Complete Hierarchical Cut-Clustering: A Case Study on Modularity and Expansion

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Institute of Theoretical Informatics, Prof. Dr. Dorothea Wagner

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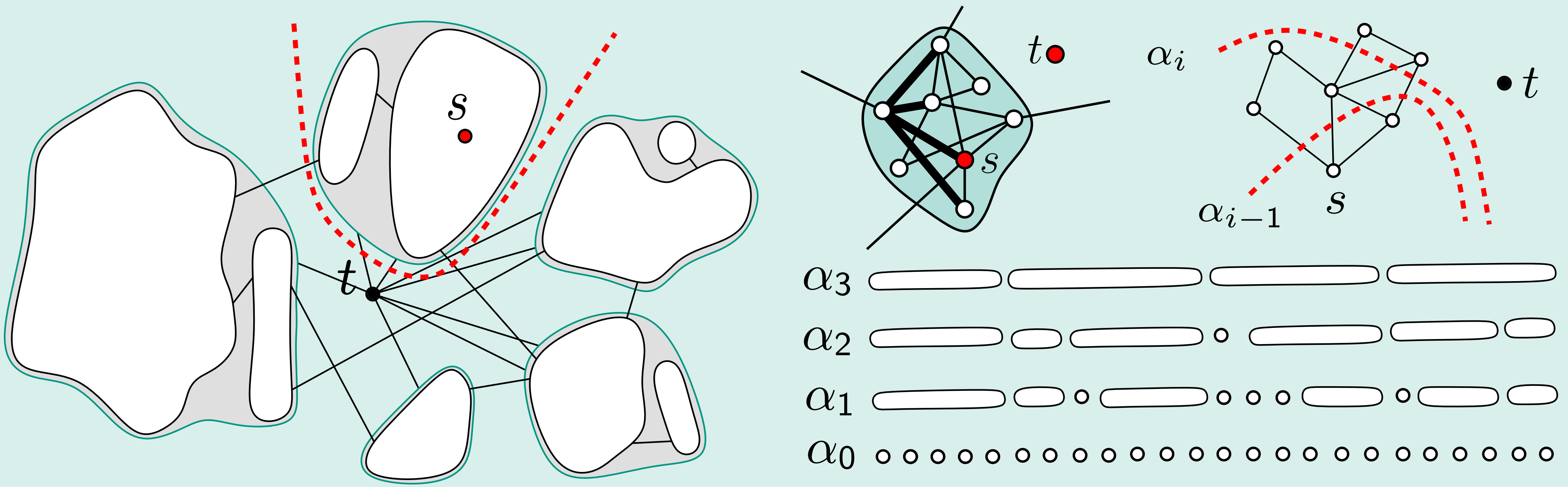
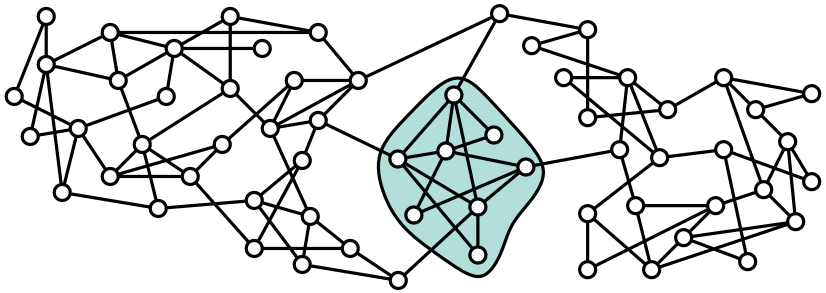


Introduction



The informal aim of graph clustering:

- (weighted) graph with a particular edge structure
- identify subsets of vertices that are significantly related
- weak external relations $\rightarrow$ more significance

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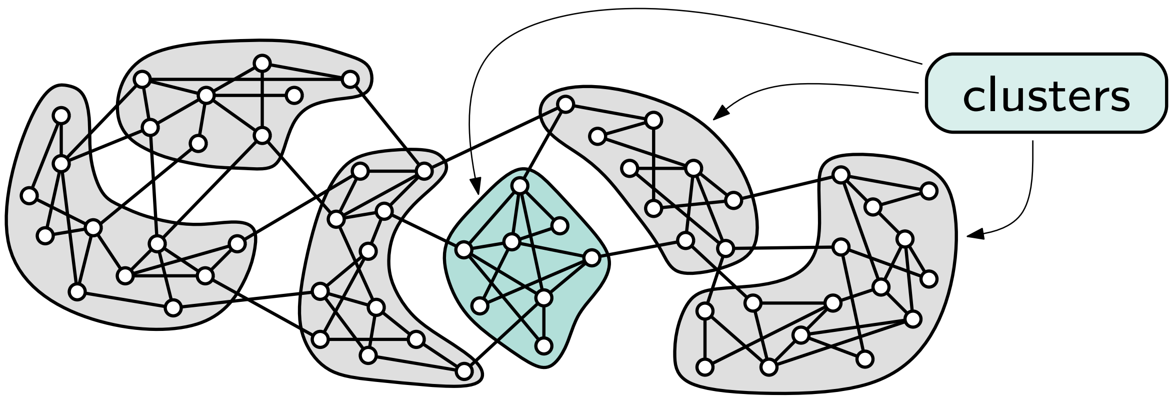

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induced subgraphs are called clusters

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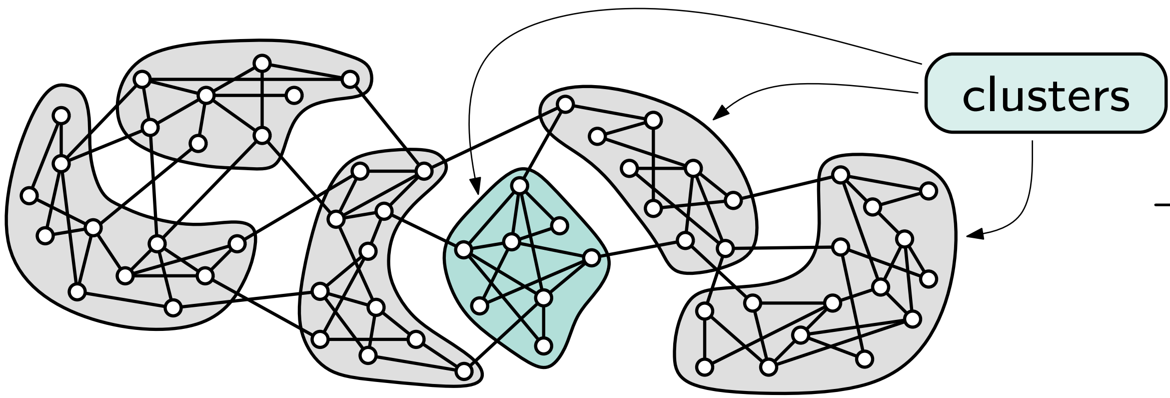

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significant relations
$\rightarrow$ many measures for quality
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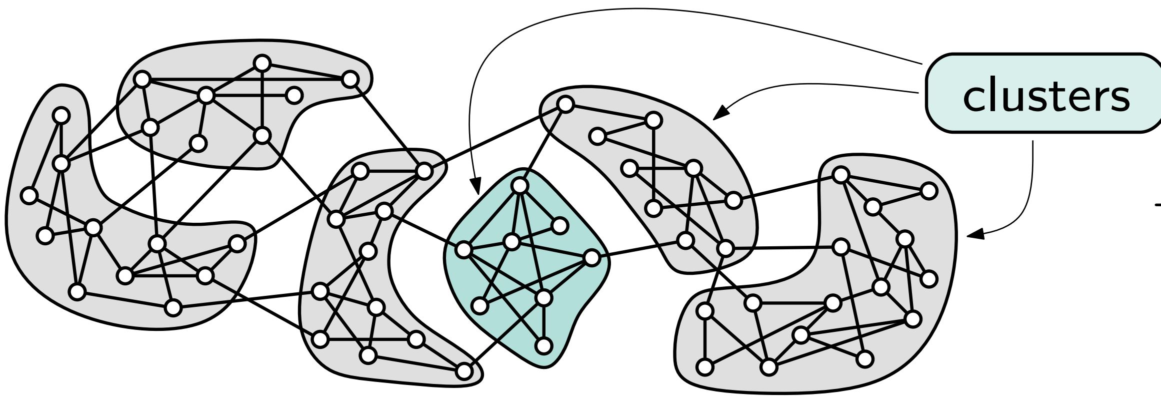

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Modularity

- widely used
- expresses significance of a clustering $\mathcal{C}$ compared to a random clustering

$$\mathcal{M}(\mathcal{C}) := \underbrace{\sum_{C\in\mathcal{C}} \frac{c(E_C)}{c(E)}}_{cov(\mathcal{C})} - \underbrace{\sum_{C\in\mathcal{C}} \frac{\left(\sum_{v\in C} d_c(v)\right)^2}{4c(E)^2}}_{\mathbb{E}(cov(\mathcal{C}))}$$

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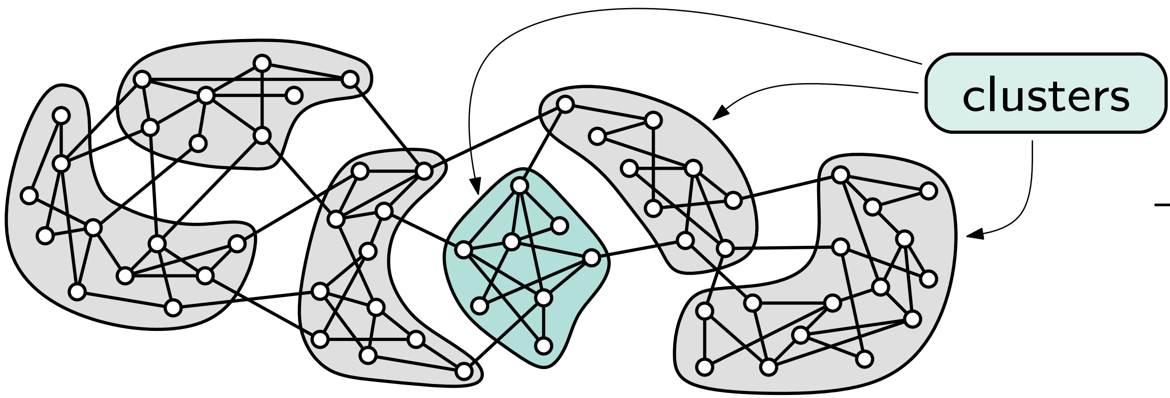

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Cut-Clustering by Flake et al. [2004]

- exploits properties of minimum s-t-cuts
- guarantees a lower bound on intra-cluster expansion (NP-hard)

$$\Psi(\mathcal{C}) := \min_{C\in\mathcal{C}}\{\min_{S\subset C} \frac{c(S,\overline{S})}{\min\{|S|,|\overline{S}|\}}\}$$

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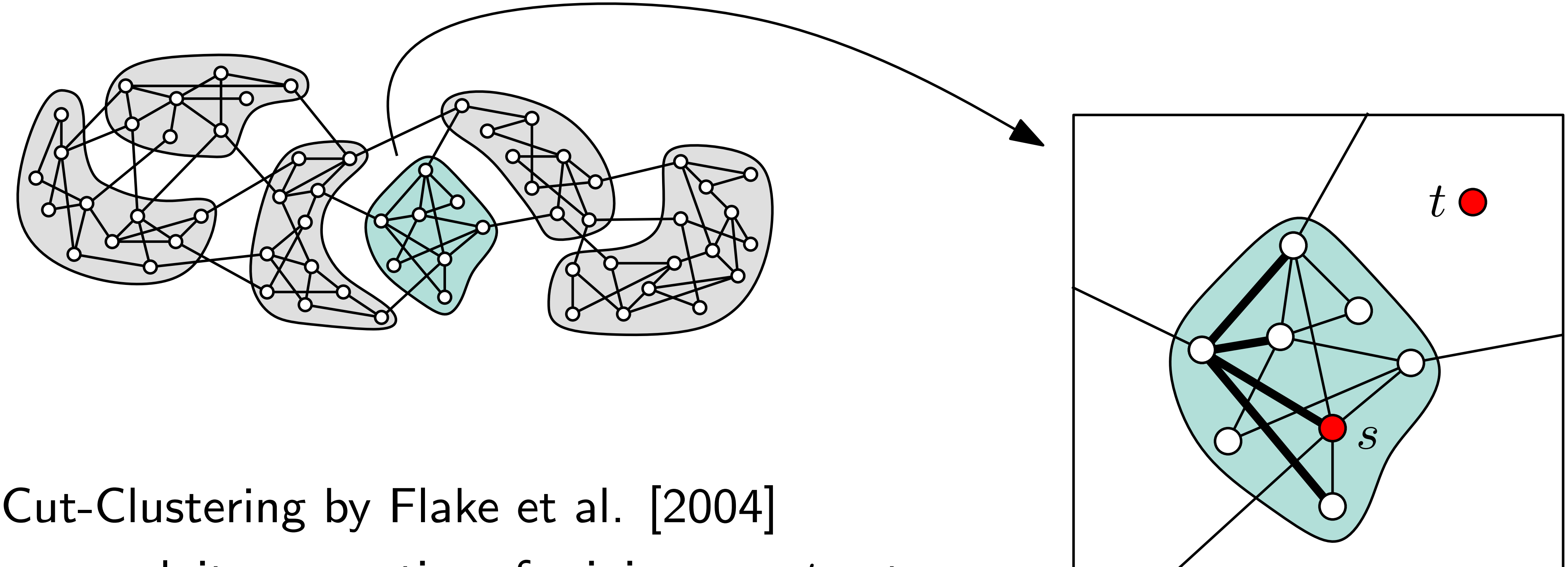

