks. We derive upper and lower bounds on the energy-limited capacity of these networks. While throughput has been shown to not scale with node density in static networks by previous studies, our results show that, depending on the energy consumption characteristics of wireless communication, the energy-limited capacity *can* scale well under both traffic models. In addition, we show that the deployment of base stations can improve the energy-limited capacity of the network, especially for networks with sensor traffic.**

I. Introduction

Wireless ad hoc networks enable devices to communicate with each other without pre-installed infrastructure. These networks have a wide range of applications such as environmental sensing, battlefield support and disaster relief [13], [19]. However, the performance of wireless ad hoc networks is often limited by the broadcasting nature of the wireless medium, which results in interference when nearby nodes attempt to simultaneously transmit. In addition, since these networks often consist of devices that are powered by batteries, power management is of great importance.

Recently there has been significant effort in studying the fundamental limits in the *throughput capacity* of wireless ad hoc networks, defined as the total data rate (bits/second) that can be transferred between nodes, accounting for interference caused by the broadcast medium. In a seminal paper [10], Gupta and Kumar study a model of ad hoc networks with stationary nodes and show that when the number of nodes n per unit area increases, the per node throughput decreases as $\Theta(\frac{1}{\sqrt{n \log n}})$. This result implies that large scale ad hoc networks would not be scalable for non-local traffic patterns. Grossglauser and Tse [8] study mobile ad hoc networks and show that with loose delay constraints, node mobility can dramatically improve network capacity. They prove that the per node throughput can be kept constant as the number of nodes per unit area increases, assuming mobile nodes that carry packets. Recently, others have investigated the throughput capacity of hybrid networks with infrastructure support [16], [15]. There is also other work in the capacity of ad hoc networks under different models [7], [1], [18], [17], [6].

While the impact of broadcast interference on network capacity has been studied extensively in the literature, the effect of energy constraints has not received as much attention. In this paper, we study the capacity limitations resulting from the energy supplies in wireless nodes. In particular, we define the *energy-limited capacity* as the maximum amount of data (bits) the network can deliver before the nodes run out of energy[1]. This type of capacity constraint is an important parameter in networks where operating lifetime is critical, such as ad hoc networks deployed in hazardous environments and sensor networks that are significantly battery-constrained.

We study two types of static (i.e., stationary) networks, pure ad hoc networks without any infrastructure support (which we call *homogeneous networks*), and *hybrid networks* where base stations with unlimited energy are deployed to support data forwarding. In both types of networks, we consider two traffic models, an *ad hoc model* intended to reflect uniform traffic patterns as might arise in ad hoc networks, and a *sensor model* motivated by sensor networks that collect data and then deliver to a single destination.

We derive upper and lower bounds on the energy-limited capacity for these two networks and two traffic models. In general, our lower bounds are within a factor of $O(\log^k n)$ from the corresponding upper bounds for some constant k. Based on the bounds, we can reach conclusions about the scaling behavior of wireless networks under energy constraints. In particular:

- We find that the energy-limited capacity of wireless networks *can* scale well under both traffic models, for certain properties of the energy consumption characteristics. This result sharply contrasts with the behavior of throughput capacity, which does not scale with increasing node density.
- The capacity of networks supporting sensor traffic is less

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[1]Nodes may certainly be involved in application-specific activities that consume energy. However, for generality, we restrict our attention to the consumption of energy that directly results from communication.

than the capacity of networks supporting ad hoc traffic, by a factor of $O(\sqrt{n}\log^c n)$ for some constant c.

- The deployment of base stations can improve the network capacity, especially with sensor traffic, where the addition of even a single base station has an immediate positive effect.
- Reducing the constant energy that is required independent of transmission distance is key to desirable scaling.

The rest of this paper is structured as follows. Section II describes the network and traffic models we use in this paper and defines the energy-limited capacity. We study the energy-limited capacity of homogeneous networks with sensor traffic in Section III. Hybrid sensor networks where base stations are deployed to support data forwarding are considered in Section IV. We study the energy-limited capacity of networks with ad hoc traffic in Section V. Related work is reviewed in Section VI and the paper is concluded in Section VII.

II. Network Models

We first describe the network models and traffic models we use in this paper. Then we define the energy-limited capacity.

A. Network Models

We consider two types of static networks, *homogeneous networks* and *hybrid networks*. In homogeneous networks, nodes rely on their own resources for communication and no infrastructure support exists. In contrast, hybrid networks consist of base stations (BSs) as well as normal nodes as in homogeneous networks. Nodes are assumed to be identical and have limited energy supplies; base stations are equipped with unlimited energy and deployed to support data forwarding for nodes. A hybrid network consists of two layers, the ad hoc layer formed by nodes, and the infrastructure or BS layer. In a homogeneous network, only the ad hoc layer exists.

In our analysis, the deployment area of the network is scaled to a unit disk. Nodes in the ad hoc layer are distributed on the unit area uniformly and independently. The number of nodes in the network is n and the initial energy of each node is a constant A.

In the infrastructure layer, m BSs are sparsely distributed in the same area as nodes. We assume that the number of base stations grows slower than the number of nodes[2], i.e., $\lim_{n\to\infty}\frac{m}{n}=0$. In addition, we assume that BSs are deployed regularly in the unit disk such that the BSs divide the area into a tessellation of hexagon cells (see Fig. 1). The BSs are at the center of each cell. Without considering the boundary effects, the area of each cell is $\frac{1}{m}$.

In the ad hoc layer, data is forwarded from the sender to the destination in a multi-hop fashion. With the support of BSs in a hybrid network, data can be transmitted in two modes: the *ad hoc mode* and the *infrastructure mode*. In the ad hoc mode, a node transmits its data to the destination in the ad hoc layer without using any base station. In the infrastructure mode, data is forwarded via the infrastructure layer. However, nodes might still use multi-hop routing in the ad hoc layer in order to reach a BS. Thus in hybrid networks, there is an issue of how to route data via these two transmission modes.

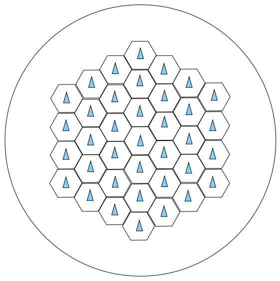

Fig. 1. A hybrid network.

B. Energy Model

To conserve energy, nodes may use different radio ranges in data transmission. In this paper, we consider only energy used in communication and do not account for the idle energy consumption. With efficient idle energy management schemes like [4], [23], [20], the impact of idle energy would be minimized.

Data transmission consumes energy in both the sender and the receiver. We define $e_t(r)$ as the minimum energy required in the sender to transmit a unit of data (called *bit*) directly over a distance r. Similarly, $e_r(r)$ is the minimum energy required in the receiver to receive a unit of data from a sender that is a distance r away. We define the *energy cost* function $e(r)$ as

$$e(r) = e_t(r) + e_r(r). \tag{1}$$

In this paper we assume that the energy consumption of the sender is $e(r)$ when the transmission range used is r and no energy is consumed in the receiver. This actually reallocates the energy consumption in a routing path by "moving" the energy consumption of each hop to its sender. Since the total energy of the routing path remains the same, this simplification would not affect our results on the energy-limited capacity[3].