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- significance due to clearly indicated membership of vertices to clusters
  - for all $U \subset C \setminus \{s\}$: $c(U,C) \geq c(U, V \setminus C)$
    $\rightarrow$ *source-community* of s

Outline

- The Cut-Clustering Algorithm
  - depends on a parameter steering the coarseness
  - clusterings are hierarchically nested for different parameter values
  - parameter constitutes guarantee on intra-cluster expansion
- Parametric Search Approach
  - returns a complete hierarchy of cut-clusterings
  - simple and efficient (brief running time experiment)
- Modularity Analysis of Cut-Clustering Algotithm
  - compared to a modularity-based greedy approach
- Expansion Analysis of Cut-Clustering Algorithm
  - comparing the guarantee to trivial and non-trivial bounds

The Cut-Clustering Algorithm

- Flake et al. [2004]
- artificial vertex, artificial edges
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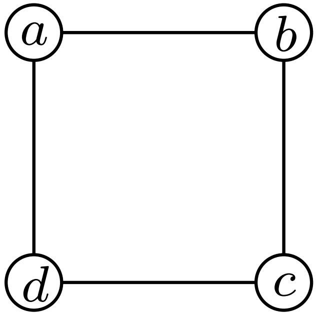

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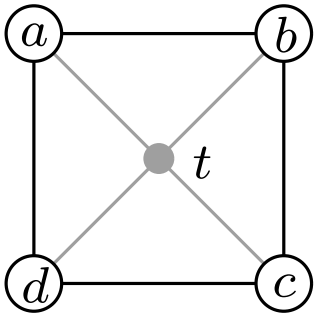

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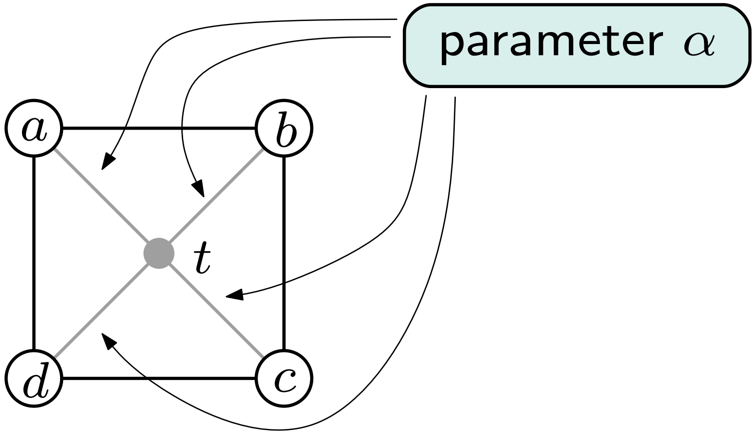

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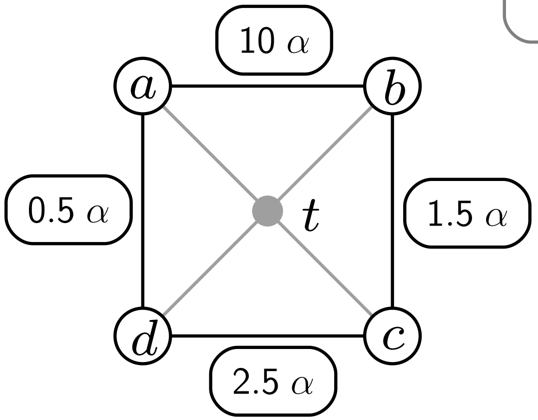

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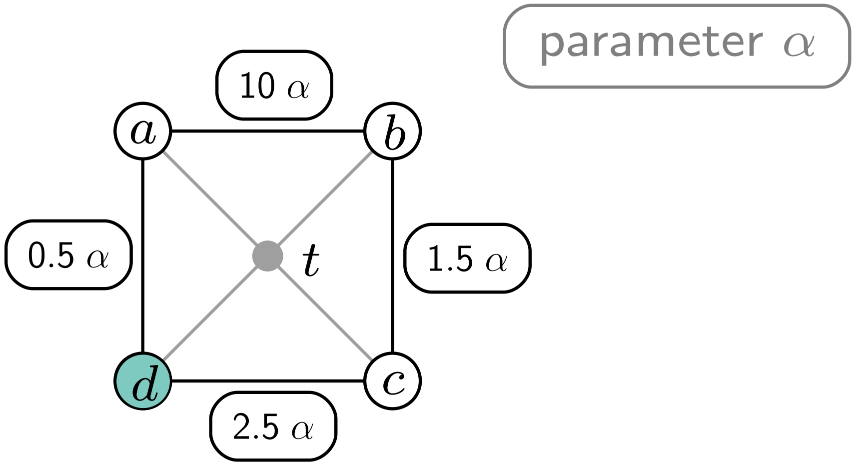

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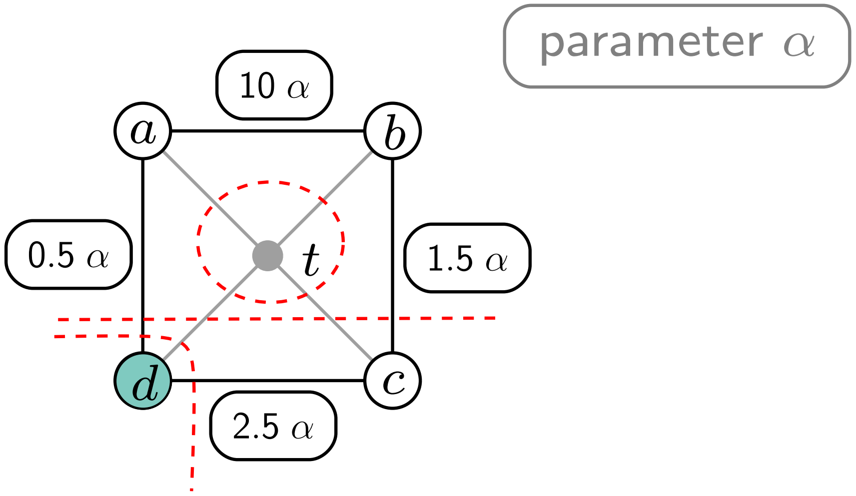

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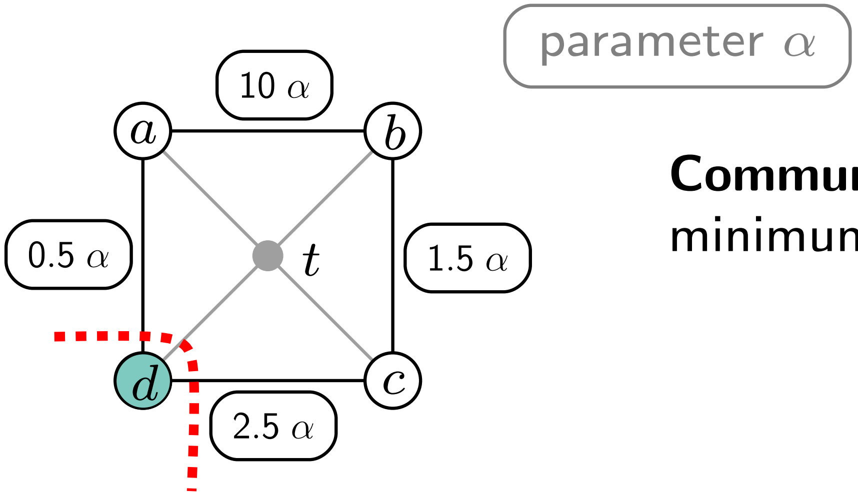

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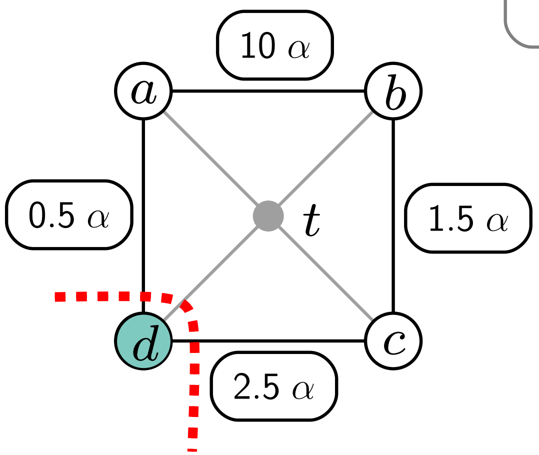

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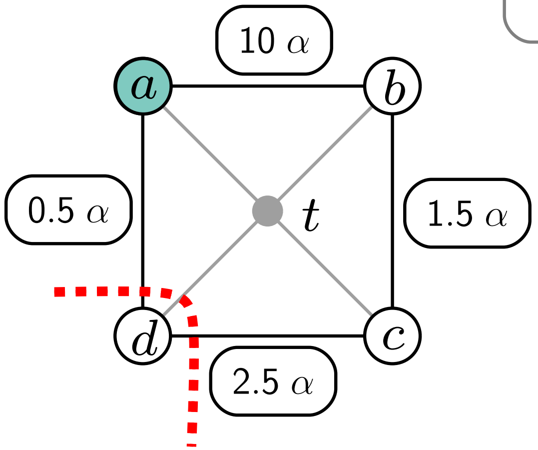

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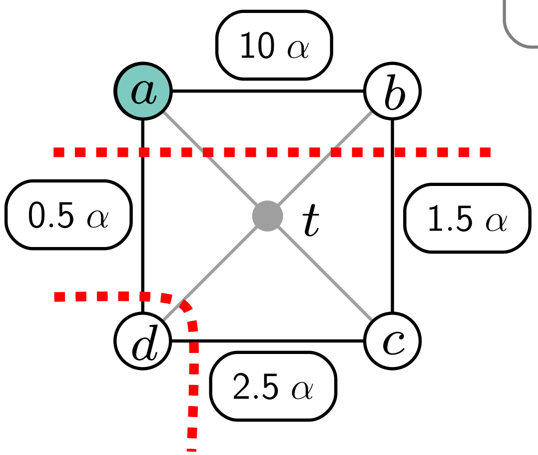

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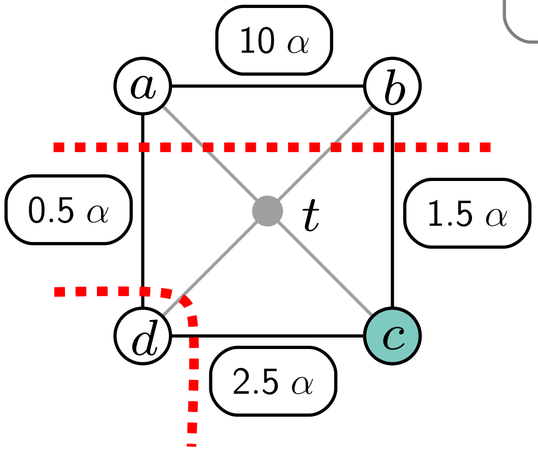

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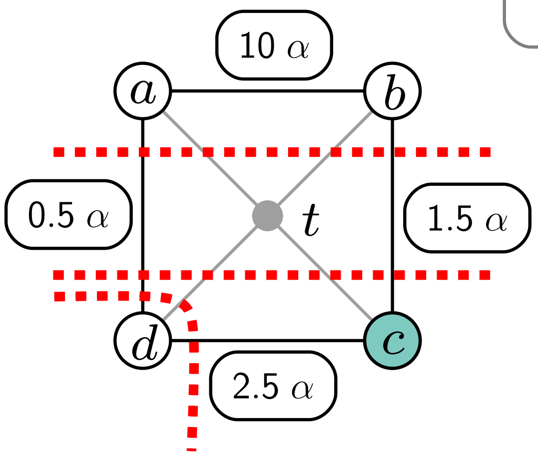

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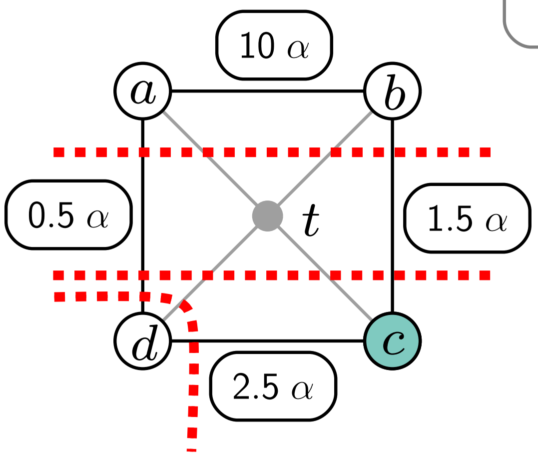