We consider two specific models which are often used in the literature. The first one is a linear model in which

$$e(r) = c_a(c_b + r^\alpha) \tag{2}$$

where c_a and c_b are constants and $\alpha \geq 2$ is the path loss component. This is the same as the form of $e(r) = c' + c'' r^\alpha$ used in [11]. We adopt this form to emphasize the impact of c_b on the scaling behavior of the transport capacity, as will be clear in the following sections. The second model is

$$e(r) = c_a r^\alpha. \tag{3}$$

[2]In Section IV, we require $m \leq n/(200\log n)$ as $n\to\infty$ in the proof of the lower bound.

[3]It can be shown that, if the receiving energy $e_r(r)$ is no more than a constant times of the transmission energy $e_t(r)$, which is normally true in practice, this simplification does not change our results by more than a constant factor.

This actually is a special case of (2) by setting c_b to zero. We consider this separately because these two models have different scaling properties and are treated differently in the analysis.

C. Traffic Models

We consider two types of traffic models which are motivated by ad hoc networks and sensor networks. In the *ad hoc traffic model*, nodes can be a sender or destination and communication occurs in a one-to-one fashion. Specifically, each node chooses a random location in the area and selects the closest node to the location as its destination. In the *sensor traffic model*, all nodes are senders of data transmission but communicate with a common entity called the *sink*. We assume that the sink is at the center of the unit disk and has infinite energy. In both traffic models, we assume nodes have an infinite amount of data to send. These traffic models try to capture the traffic patterns normally present in ad hoc networks and sensor networks, which have significant impact on the energy-limited capacity of the network[4].

D. Definition of Energy-Limited Capacity

Now we define the energy-limited capacity. Since we are considering the maximum amount of data the network can deliver under node energy limits, we assume that an ideal transmission schedule is used, i.e., no collision occurs and nodes can avoid idle energy consumption. We first define the *feasible transfer volume* of networks.

Definition 2.1 (Feasible transfer volume): For a network consisting of n nodes, a transfer volume of $\lambda(n)$ bits is *feasible* if each node can send $\lambda(n)$ bits to its corresponding destination under energy limitation of nodes.

The definition of the feasible transfer volume requires that every node is able to transmit $\lambda(n)$ bits of data. Therefore, it measures the transport capacity of the network until the first node runs out of energy. We now define the energy-limited capacity which is similar to the definition of throughput capacity in [10].

Definition 2.2 (Energy-Limited Capacity): The *Energy-Limited Capacity* (or *capacity* for short[5]) of networks is of order $\Theta(f(n))$ bits if there are deterministic constants $c > 0$ and $c' < +\infty$ such that

$$\lim_{n\to\infty} \text{Prob}\{\lambda(n) = cf(n) \text{ is feasible}\} = 1, \text{ and}$$
$$\lim_{n\to\infty} \inf \text{Prob}\{\lambda(n) = c'f(n) \text{ is feasible}\} < 1.$$

The definition of energy-limited capacity does not consider throughput or delay. As we will see in later sections, minimum energy consumption in general requires using shorter transmission ranges, which tends to improve throughput because of the reduced interference [10]. So achieving optimal transfer volume does not pose limitations on the achievable throughput. However, it might result in larger delay because of the increased number of hops. We will study this tradeoff in future work.

III. Energy-Limited Capacity of Homogeneous Networks with Sensor Traffic

In this section, we study the energy-limited capacity of homogeneous networks with sensor traffic. In these networks, all nodes transmit data to a single node called the sink which is at the center of the unit area. We now obtain upper and lower bounds on the capacity.

A. Upper Bound on Energy-Limited Capacity

We first derive an upper bound on the energy-limited capacity which is stated in the following lemma.

Lemma 3.1: In a homogeneous network with n nodes and sensor traffic, the feasible transfer volume $\lambda(n)$ is no more than

$$c_0 \int_{\frac{1}{\phi_n\sqrt{n}}}^{\frac{1}{\sqrt{\pi}}} \frac{2\pi A x}{e(x)}\, dx \tag{4}$$

with high probability where ϕ_n is a sequence that satisfies $\lim_{n\to\infty}\phi_n \to \infty$ and c_0 is a constant.

Proof: We prove the upper bound of the transfer volume by bounding the total amount of data that nodes can transmit directly to the sink. We first consider how close nodes are to the sink when the network consists of n nodes. Let $P(d)$ be the probability that there is no node within a distance d to the sink. Since nodes are distributed randomly in the unit disk, we have $P(d) = (1-\pi d^2)^n$. Let ϕ_n be a sequence that satisfies $\lim_{n\to\infty}\phi_n \to \infty$. Then we have

$$P(\frac{1}{\phi_n\sqrt{n}}) = \left(1 - \frac{\pi}{\phi_n^2 n}\right)^n \to e^{-\frac{\pi}{\phi_n^2}}$$

which approaches 1 as $n \to \infty$. Thus with high probability, no node lies within a distance $\frac{1}{\phi_n\sqrt{n}}$ to the sink.

Consider any node X_i. Let d_i be the distance between X_i and the sink. Due to the uniform distribution of X_i, the probability density function of d_i is $P\{d_i = x\} = 2\pi x$. Let T_i be the amount of data that X_i can send directly to the sink under the energy limit. The mean of T_i is

$$E[T_i] = \int_{\frac{1}{\phi_n\sqrt{n}}}^{\frac{1}{\sqrt{\pi}}} P\{d_i = x\}\frac{A}{e(x)}\, dx = \int_{\frac{1}{\phi_n\sqrt{n}}}^{\frac{1}{\sqrt{\pi}}} \frac{2\pi A x}{e(x)}\, dx.$$

Here we take into account the fact that X_i is not within a distance of $\frac{1}{\phi_n\sqrt{n}}$ to the sink with high probability. By the Strong Law of Large Numbers, for a network of n nodes, we have

$$\frac{\sum_{i=1}^n T_i}{n} \to E[T_i]$$

with probability 1 as $n \to \infty$. In other words, the total amount of data that all nodes can send directly to the sink is no more than $c_0 n E[T_i]$ with high probability. Thus the feasible transfer volume per node $\lambda(n)$ is no more than $c_0 E[T_i]$. This completes the proof. ■

[4]In sensor networks, data may be aggregated within the network to reduce traffic load. However, we do not consider the effect of data aggregation in this paper and leave it as a topic of future work.

[5]We will use the term capacity in the rest of this paper to refer to energy-limited capacity unless stated otherwise.