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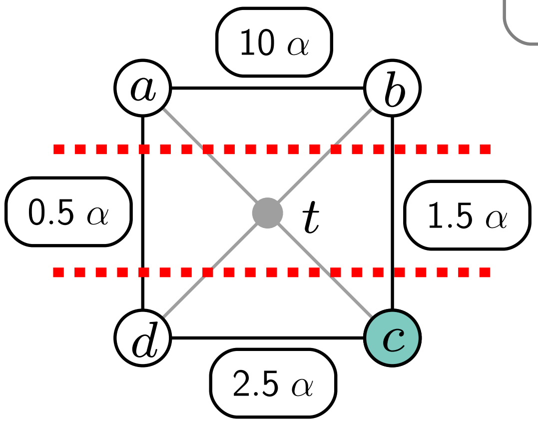

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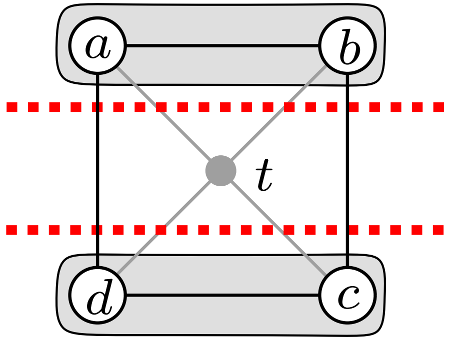

Community Cut:
minimum s-t-cut with source side of minimum size
$\longrightarrow$ Cut-Clustering with two clusters

The Cut-Clustering Algorithm

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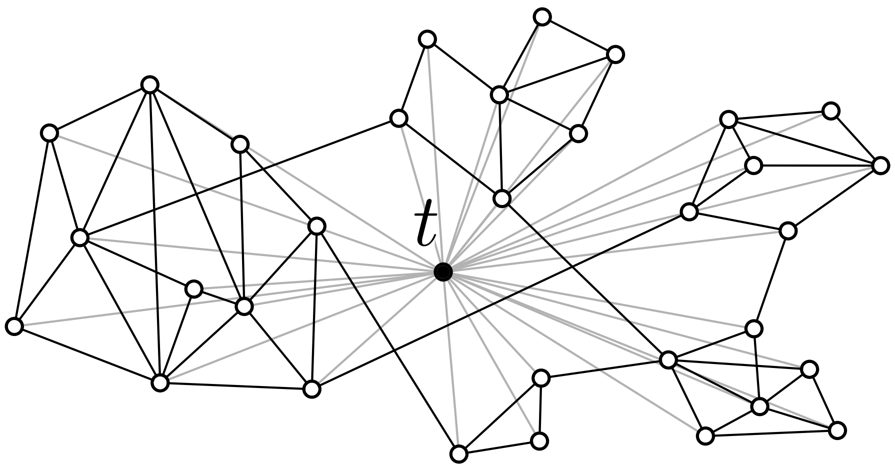
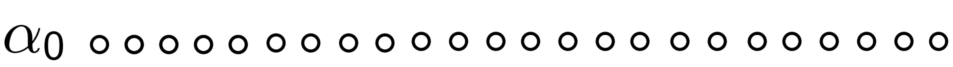
- $\alpha_0 = 0$, singleton clustering $\mathcal{C}_0$





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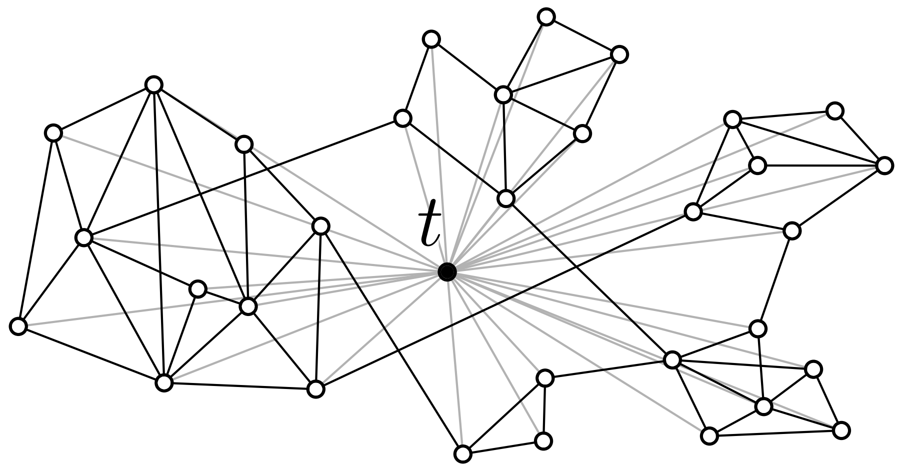

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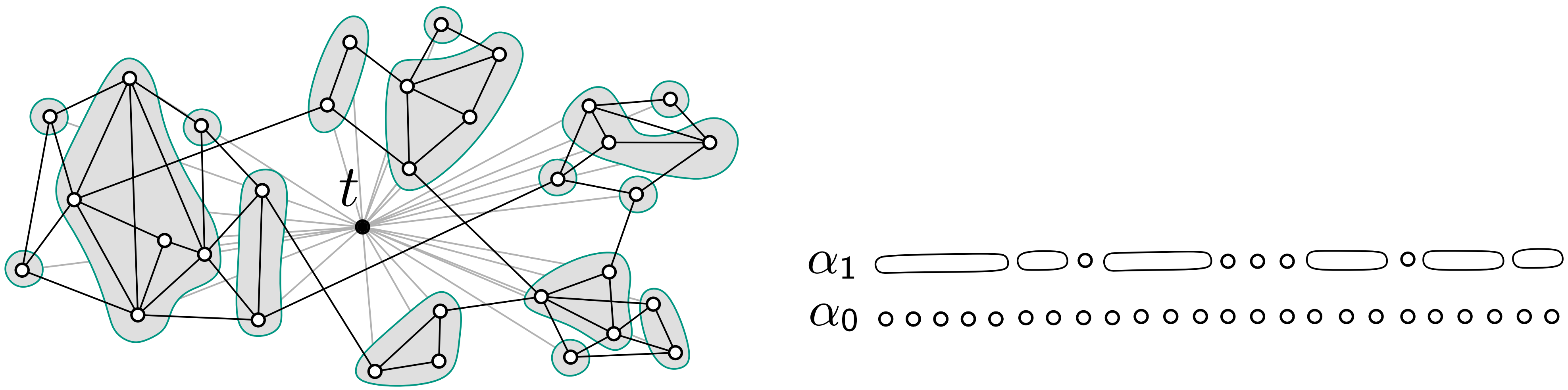

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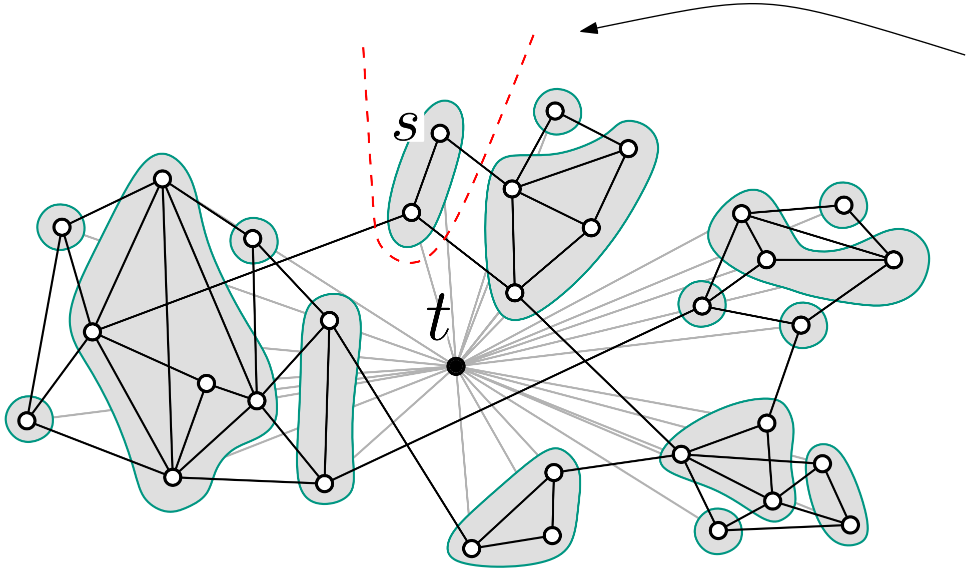
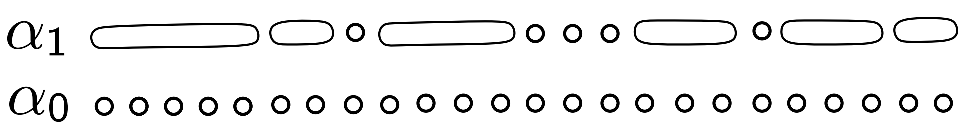


Source Community of s in **underlying** graph



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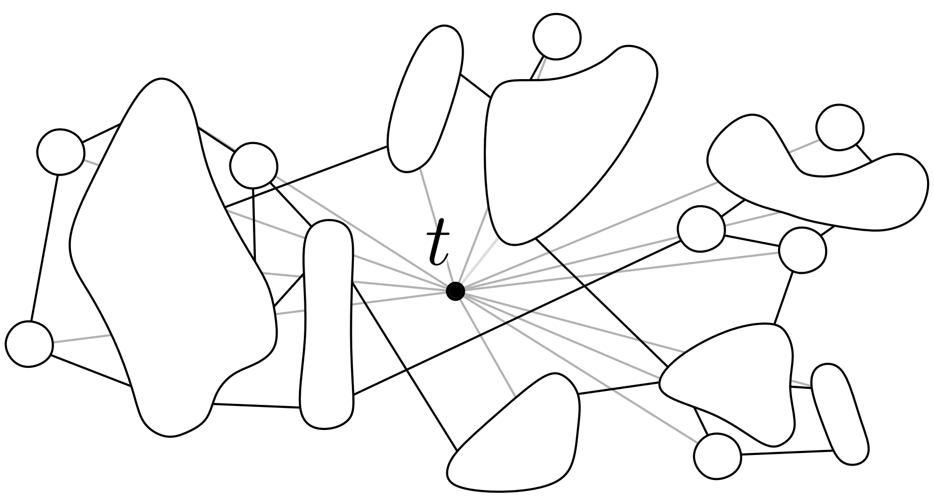

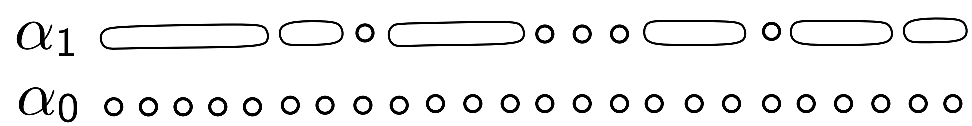

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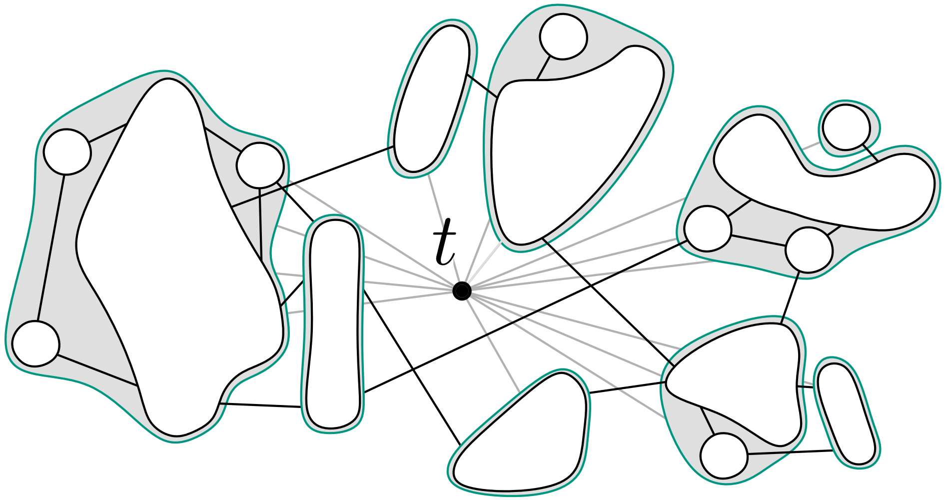

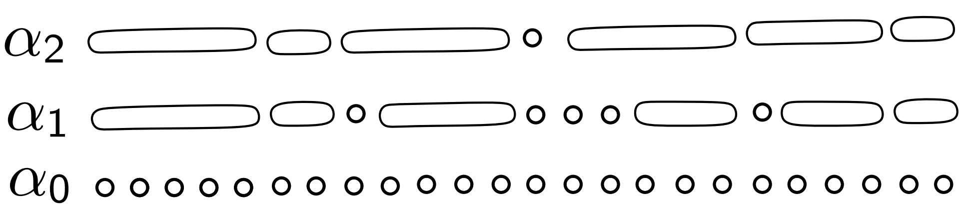

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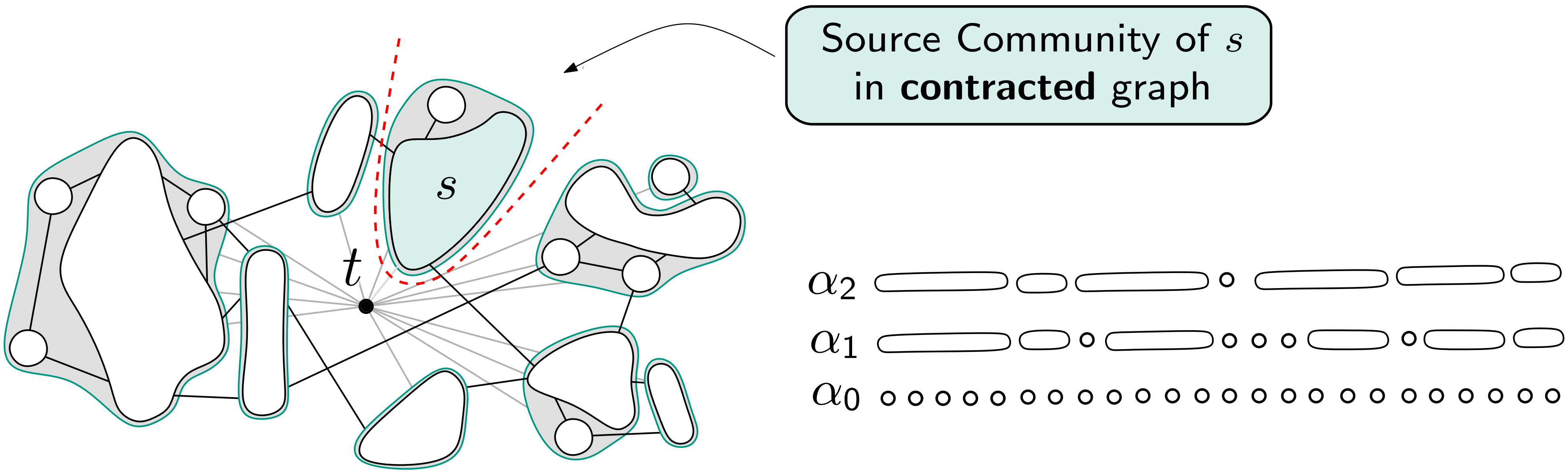

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community-cuts are
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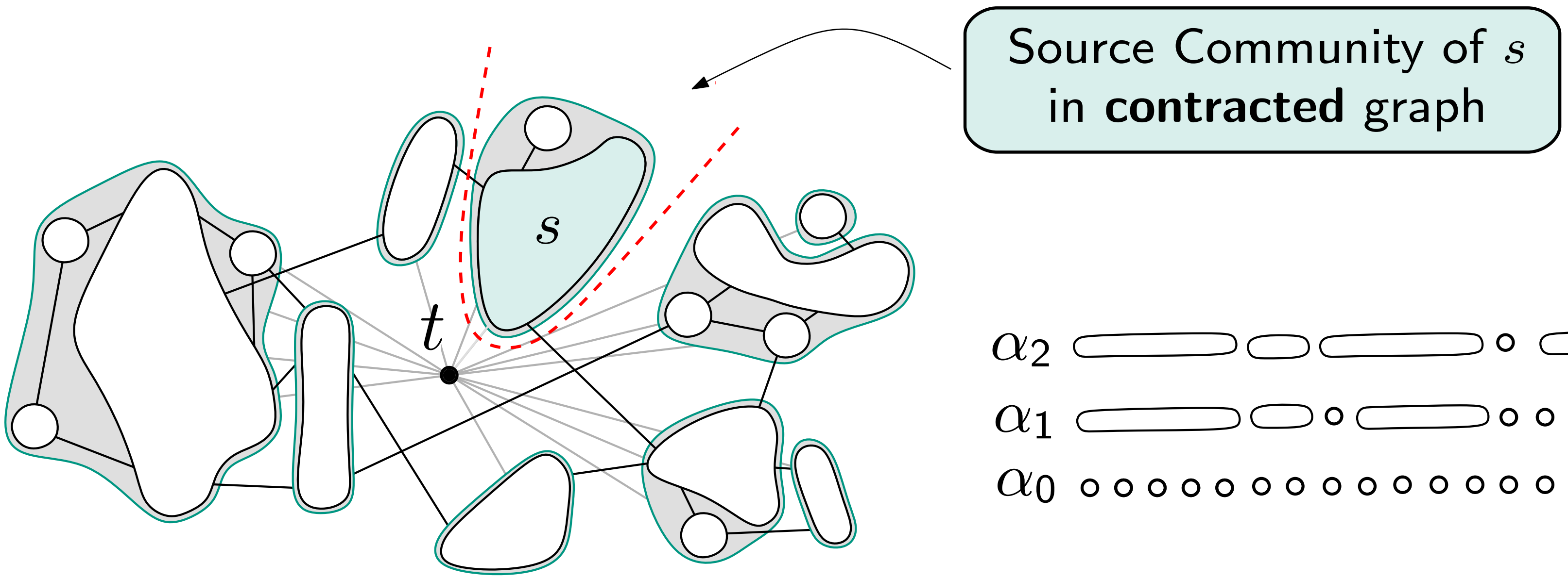

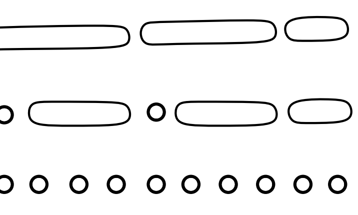

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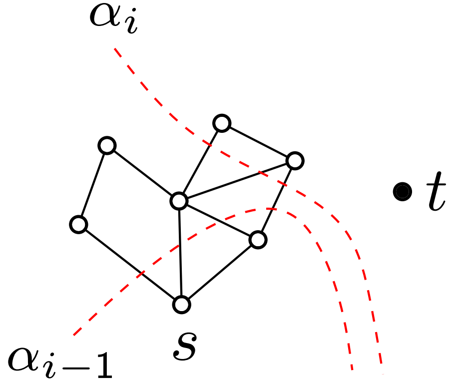

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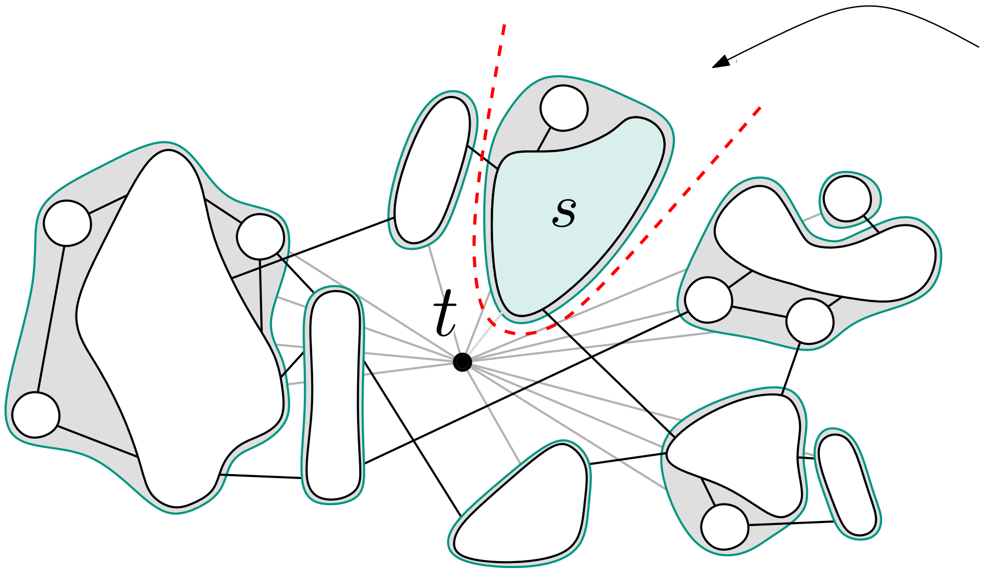

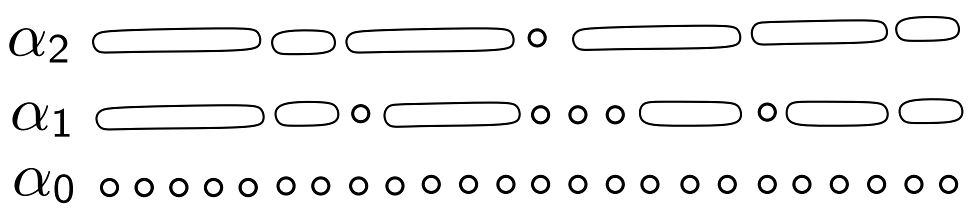

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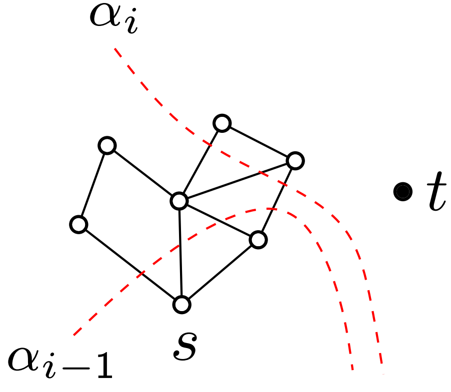

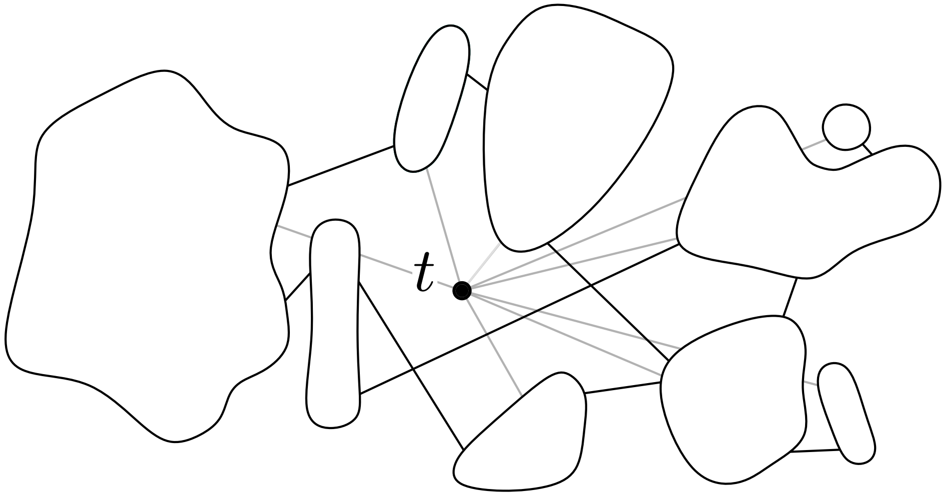

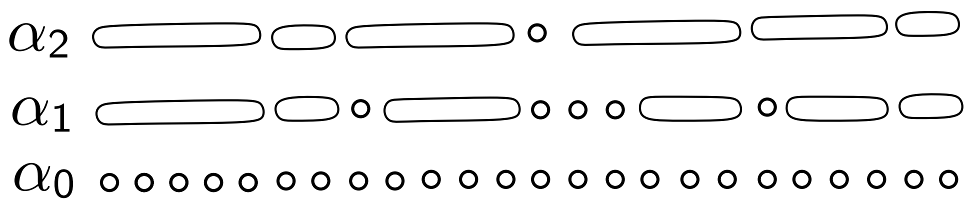

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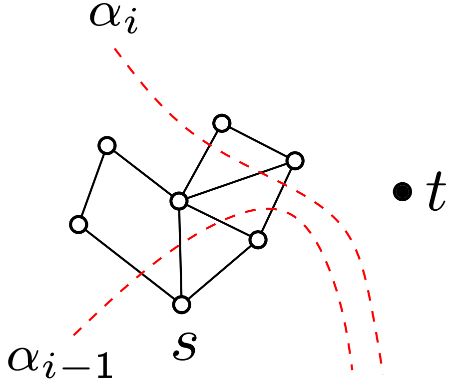