In this lemma, we assume that $\frac{x}{e(x)}$ is integrable over $(0, \frac{1}{\sqrt{\pi}})$. This does not pose much limitation since most energy cost functions in practice fall into this category. We now obtain upper bounds on the feasible transfer volume for the two specific cases.

Lemma 3.2: In a homogeneous network with sensor traffic, if the energy cost function is $e(r) = c_a r^\alpha$, the feasible transfer volume $\lambda(n)$ satisfies

$$\lambda(n) \leq \begin{cases} \frac{c_1 A}{c_a} \log(\phi_n^2 n) & \text{if } \alpha = 2; \\ \frac{c_2 A}{c_a(\alpha-2)} \phi_n^{\alpha-2} n^{\frac{\alpha}{2}-1} & \text{if } \alpha > 2 \end{cases} \tag{5}$$

with high probability.

Lemma 3.3: In a homogeneous network with sensor traffic, if the energy cost function is $e(r) = c_a(c_b + r^\alpha)$, the feasible transfer volume $\lambda(n)$ satisfies

$$\lambda(n) \leq \begin{cases} \frac{c_3 A}{2c_a} \left(\ln(c_b + \frac{1}{\pi}) - \ln(c_b + \frac{1}{\phi_n^2 n}) \right) & \text{if } \alpha = 2; \\ \frac{c_3 A}{c_a} \int_{\frac{1}{\phi_n \sqrt{n}}}^{\frac{1}{\sqrt{\pi}}} \frac{x\, dx}{c_b + x^\alpha} & \text{if } \alpha > 2 \end{cases} \tag{6}$$

with high probability.

In the case of $e(r) = c_a(c_b + r^\alpha)$, it is easy to prove $\lambda(n) = O(1)$. This is because for any transmission, the minimum energy required is $c_a c_b$. Given energy supply A, each node can transmit no more than $\frac{A}{c_a c_b}$ units of data. Therefore, the capacity of these networks is $O(1)$. However, this does not rule out the possibility that the feasible transfer volume can increase with the node density. To understand how $\lambda(n)$ evolves with n, we explicitly condition the feasible transfer volume $\lambda(n)$ on n in Lemma 3.3, which holds as long as n is large.

B. Lower Bound on Energy-Limited Capacity

In this section, we provide a lower bound on the energy-limited capacity. Our proof constructs a spatial tessellation and a transmission strategy that achieves the lower bound. In our proof, data transmissions use the same transmission range.

We first describe the spatial tessellation used in the proof. Let r_n be the data transmission range and define $w_n = r_n/8$. We set r_n to be at least $c_4\sqrt{\frac{\log n}{\pi n}}$ where c_4 is a constant. c_4 is set to be 160 which will be clear in the proof of Lemma 3.4. So the network is connected with high probability according to [9]. We partition the unit disk into regions using circles centered at the sink with radii iw_n for $1 \leq i \leq \frac{1}{w_n\sqrt{\pi}}$. Let B_i be the region between circles with radii iw_n and $(i+1)w_n$. For each i, we further divide region B_i evenly into $6i$ smaller cells (see Fig. 2). So the angle subtended by a cell at the sink is $\frac{\pi}{3i}$ and the area of a cell is $\Theta(w_n^2)$. Intuitively, nodes are distributed evenly in the unit disk due to the uniform distribution of nodes, i.e., every cell contains $\Theta(nw_n^2)$ nodes. By using similar techniques as in [10], we prove this claim in the following lemma. The proof is omitted here due to space limits and can be found in [24].

Lemma 3.4: Every cell contains $\Theta(nw_n^2)$ nodes with high probability.

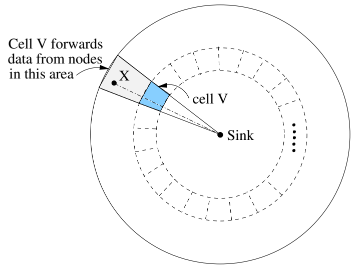

Fig. 2. Spatial tessellation in homogeneous networks with sensor traffic.

Given the spatial tessellation, data is forwarded geographically from the sender towards the sink. More precisely, consider any node X. Let L be the line segment connecting X and the sink. Data from X is forwarded along the sequence of cells intersecting L until reaching the sink. Since the transmission range r_n equals to $8w_n$ and each cell can be contained in a disk of radius $2w_n$, a node in any cell can transmit directly to nodes in adjacent cells. In addition, each cell contains $\Theta(nw_n^2)$ nodes. Therefore, for any node, a routing path along the sequence of cells described above exists with high probability.

Now we study the transfer volume achieved using this routing strategy. For a transfer volume of $\lambda(n)$, each cell must be able to forward the offered traffic load with high probability. Consider a cell V in region B_i. If data from a node, say X, is forwarded by cell V, the line segment connecting X and the sink must intersect V. So X must lie within the sector subtended by V at the sink and outside cell V, i.e., in a region B_j, $j > i$ (see Fig. 2). Noting that the angle of the sector is $\frac{\pi}{3i}$, the total number of nodes whose traffic is handled by V, including nodes in V itself, is no more than $\frac{c_5(1-\pi i^2 w_n^2)n}{i}$ with high probability. So for a transfer volume of $\lambda(n)$, the amount of traffic that cell V needs to transmit is no more than $\frac{c_5(1-\pi i^2 w_n^2)n\lambda(n)}{i}$.

Since nodes in a cell can transmit to any node in adjacent cells, nodes within a cell are exchangeable in forwarding traffic. Given that there are $\Theta(nw_n^2)$ nodes in V and the transmission range is r_n, the total amount of traffic that cell V can transmit is $\Theta(\frac{nw_n^2 A}{e(r_n)})$. Thus, for a transfer volume of $\lambda(n)$ to be feasible, we have

$$\frac{c_5(1-\pi i^2 w_n^2)n\lambda(n)}{i} \leq \frac{c_6 n w_n^2 A}{e(r_n)} \tag{7}$$

for all $1 \leq i \leq \frac{1}{w_n\sqrt{\pi}}$. In addition, nodes in the region that is within the circle of radius r_n need to transmit all traffic. So we have

$$n\lambda(n) \leq \frac{c_6 n w_n^2 A}{e(r_n)}. \tag{8}$$

Note that if $\lambda(n)$ satisfies (8), the minimum of $\lambda(n)/c_5$ and $\lambda(n)$ would satisfy both (8) and (7) for all i. That means either $\lambda(n)/c_5$ or $\lambda(n)$ is feasible. So combining with $w_n = r_n/8$, we prove that

$$\lambda(n) = \frac{c_7 r_n^2 A}{e(r_n)}$$

is feasible for some constant c_7. To maximize $\lambda(n)$, we should choose the transmission range r_n such that $\frac{r_n^2 A}{e(r_n)}$ is maximized. Taking into account the fact that $r_n \geq c_4\sqrt{\frac{\log n}{\pi n}}$, we have the following result.

Lemma 3.5: In a homogeneous network with sensor traffic, a transfer volume of

$$\lambda(n) = \max\{\frac{c_7 x^2 A}{e(x)} : c_4\sqrt{\frac{\log n}{\pi n}} \leq x \leq \frac{1}{\sqrt{\pi}}\}$$

is feasible with high probability.

Using the above lemma, we obtain lower bounds of $\lambda(n)$ for the specific cases of energy cost functions.

Lemma 3.6: In a homogeneous network with sensor traffic, if the energy cost function is $e(r) = c_a r^\alpha$, a transfer volume of

$$\lambda(n) = \begin{cases} \Omega(1) & \text{if } \alpha = 2; \\ \Omega((\frac{n}{\log n})^{\frac{\alpha}{2}-1}) & \text{if } \alpha > 2 \end{cases}$$

is feasible with high probability.

Proof: Let $f(x) = \frac{x^2 A}{e(x)}$. When $\alpha = 2$, $f(x)$ is a constant independent of x. So by Lemma 3.5, we have $\lambda(n) = \Omega(1)$. When $\alpha > 2$, $f(x)$ is a decreasing function of x. So we have $\lambda(n) = \Omega((\frac{n}{\log n})^{\frac{\alpha}{2}-1})$. ■

Lemma 3.7: In a homogeneous network with sensor traffic, if the energy cost function is $e(r) = c_a(c_b + r^\alpha)$, the feasible transfer volume is $\Omega(1)$ when $\alpha = 2$. If $\alpha > 2$, a transfer volume of

$$\lambda(n) = \begin{cases} \frac{c_8 A}{c_a} f((\frac{2c_b}{\alpha-2})^{\frac{1}{\alpha}}) & \text{if } c_4\sqrt{\frac{\log n}{\pi n}} \leq (\frac{2c_b}{\alpha-2})^{\frac{1}{\alpha}}; \\ \frac{c_8 A}{c_a} f(c_4\sqrt{\frac{\log n}{\pi n}}) & \text{if } c_4\sqrt{\frac{\log n}{\pi n}} > (\frac{2c_b}{\alpha-2})^{\frac{1}{\alpha}} \end{cases}$$

is feasible with high probability where $f(x) = \frac{x^2}{c_b + x^\alpha}$.

Proof: When $\alpha = 2$, $f(x)$ is an increasing function of x. So by Lemma 3.5, we have $\lambda(n) = \Omega(1)$.

We now consider the case of $\alpha > 2$. In this case, $f(x)$ is maximized when $x^* = (\frac{2c_b}{\alpha-2})^{\frac{1}{\alpha}}$. That is, the transfer volume is maximized when using the optimal radio range x^*. However, for a given n, the minimum transmission range used is $x' = c_4\sqrt{\frac{\log n}{\pi n}}$. Thus when x' is larger than the optimal range x^*, nodes can not use range x^*. In this case, the feasible transfer volume is $\frac{c_8 A}{c_a} f(x')$. When n is large enough such that $x' \leq x^*$, the maximum transfer volume is achieved which is $\frac{c_8 A}{c_a} f(x^*)$ by using the optimal range x^*. This completes the proof. ■

C. Scaling Properties

In this section, we discuss the scaling properties of the energy-limited capacity in homogeneous networks with sensor traffic. We focus on the class of networks where the energy cost function is $e(r) = c_a r^\alpha$ or $e(r) = c_a(c_b + r^\alpha)$. As we can see from previous results, the achievable lower bounds are within a factor of $O(\log^k n)$ from the corresponding upper bounds for some constant k. Thus we present only the results for the upper bounds here.

Based on our results, we classify these networks into two categories, depending on the scaling properties of the capacity. In the first category, the capacity of the network increases infinitely with the density of the network. These networks are said to have infinite scaling regions and include networks with $e(r) = c_a r^\alpha$. For these networks, minimum energy consumption is achieved by transmitting data using the shortest range possible, even at the cost of forwarding data in more hops. Thus the capacity benefits from the increase of node density.

In the second category, the capacity of the network can only increase finitely. These networks are said to have finite scaling regions and include networks with $e(r) = c_a(c_b + r^\alpha)$. The capacity of these networks is $O(1)$. However, this does not rule out the possibility that the feasible transfer volume can increase with node density. Intuitively, for these networks, there is an optimal transmission range which achieves minimum energy consumption. On the other hand, to ensure network connectivity, the transmission range must be large enough. So the transfer volume increases with the node density n when n is small, and saturates when n is large such that nodes can use the optimal transmission range while maintaining network connectivity.

As we can see in the next section, the scaling behavior of networks with $e(r) = c_a(c_b + r^\alpha)$ is similar to that of $e(r) = c_a r^\alpha$, but in a finite region. Thus we can apply the results for the case of $e(r) = c_a r^\alpha$, which is self-explained, to help understand the behavior of networks with $e(r) = c_a(c_b + r^\alpha)$.

D. Numerical Results

We now study in more detail the scaling properties of networks with $e(r) = c_a(c_b + r^\alpha)$ via numerical results. Specifically, we would like to answer the following questions.

1) What is the upper bound of energy-limited capacity? How does it relate to the parameters c_a, c_b and α?
2) How does the transfer volume evolve with the node density? In other words, what is the scaling region in which the capacity improves as the node density increases?

Our analytic results shows that the feasible transfer volume is inversely proportional to c_a. To quantify the effects of c_b and α, we provide numerical results of $\lambda(n)$ using Lemma 3.3. We choose $\phi_n = \ln n$ and set all constants in Lemma 3.3 to 1. The real upper bounds should be scaled by a constant factor. However, this does not affect the scaling behavior.

Fig. 3 shows the optimal transfer volume, i.e., $\lambda(n)$ when n is infinity, with different c_b and α. We can see that the feasible transfer volume increases as c_b decreases. This result suggests that reducing the constant energy consumption can lead to significant capacity improvement. Fig. 3 also shows that $\lambda(n)$ increases with α when c_b is small.

Fig. 4 shows how the transfer volume $\lambda(n)$ grows with the node density when α is 3. As the node density increases, $\lambda(n)$ also increases until saturating at the maximum value. We can see that with a smaller c_b, $\lambda(n)$ increases over a larger region of node density. Fig. 4 also shows that the scaling behavior of $e(r) = c_a(c_b + r^\alpha)$ is similar to that of $e(r) = c_a r^\alpha$ (the line with label "$c_b = 0$" in the figure) before $\lambda(n)$ reaches the

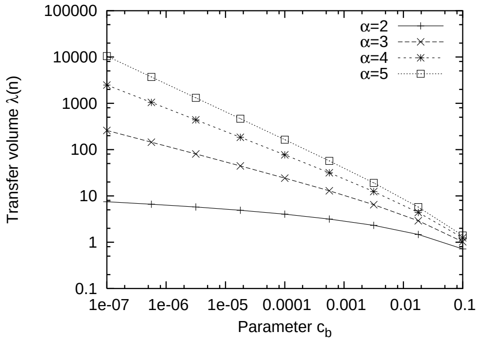

Fig. 3. Effects of parameters c_b and α on the transfer volume in homogeneous networks with sensor traffic.