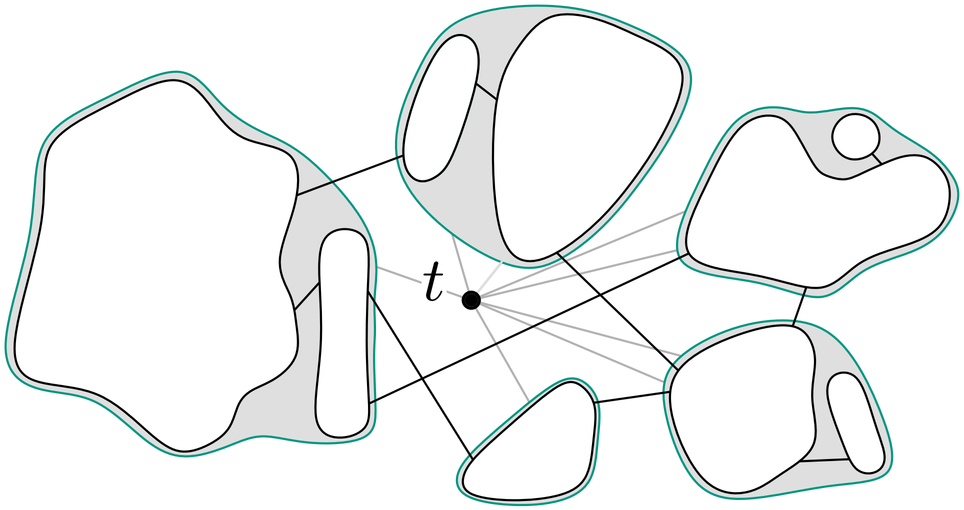

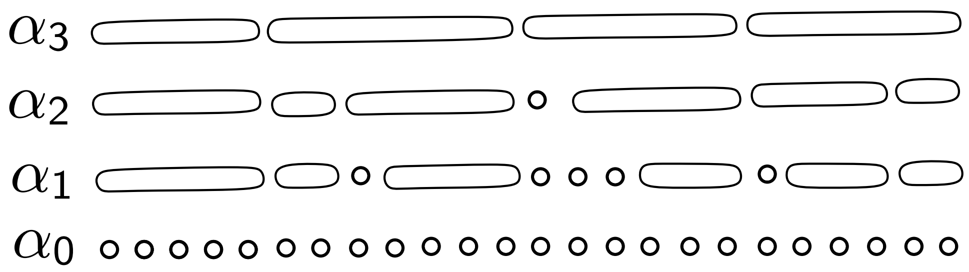

The Cut-Clustering Algorithm

- Flake et al. [2004]
- iterative application of the Cut-Clustering approach with different values $\alpha_1 > \cdots > \alpha_k$
- $\alpha_0 = 0$, singleton clustering $\mathcal{C}_0$

For $i = 1, \ldots, k$ do

- replace α_{i-1} by α_i
- contract each cluster $C \in \mathcal{C}_{i-1}$
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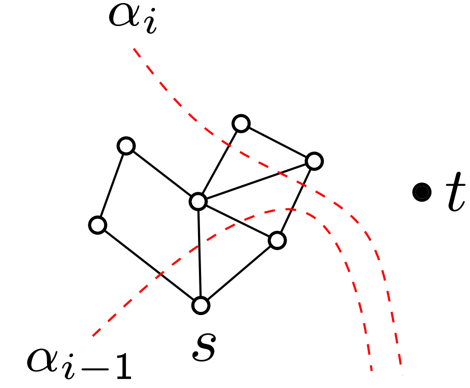
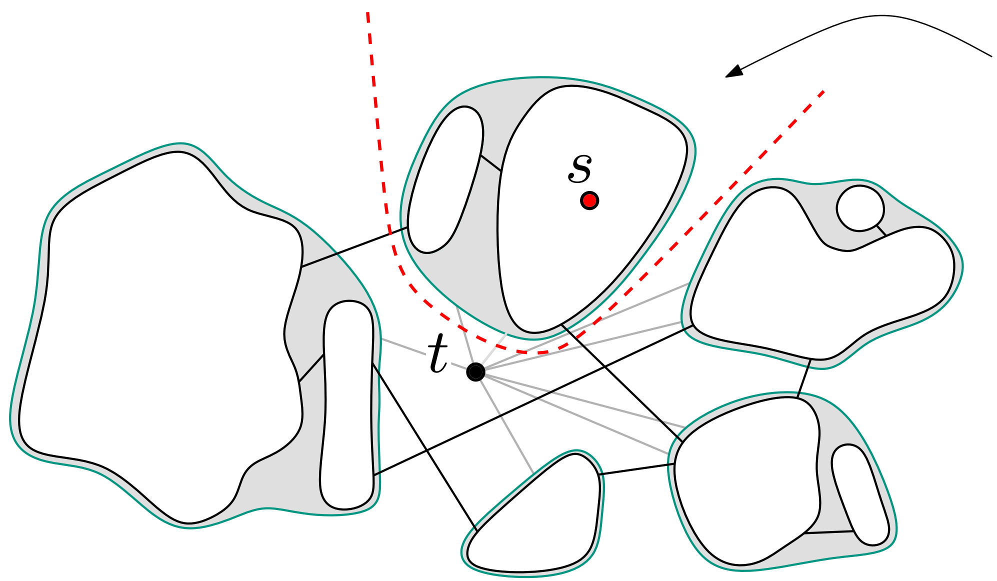
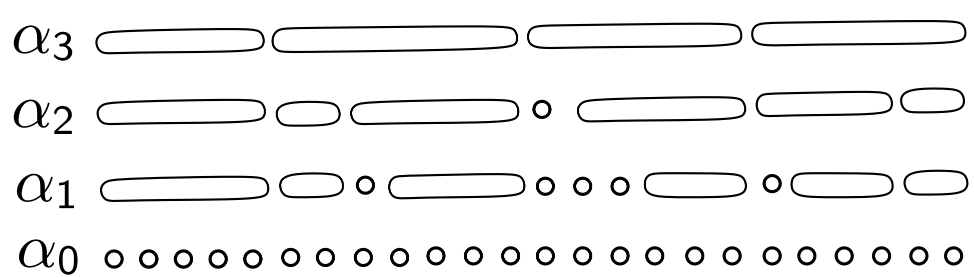
community-cuts are

- unique
- non-crossing
- vertically nested





Source Community of s in **underlying** graph



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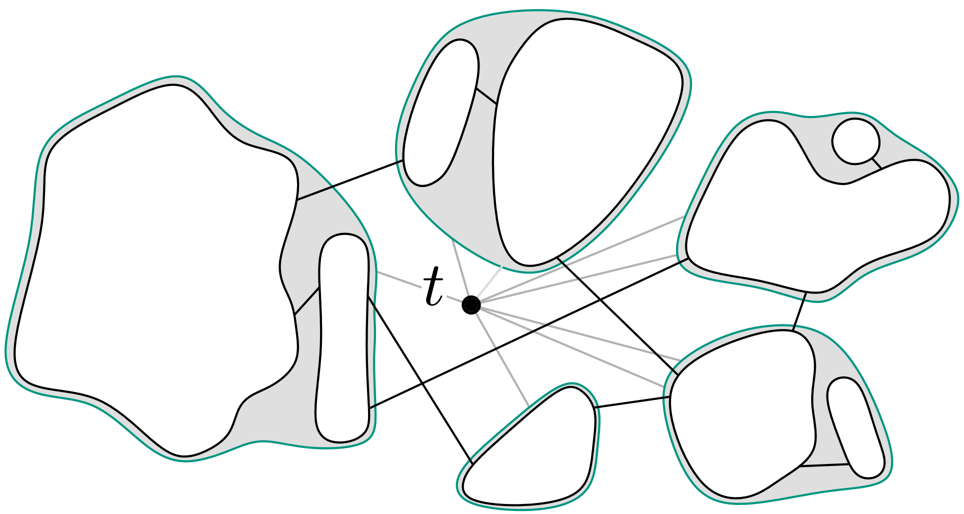
Problem: Choice of α

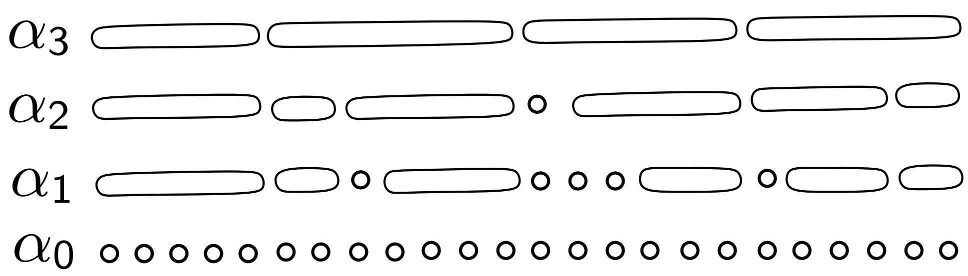
Naive idea: Binary search

needs discretization

too low $\rightarrow$ missing levels

too high $\rightarrow$ extensive running time





New Parametric Search Approach

- bases on cut-cost function

$$\omega_C : \mathbb{R}_0^+ \to [c(C, V \setminus C), \infty) \subset \mathbb{R}_0^+$$

$$\omega_C(\alpha) := c(C, V \setminus C) + |C|\alpha$$

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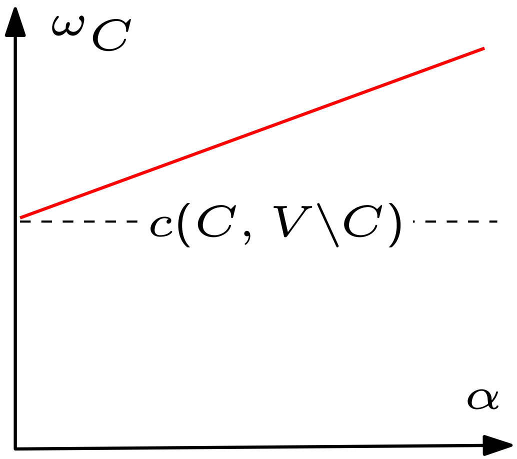

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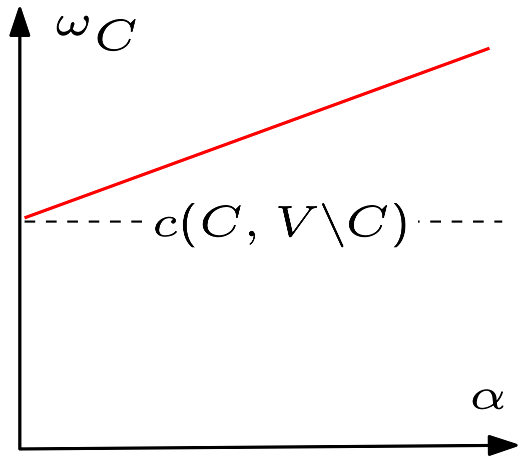

Theorem:
Let $\mathcal{C}_i < \mathcal{C}_j$ denote two different clusterings with associated parameter values $\alpha_i > \alpha_j$.
In time $O(|\mathcal{C}_i|)$ a parameter value α_m with $\alpha_j < \alpha_m \leq \alpha_i$ can be computed such that

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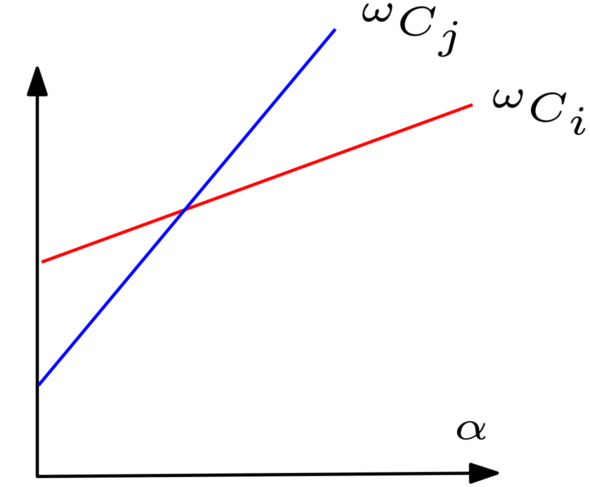

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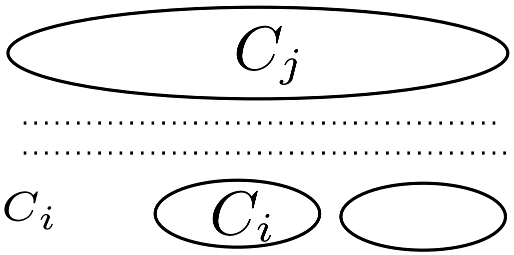
with α_i' the intersection point of ω_{C_j} and ω_{C_i}



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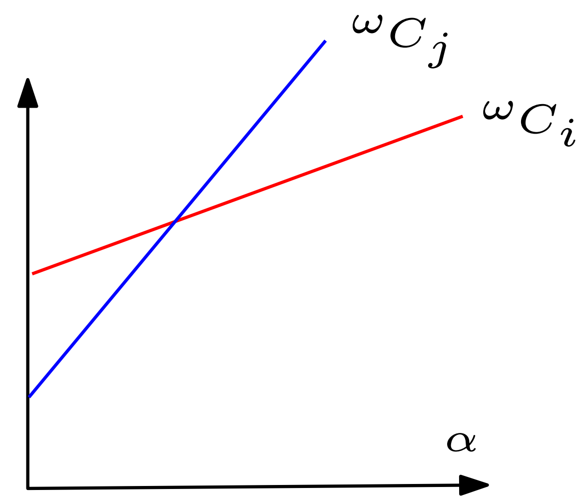

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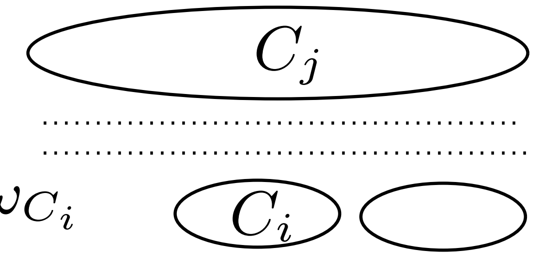

$\Rightarrow$ at most $2h$ iterations, returns **complete** hierarchy

Running Time Experiment

graph	n	m	h	ParS [m:s]	BinS [fac]
jazz	198	2742	3	0.062	7.871
celegans_metabolic	453	2025	8	0.300	8.380
celegansneural	297	2148	17	0.406	9.919
email	1133	5451	4	1.116	8.758
netscience	1589	2742	38	4.310	11.952
bo_cluster	2114	2277	19	4.355	12.800
data	2851	15093	4	11.506	9.620
dokuwiki_org	4416	12914	18	39.815	15.571
power	4941	6594	66	1:25.736	15.742
hep-th	8361	15751	56	6:26.213	18.183
PGPgiantcompo	10680	24316	94	13:25.121	18.575
as-22july06	22963	48436	33	39:54.495	20.583
cond-mat	16726	47594	80	44:15.317	27.425
astro-ph	16706	121251	60	98:25.791	24.825
rgg_n_2_15	32768	160240	46	245:25.644	22.573
cond-mat-2003	31163	120029	74	268:14.601	20.933
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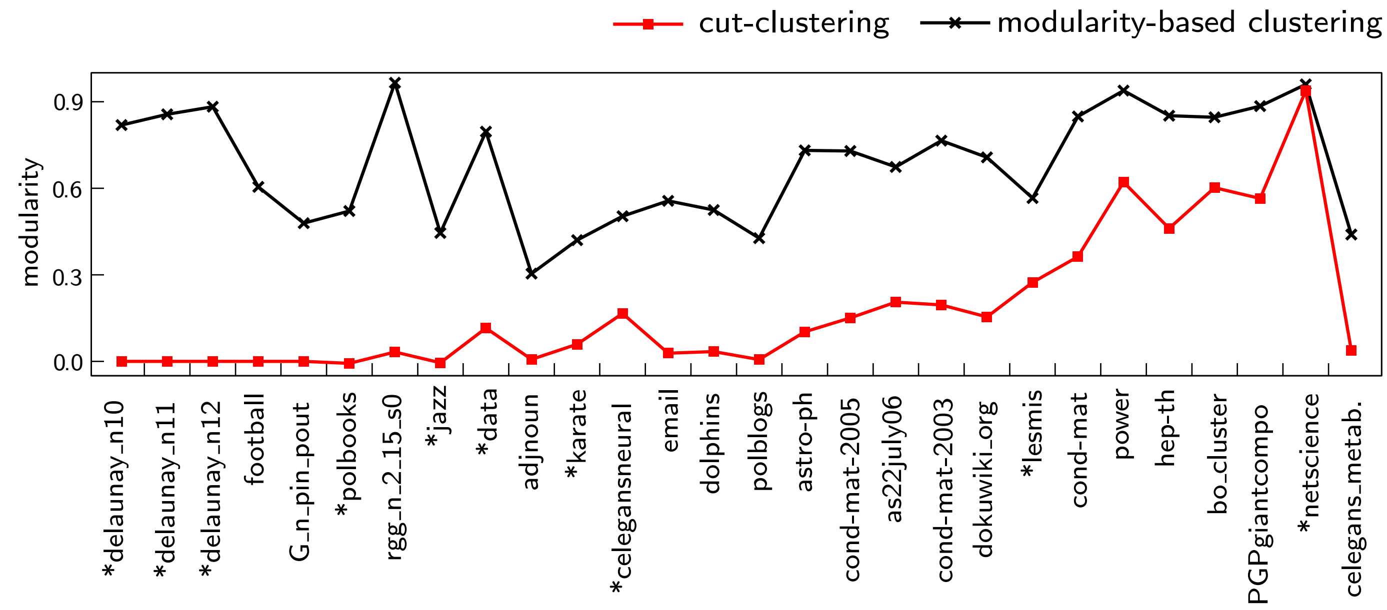
cut-clustering algorithm vs. modularity-based greedy approach

- modClus $\geq$ cutClus



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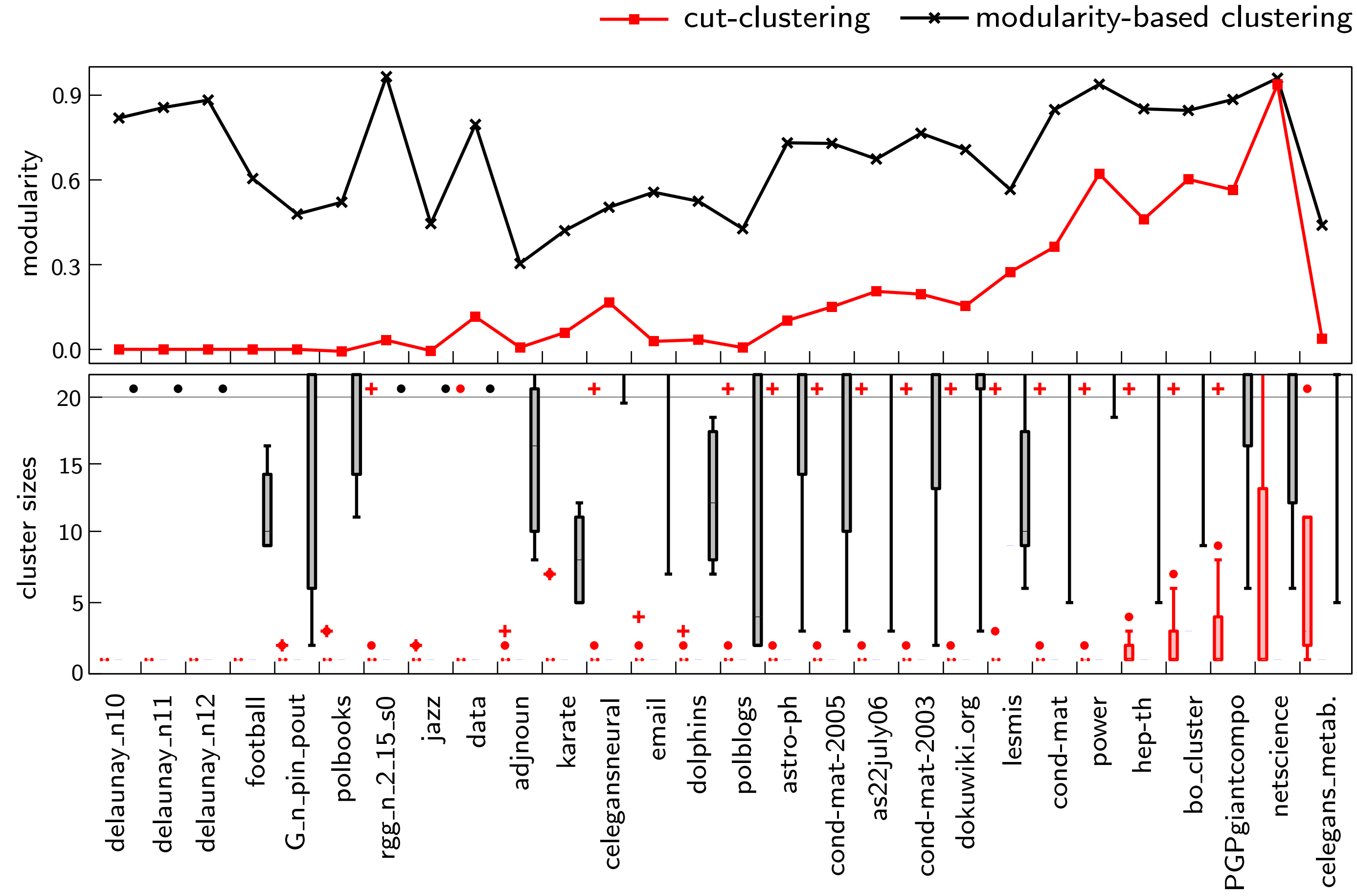

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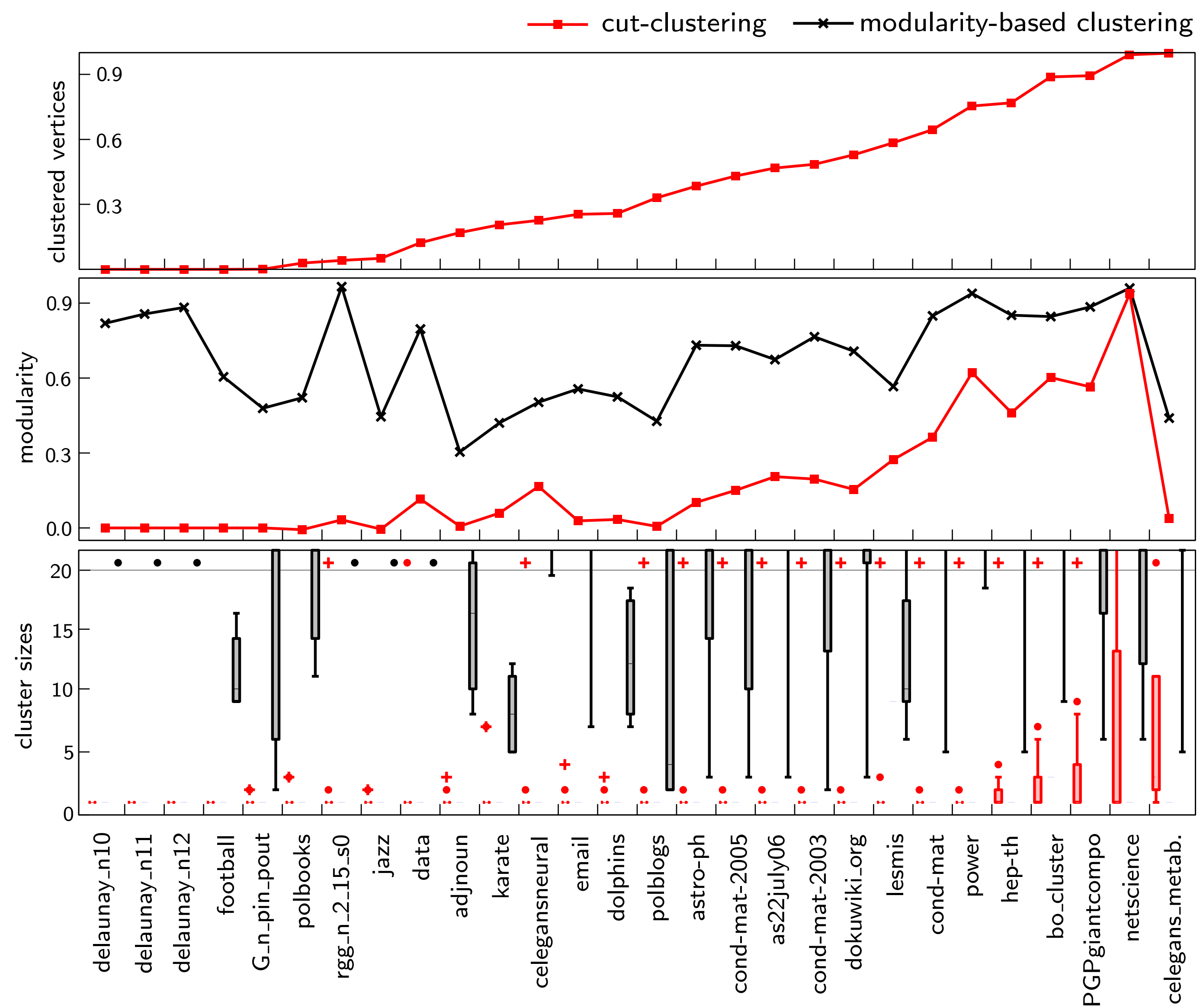