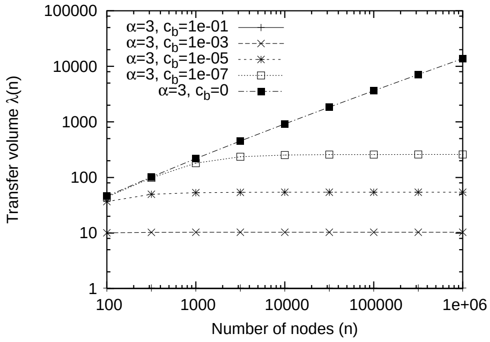

Fig. 4. Transfer volume vs. number of nodes in homogeneous networks with sensor traffic.

maximum capacity. We observe similar results with different values of α.

In summary, while the upper bound is $O(1)$ when $e(r) = c_a(c_b + r^{\alpha})$, the energy-limited capacity can scale with the node density, although within a finite region. In addition, reducing the constant energy both improves the network capacity and increases the scaling region.

IV. Energy-Limited Capacity of Hybrid Networks with Sensor Traffic

In this section, we consider the energy-limited capacity in hybrid networks where base stations (BSs) with unlimited energy are deployed to support data forwarding. We study the capacity gain obtained with the deployment of base stations. Since the base stations have unlimited energy, the capacity limitation lies in the nodes due to their energy constraints. Intuitively, by transmitting data via the BSs, the traffic load on the nodes is reduced, resulting in better energy-limited capacity. On the other hand, the use of the BSs may cause hot spots and overload nodes near the BSs, leading to sup-optimal transfer volume. Thus it is important to balance these two factors and make efficient use of the BSs.

In hybrid networks, data can be forwarded in the ad hoc mode where the BSs are not involved in data forwarding, or in the infrastructure mode. Node energy is shared by these two transmission modes. In this paper, we consider an energy allocation strategy where each node uses p fraction of its energy for the ad hoc mode and the remaining $(1-p)$ fraction for the infrastructure mode, $0 \leq p \leq 1$. In the infrastructure mode, nodes should transmit data to the closest BS, i.e., the BS in the same cell as the node, in order to conserve energy. In addition, data should enter and exit the BS layer once. Thus, routing in the infrastructure mode consists of three phases, from the sender to the closest BS, forwarding within the BS layer, and from the BS in the destination's cell to the destination. Node energy is only consumed in the first phase, i.e., forwarding data from the sender to the BS, possibly in multiple hops.

Let $\lambda_a(n)$ be the transfer volume of a network when all energy is used for the ad hoc mode. Similarly, define $\lambda_i(n, m)$ as the transfer volume if all energy is used for the infrastructure mode. Because the transfer volume is proportional to node energy, given any energy allocation with parameter p, the total transfer volume is

$$\Lambda(n, m) = p\lambda_a(n) + (1-p)\lambda_i(n, m). \tag{9}$$

Note that energy consumption in the ad hoc or infrastructure mode is isolated. So we can obtain an upper (lower) bound for $\Lambda(n, m)$ by summing up the upper (lower) bounds of $\lambda_a(n)$ and $\lambda_i(n, m)$.

A. Area Scaling

We first determine the transfer volume in both transmission modes, namely $\lambda_a(n)$ and $\lambda_i(n, m)$. In the ad hoc mode, nodes forward data without using any base station. We can obtain $\lambda_a(n)$ by applying the results of homogeneous networks in previous sections. In the infrastructure mode, all nodes in a cell transmit data to the BS within the cell. To compute $\lambda_i(n, m)$, we extend the analysis developed for homogeneous networks with sensor traffic in Section III. The difference is that $\lambda_i(n, m)$ considers only nodes within a cell, instead of the unit area in previous analysis.

For the upper bound of $\lambda_i(n, m)$, it is sufficient to consider a single cell. This is because, by definition, a transfer volume of $\lambda_i(n, m)$ requires every node to be able to transmit that amount of data to its destination. Let V be any cell. Because the BSs are placed regularly, the area of V is $\frac{1}{m}$. Since nodes are randomly distributed, the probability that a node is in V is $\frac{1}{m}$. By the Strong Law of Large Numbers, the number of nodes in V is $\frac{n}{m}$ with probability 1 as $n \to \infty$. To determine $\lambda_i(n, m)$, we adapt the proof in Lemma 3.1 to account for the fact that nodes are within a cell[6]. Specifically, the claim that no node lies within a distance of $\frac{1}{\phi_n \sqrt{n}}$ to the BS with high probability remains true. The probability density function of a node's distance to the BS becomes $P\{d_i = x\} = 2\pi m x$ by conditioning on the fact that the node is within the cell. So

[6]We approximate a hexagon cell as a disk of the same area to apply the analysis in Section III. We argue that this would not impact our results. Specifically, if the energy cost function $e(r)$ satisfies $e(c'r) \leq c''e(r)$ for some constants c', c'' and $0 \leq r \leq 1/\sqrt{\pi}$, which is the case in practice, it can be shown that the results obtained by approximation differ only by a constant factor.

the upper bound $\lambda_i^u(n)$ of the feasible transfer volume is

$$\lambda_i^u(n,m) = c_9 m \int_{\frac{1}{\phi_n \sqrt{n}}}^{\frac{1}{\sqrt{m\pi}}} \frac{2\pi A x}{e(x)}\, dx. \tag{10}$$

For the lower bound of $\lambda_i(n,m)$, we need to prove that nodes in every cell can transmit enough data to the sink. To bound the number of nodes in each cell, we assume that $m \leq \frac{n}{200 \log n}$. So the area of each cell is at least $\frac{200 \log n}{n}$. Using the same techniques as in Lemma 3.4, the number of nodes in every cell can be shown to be $\Theta(\frac{n}{m})$. So to ensure connectivity among node within a cell, the transmission range r_n must be at least of order $\sqrt{\frac{\log(n/m)}{\pi n}}$. Following the same arguments in the proof of Lemma 3.5, we can prove a lower bound $\lambda_i^l(n,m)$ of the feasible transfer volume below.

$$\lambda_i^l(n,m) = \max\{\frac{c_{10} m x^2 A}{e(x)} : c_{11}\sqrt{\frac{\log(n/m)}{\pi n}} \leq x \leq \frac{1}{\sqrt{m\pi}}\}. \tag{11}$$

B. Upper Bound on Energy-Limited Capacity

We now study the capacity of hybrid networks with sensor traffic. For this class of networks, transmissions in the ad hoc mode and the infrastructure mode are the same except at different scales, i.e., in the unit disk or in a cell. In the following we present the upper bounds on the energy-limited capacity. We obtain the lower bounds using similar techniques, which we omit here due to space limits. In general, the achievable lower bounds are within a factor of $O(\log^k n)$ from the corresponding upper bounds for some constant k.