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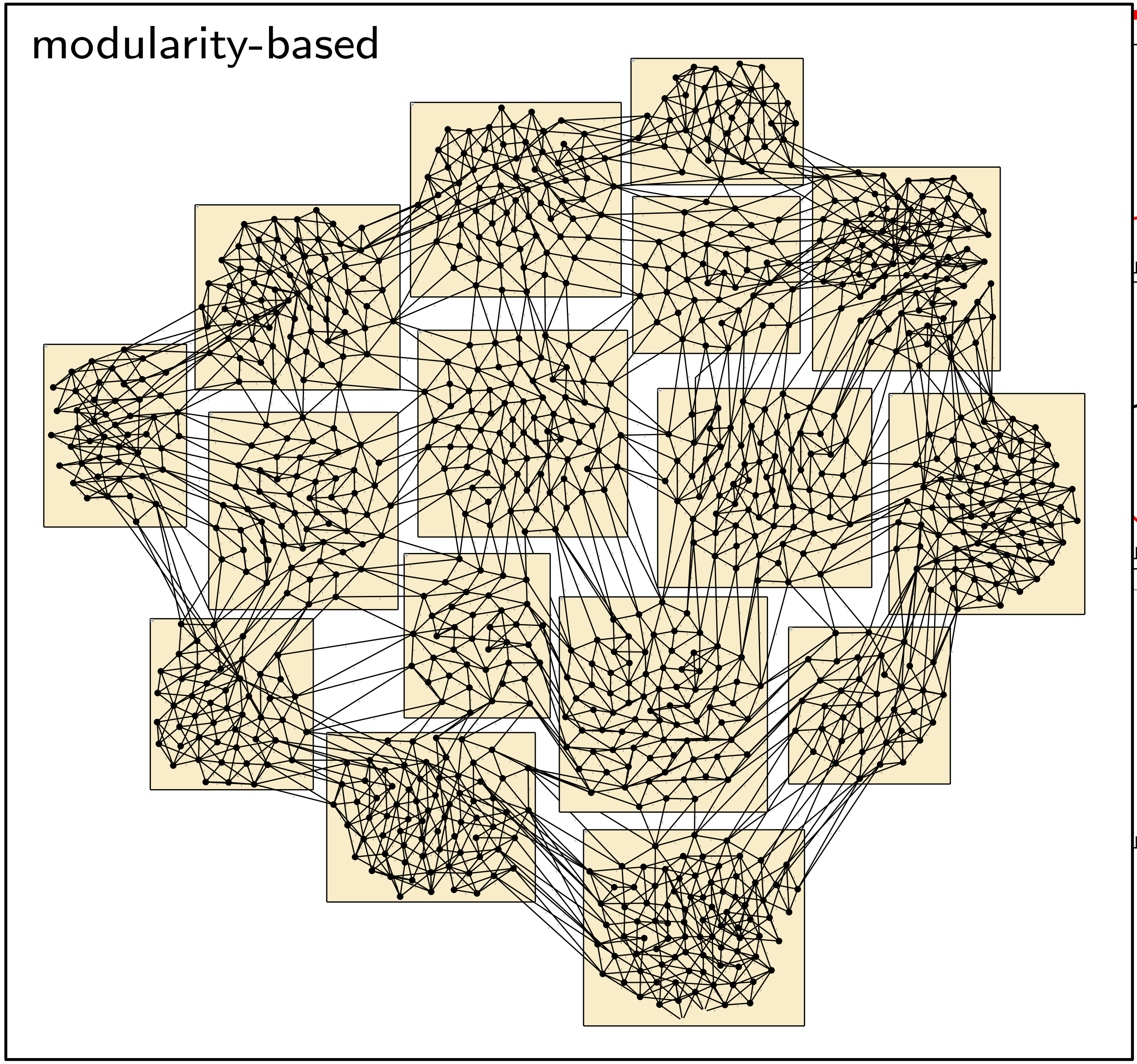

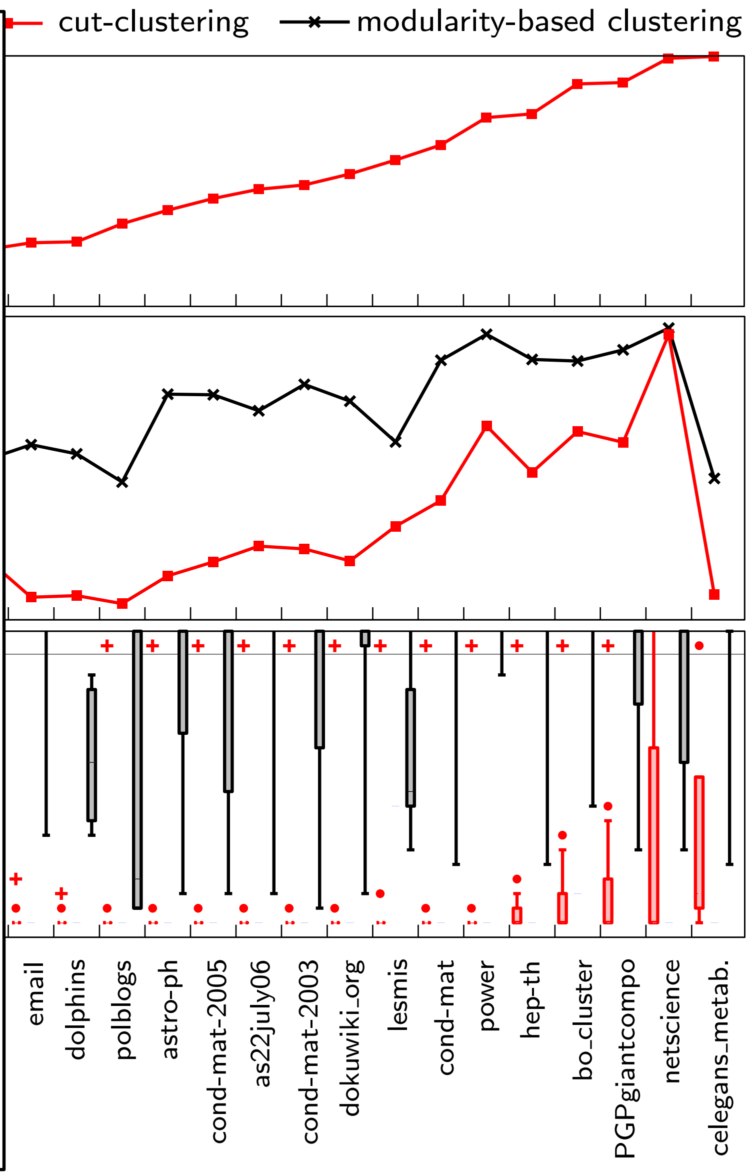

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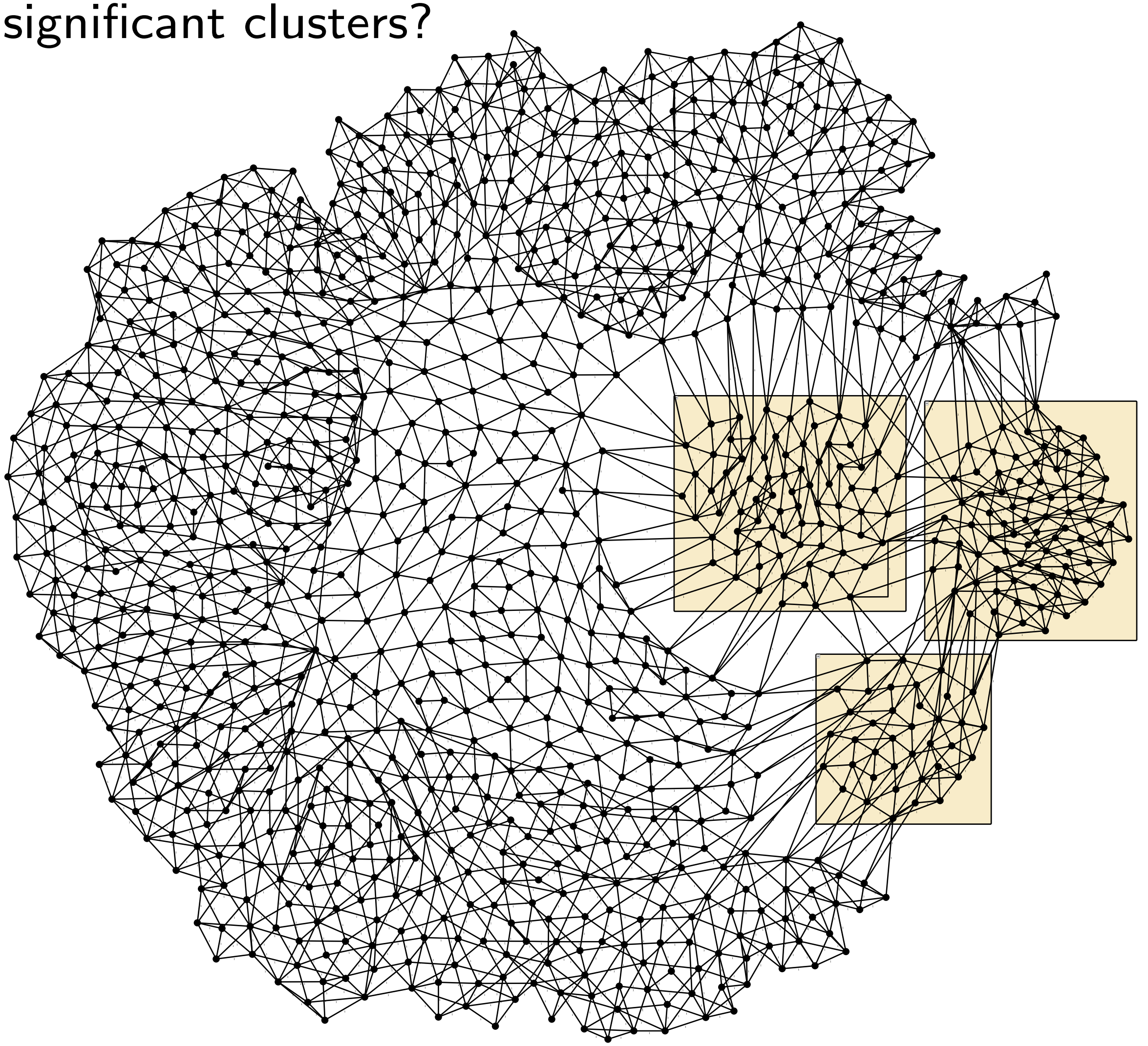
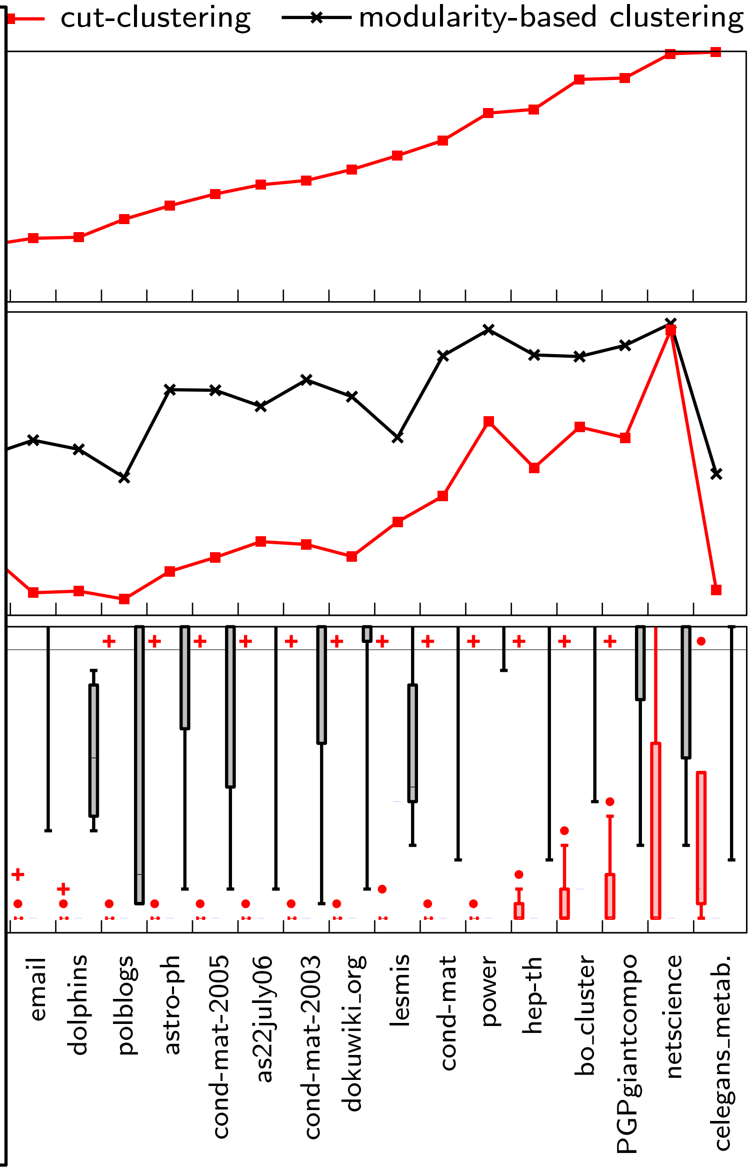
significant clusters?