Given the energy allocation parameter p, we can obtain the total transfer volume by combining $\lambda_a(n)$ and $\lambda_i(n,m)$. We now derive upper bounds for the specific cases of energy cost functions.

Case 1. $e(r) = c_a r^\alpha$.

Noting that $\lim_{n\to\infty} \frac{m}{n} = 0$, we derive $\lambda_i^u(n,m)$ from (10).

$$\lambda_i^u(n,m) = \begin{cases} O(m \log n) & \text{if } \alpha = 2; \\ O(m n^{\frac{\alpha}{2}-1} \phi_n^{\alpha-2}) & \text{if } \alpha > 2. \end{cases} \tag{12}$$

When $\alpha > 2$, the infrastructure mode transfer volume $\lambda_i^u(n,m)$ is $O(mn^{\frac{\alpha}{2}-1}\phi_n^{\alpha-2})$ by (12). And the ad hoc mode transfer volume $\lambda_a(n)$ is $O(n^{\frac{\alpha}{2}-1}\phi_n^{\alpha-2})$. $\lambda_i^u(n,m)$ dominates the total transfer volume. So $\Lambda(n,m)$ is maximized if $p \to 0$, i.e., traffic should be forwarded via the BSs. In this case, the transfer volume scales with the number of BSs. Similarly, when $\alpha = 2$, the total capacity is $O(m \log n)$ as compared to the $O(\log n)$ capacity in networks without BSs. So the capacity increases as m increases.

In both cases, the deployment of BSs in networks with sensor traffic has immediate improvement on the network capacity. The capacity gain scales linearly with the number of BSs. This suggests that BS deployment in sensor networks is a promising approach to improve capacity.

Case 2. $e(r) = c_a(c_b + r^\alpha)$.

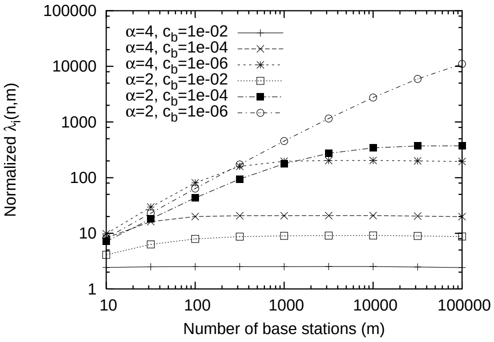

Fig. 5. Normalized $\lambda_i(n,m)$ vs. number of base stations in hybrid networks with sensor traffic.

We first determine $\lambda_i^u(n,m)$ by (10).

$$\lambda_i^u(n,m) = \begin{cases} \frac{c_{12} m A}{2 c_a} \ln \frac{c_b + \frac{1}{m\pi}}{c_b + \frac{1}{\phi_n^2 n}} & \text{if } \alpha = 2; \\ \frac{c_{12} m A}{c_a} \int_{\frac{1}{\phi_n \sqrt{n}}}^{\frac{1}{\sqrt{m\pi}}} \frac{x\, dx}{c_b + x^\alpha} & \text{if } \alpha > 2. \end{cases} \tag{13}$$

It is easy to see that $\lambda_i^u(n,m)$ is $O(1)$ regardless of α and m. This is because for any transmission, the minimum energy required is $c_a c_b$. Given energy supply A, each node can transmit no more than $\frac{A}{c_a c_b}$ units of data. Therefore, this class of networks has finite scaling regions.

We now study the scaling properties of networks with $e(r) = c_a(c_b + r^\alpha)$ by numerical results. In the calculation, we set all constants to 1. Fig. 5 shows how the infrastructure mode capacity grows with the number of BSs when n is ∞. We normalize $\lambda_i^u(n,m)$ to the ad hoc mode transfer volume $\lambda_a(n)$. We can see that the normalized $\lambda_i^u(n,m)$ is always greater than 1, suggesting that $\lambda_i^u(n,m)$ always dominates the total transfer volume. $\lambda_i^u(n,m)$ increases with the number of BSs until it saturates. In addition, the maximum normalized $\lambda_i^u(n,m)$ increases with smaller c_b and smaller α. So the capacity gain is more significant when c_b and α is small.

Fig. 6 shows how the total transfer volume grows with the number of nodes when c_b is 10^{-5}. We compare $\Lambda(n,m)$ when the number of BSs m is 0, $\ln n$, $\sqrt{n}$ and $\frac{n}{\ln n}$. Specifically, we calculate $\lambda_a(n)$ using (6) and $\lambda_i(n,m)$ using (13), and set $\Lambda(n,m)$ as the larger of these two. We can see that these networks have finite scaling regions, i.e., the transfer volume saturates when n is large enough. Fig. 6 also confirms that the total transfer volume increases with the number of BSs, even the number of BSs is relatively small, e.g., $\ln n$.

V. Energy-Limited Capacity of Networks with Ad Hoc Traffic

In this section, we briefly present the energy-limited capacity of networks with ad hoc traffic. In these networks, nodes communicate with randomly chosen nodes, instead of a common sink. We will present the upper bounds of the energy-limited capacity. We also obtain the lower bounds which are within a factor of $O(\log^k n)$ from the corresponding upper bounds. Please refer to [24] for more details.

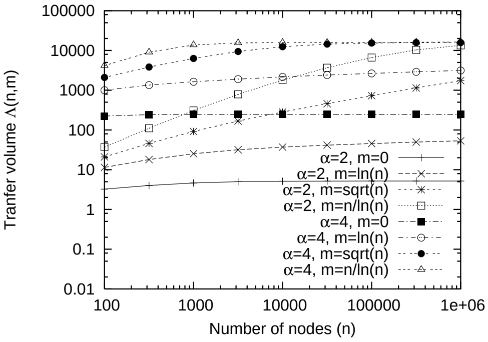

Fig. 6. Transfer volume $\Lambda(n, m)$ vs. number of nodes in hybrid networks with sensor traffic.

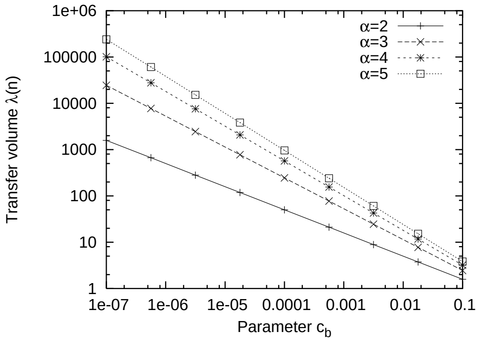

Fig. 7. Effect of parameter c_b on transfer volume in homogeneous networks with ad hoc traffic.

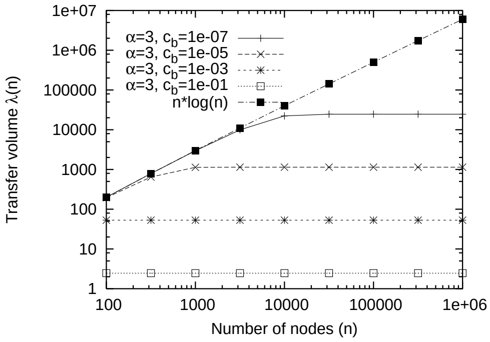

Fig. 8. Transfer volume vs. number of nodes in homogeneous networks with ad hoc traffic.

A. Homogeneous Networks

For homogeneous networks, we extend the techniques used in [17] to obtain the upper bound on the energy-limited capacity. The work in [17] studies the throughput capacity of networks where bandwidth is infinite but the maximum transmit power of each node is limited. Under these assumptions, the authors show that the interference is negligible, which renders the transmit power of nodes as the main constraint. This is similar to our analysis of energy-limited capacity where the transfer volume is limited by node energy supplies. We obtain the following results for the specific cases of energy cost functions.

Lemma 5.1: In a homogeneous network with ad hoc traffic, if the energy cost function is $e(r) = c_a r^\alpha$, the feasible transfer volume $\lambda(n)$ satisfies

$$\lambda(n) = O(\frac{A}{c_a}(n \log n)^{\frac{\alpha-1}{2}}) \tag{14}$$

with high probability.

Lemma 5.2: In a homogeneous network with ad hoc traffic, if the energy cost function is $e(r) = c_a(c_b + r^\alpha)$, the feasible transfer volume $\lambda(n)$ satisfies

$$\lambda(n) \le \begin{cases} \frac{c_{13}A}{c_a f(k^*)} & \text{if } \sqrt{n \log n} \ge \frac{k^*}{c_{14}}; \\ \frac{c_{13}A}{c_a f(c_{14}\sqrt{n \log n})} & \text{if } \sqrt{n \log n} < \frac{k^*}{c_{14}} \end{cases} \tag{15}$$

where $k^* = (\frac{\alpha-1}{c_b})^{\frac{1}{\alpha}}$ and $f(x) = c_b x + \frac{1}{x^{\alpha-1}}$ for some constants c_{13} and c_{14}.

We can see that the energy-limited capacity can scale well in homogeneous networks with ad hoc traffic. For example, when $e(r) = c_a r^\alpha$, the transfer volume scales as $O((n \log n)^{\frac{\alpha-1}{2}})$. When $e(r) = c_a(c_b + r^\alpha)$, the transfer volume scales with the number of nodes, but within a finite region, which parallels the results for networks with sensor traffic. Comparing networks with ad hoc and sensor traffic, we can see that the capacity of networks supporting sensor traffic is less than the capacity of networks supporting ad hoc traffic, by a factor of $O(\sqrt{n} \log^c n)$. This capacity penalty is due to the effect of traffic aggregation near the sink in networks with sensor traffic.

Now we provide some numerical results for the case where $e(r) = c_a(c_b + r^\alpha)$. We set all constants to 1. Fig. 7 depicts how the transfer volume $\lambda(n)$ changes with c_b. We can see that $\lambda(n)$ increases as c_b decreases. So reducing the constant energy consumption can increase the capacity of the network. In addition, $\lambda(n)$ increases with α. This behavior is similar to that of networks with sensor traffic.

Fig. 8 shows how the transfer volume $\lambda(n)$ grows with the node density when $\alpha = 3$. We can see that $\lambda(n)$ can increases with node density, as in networks with sensor traffic. In addition, with a smaller c_b, the transfer volume increases over a larger region of node density. Fig. 8 also shows that the scaling behavior for the case of $e(r) = c_a(c_b + r^\alpha)$ is similar to that of $e(r) = c_a r^\alpha$, which scales as $O(n \log n)$, before $\lambda(n)$ reaches the maximum transfer volume.

In summary, while the upper bound is $O(1)$ when $e(r) = c_a(c_b + r^\alpha)$, the energy-limited capacity can scale with the node density, although within a finite region. In addition, reducing the constant energy both improves the network capacity and increases the scaling region.

B. Hybrid Networks

We now study hybrid networks with ad hoc traffic. In these networks, base stations with unlimited energy are deployed to help data forwarding and data can be routed via the ad hoc mode or the infrastructure mode. As in Section IV, we obtain the total transfer volume by combining the transfer volume contributed by the ad hoc and infrastructure modes. In the following, we focus on the specific cases of energy cost functions.

Case 1. $e(r) = c_a r^\alpha$.

When $\alpha > 2$, by (14) and (12), the total transfer volume is

$$\Lambda(n,m) = pO((n\log n)^{\frac{\alpha-1}{2}}) + (1-p)O(mn^{\frac{\alpha}{2}-1}\phi_n^{\alpha-2}).$$

If $m = o(\sqrt{n}(\log n)^{\frac{\alpha-1}{2}})$, the ad hoc mode transfer volume $\lambda_a(n)$ dominates $\Lambda(n,m)$. So $\Lambda(n,m)$ is maximized if $p \to 1$. In other words, traffic should be forwarded using the ad hoc mode. Therefore, there is no significant performance gain when m is small, i.e., $m = o(\sqrt{n}(\log n)^{\frac{\alpha-1}{2}})$. If $m = \omega(\sqrt{n}(\log n)^{\frac{\alpha-1}{2}})$, the infrastructure mode transfer volume will be the dominating term in $\Lambda(n,m)$. So $\Lambda(n,m)$ is maximized if $p \to 0$, i.e., traffic should be routed via the BS layer. In this case, the transfer volume increases linearly with m.

Similarly, when $\alpha = 2$, we have

$$\Lambda(n,m) = pO(\sqrt{n\log n}) + (1-p)O(m\log n).$$

So the total transfer volume is $O(\sqrt{n\log n})$ if $m = o(\sqrt{\frac{n}{\log n}})$ and $O(m\log n)$ if $m = \omega(\sqrt{\frac{n}{\log n}})$.

In summary, when the energy cost function is $e(r) = c_a r^\alpha$, the capacity gain due to the deployment of BSs is bimodal. Specifically, when the number of BSs grows asymptotically slower than some threshold which is of the order $\sqrt{n}\log^k n$ for some constant k, the total capacity is dominated by the ad hoc mode capacity. Thus the improvement due to the deployment of BSs is insignificant. When the number of BSs grows faster than that threshold, the total capacity is dominated by the infrastructure mode capacity which increases linearly with the number of BSs. This suggests that to achieve non-negligible capacity gain, the number of BSs deployed should be large enough.

Case 2. $e(r) = c_a(c_b + r^\alpha)$.

Similarly, we can obtain the total transfer volume by combining the transfer volume contributed by the ad hoc and infrastructure modes using (15) and (13).

VI. Related Work

In this section, we review some related work in throughput capacity and energy conservation. Previous work on throughput capacity studies the performance limits caused by the interference. In this paper, we focus on the capacity limitations resulting from the energy supplies in nodes. So our work complements previous research on throughput capacity.

In [10], Gupta and Kumar study the throughput capacity of static random networks. They show that the per node throughput decreases as $\Theta(\frac{1}{\sqrt{n\log n}})$ as the number of nodes n increases. This implies that large scale wireless networks may have performance problems for non-local traffic patterns. To address this performance limitation, different approaches have been proposed. Grossglauser and Tse [8] show that node mobility can dramatically improve network capacity and achieve $O(1)$ throughput under loose delay constraints. The work in [15] and [16] considers hybrid networks where base stations connected with high-bandwidth wired networks are deployed to support data forwarding in the wireless network. The results in [16] show that the performance gain due to BS deployment is bimodal, which is similar to our results in energy-limited capacity.

There is also work in the capacity of sensor networks. In [6], Duarte-Melo and Liu study the throughput capacity of sensor networks and present the conditions under which the trivial upper bound $\frac{1}{n}$ can not be achieved.

In [17], Negi and Rajeswaran study the throughput capacity in networks where each node is constrained to a maximum transmit power but can utilize unlimited bandwidth. Under these assumptions, the interference is negligible, which renders the transmit power of nodes as the main constraint. The authors show that throughput capacity scales well under this setting. This is similar to our analysis of energy-limited capacity. In this paper, we use the method in [17] in our analysis. There is also other work in the capacity of wireless networks [22], [14], [12], [7].

In wireless environments, nodes are often equipped with limited energy. Thus energy management is important. The work in [21] and [3] considers energy-conserving routing in ad hoc networks. To achieve optimal performance, it is also important to reduce idle energy consumption which is shown to dominate the overall energy consumption if nodes are idle most of the time. The work in [4] and [23] considers sensor networks and exploits the redundancy in nodes to minimize idle energy while maintaining network connectivity. The work in [20] proposes a scheme to further reduce idle energy consumption by aggressively putting nodes into sleep mode, thus trading off latency for energy saving. There is also work on the network lifetime of sensor networks. In [2], Bhardwaj and Chandrakasan derive upper bounds on the lifetime of sensor networks that transmit data from a point, a line or an area source. In [5], the authors examine the energy consumption and lifetime of hybrid sensor networks based on a clustering mechanism and study the optimal number of clusters used. While the above studies consider energy consumption and network lifetime in sensor network, our work is significantly different. We focus on the scaling properties of the energy-limited capacity, which is important in large scale networks, and formally derive upper and lower bounds on the capacity. In addition we analyze the capacity in both homogeneous and hybrid networks, with ad hoc and sensor traffic models.

VII. Conclusion

In this paper, we studied the energy-limited capacity in wireless networks. We derived upper and lower bounds for homogeneous and hybrid networks, under both ad hoc and sensor traffic models. We showed that the energy-limited capacity can scale well in all these environments. This implies that large scale wireless networks can be scalable in terms of energy consumption, despite the decrease of throughput capacity. Table I summarizes the results for networks with energy cost function $e(r) = c_a r^\alpha$. The scaling behavior of networks with energy cost function $e(r) = c_a(c_b + r^\alpha)$ is

α	Capacity Bound	Homogeneous network with ad hoc traffic	Hybrid network with ad hoc traffic	Homogeneous network with sensor traffic	Hybrid network with sensor traffic
$\alpha = 2$	Upper	$O(\sqrt{n \log n})$	$O(\sqrt{n \log n})$ if $m = o(\sqrt{\frac{n}{\log n}})$ $O(m \log n)$ if $m = \omega(\sqrt{\frac{n}{\log n}})$	$O(\log n)$	$O(m \log n)$
	Lower	$\Omega(\sqrt{\frac{n}{\log n}})$	$\Omega(\sqrt{\frac{n}{\log n}})$ if $m = o(\sqrt{\frac{n}{\log n}})$ $\Omega(m)$ if $m = \omega(\sqrt{\frac{n}{\log n}})$	$\Omega(1)$	$\Omega(m)$
$\alpha > 2$	Upper	$O((n \log n)^{\frac{\alpha-1}{2}})$	$O((n \log n)^{\frac{\alpha-1}{2}})$ if $m = o(\sqrt{n}(\log n)^{\frac{\alpha-1}{2}})$ $O(mn^{\frac{\alpha}{2}-1}\phi_n^{\alpha-2})$ if $m = \omega(\sqrt{n}(\log n)^{\frac{\alpha-1}{2}})$	$O(\phi_n^{\alpha-2} n^{\frac{\alpha}{2}-1})$	$O(mn^{\frac{\alpha}{2}-1}\phi_n^{\alpha-2})$
	Lower	$\Omega((\frac{n}{\log n})^{\frac{\alpha-1}{2}})$	$\Omega((\frac{n}{\log n})^{\frac{\alpha-1}{2}})$ if $m = o(\sqrt{\frac{n}{\log n}})$ $\Omega(m(\frac{n}{\log n})^{\frac{\alpha}{2}-1})$ if $m = \omega(\sqrt{\frac{n}{\log n}})$	$\Omega((\frac{n}{\log n})^{\frac{\alpha}{2}-1})$	$\Omega(m(\frac{n}{\log n})^{\frac{\alpha}{2}-1})$

TABLE I

ENERGY-LIMITED CAPACITY OF NETWORKS WITH $e(r) = c_a r^\alpha$. ϕ_n IS A SEQUENCE THAT SATISFIES $\lim_{n\to\infty} \phi_n \to \infty$.

similar, but within a finite region. We found that reducing c_b can both improve the capacity and increase the scaling region. Thus reducing the constant energy consumption is key to desirable scaling. In addition, the capacity of networks supporting sensor traffic is less than the capacity of networks supporting ad hoc traffic, by a factor of $O(\sqrt{n}\log^c n)$ for some constant c. This is because of the effects of traffic concentration at the sink.

We also analyzed the capacity gain due to the deployment of base stations in hybrid networks. For ad hoc traffic, we showed that the capacity gain is bimodal. Specifically, when $e(r) = c_a r^\alpha$, to achieve non-negligible capacity gain, the number of base stations needs to grow faster than $\sqrt{n}\log^k n$. So a minimum investment in base stations is required to improve capacity in this case. This parallels the result in throughput capacity [16]. For sensor traffic, however, the deployment of base stations has immediate improvement on the network capacity. This suggests that hybrid deployment would be a promising approach to extend the lifetime of sensor networks.

In this paper, we obtained lower bounds that are a factor of $O(\log^k n)$ from the corresponding upper bounds. This indicates effective scaling of the capacity is achievable. In the future, we would like to extend this study to consider other kinds of energy consumption in the network. For example, it would be interesting to investigate the impact of idle energy consumption and energy conservation schemes on the network capacity. In addition, we would like to design decentralized routing algorithms to achieve the energy-limited capacity.

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