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Source-community S of s:
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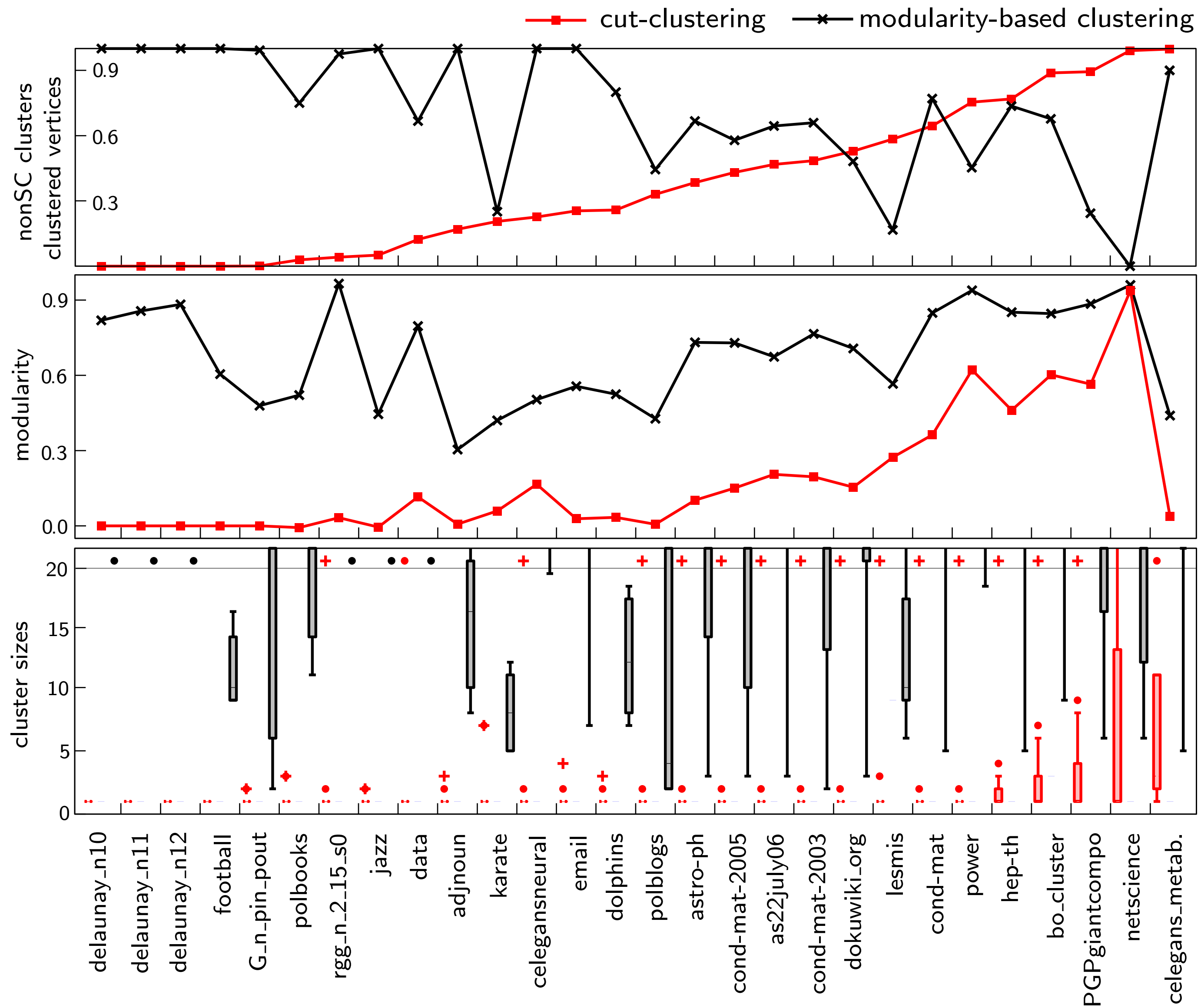

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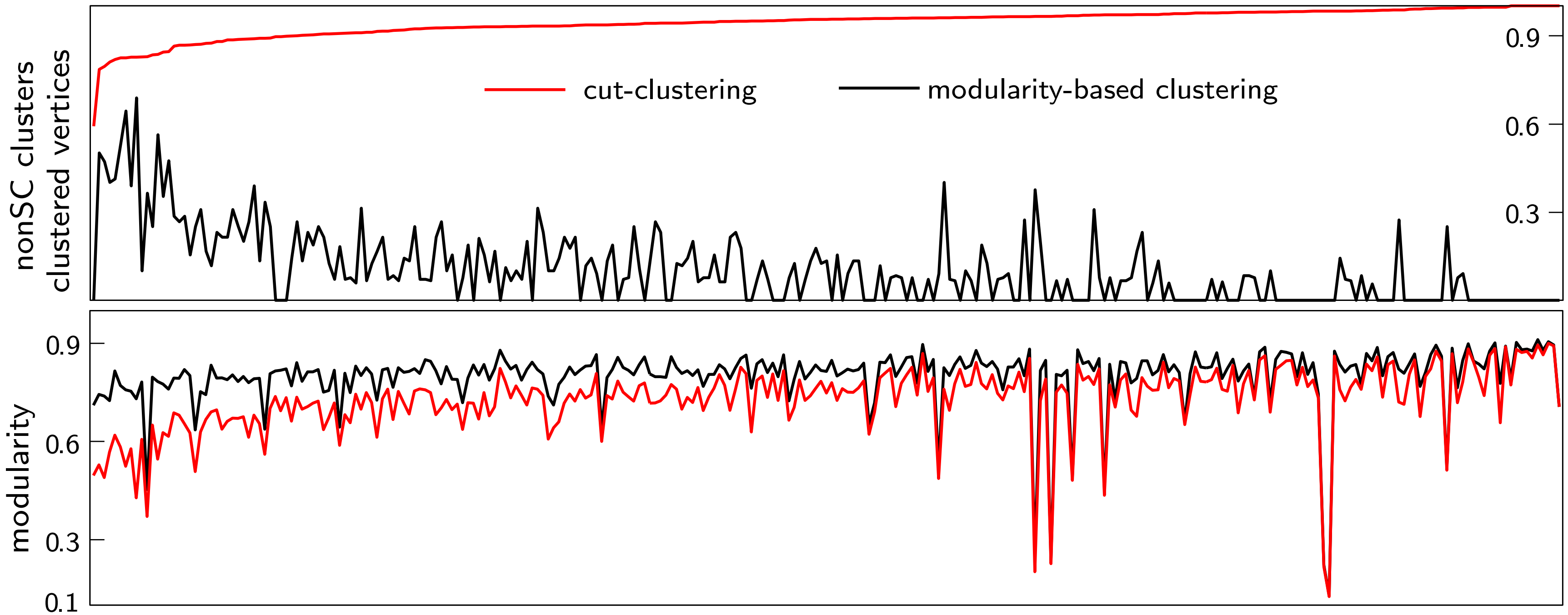
275 snapshots of the email network of the Department of Informarics at KIT.



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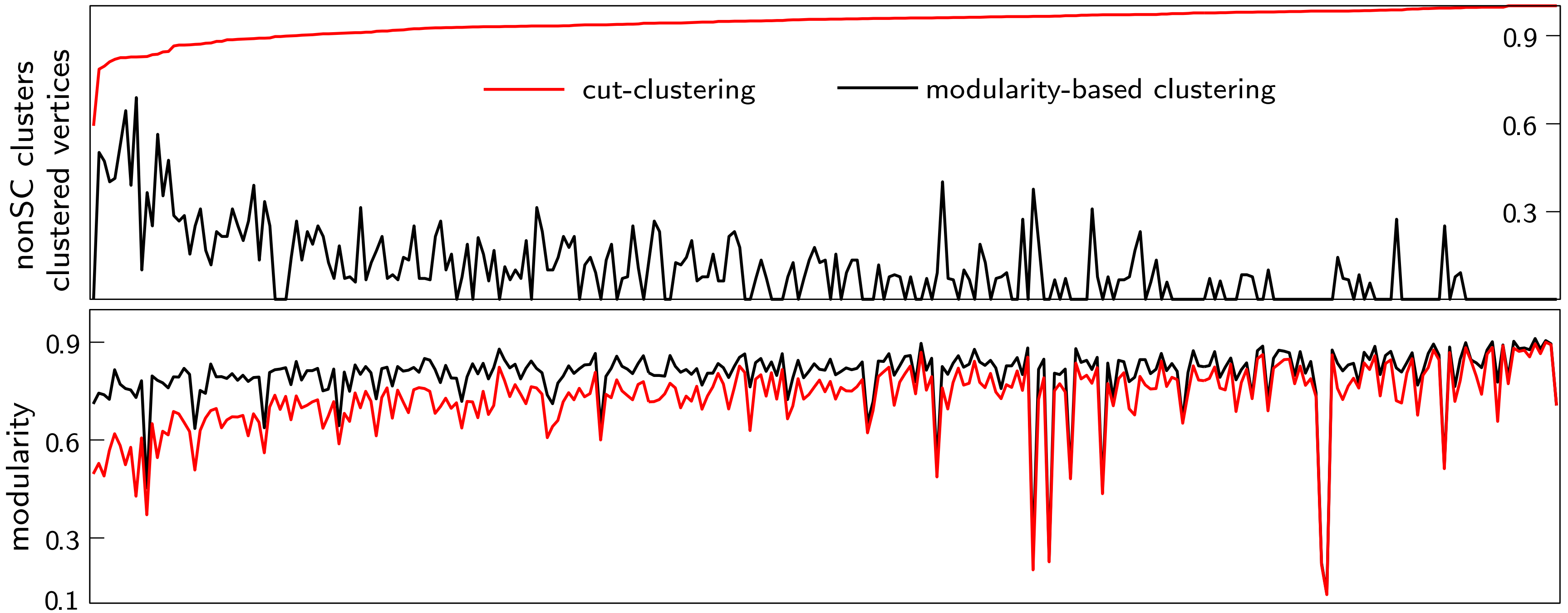

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- high dependence on graph structure
- Conjecture: Good cutClus if significant clustering exists

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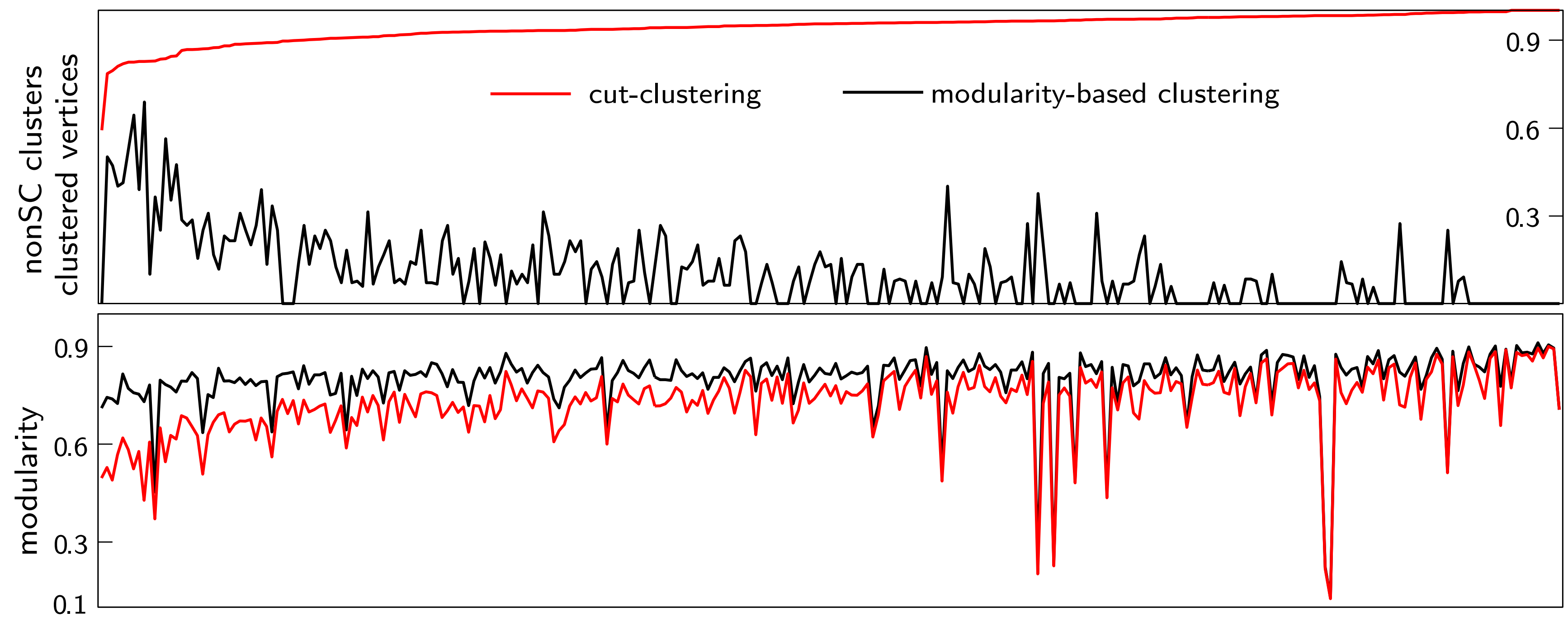

Expansion Analysis

intra-cluster expansion is
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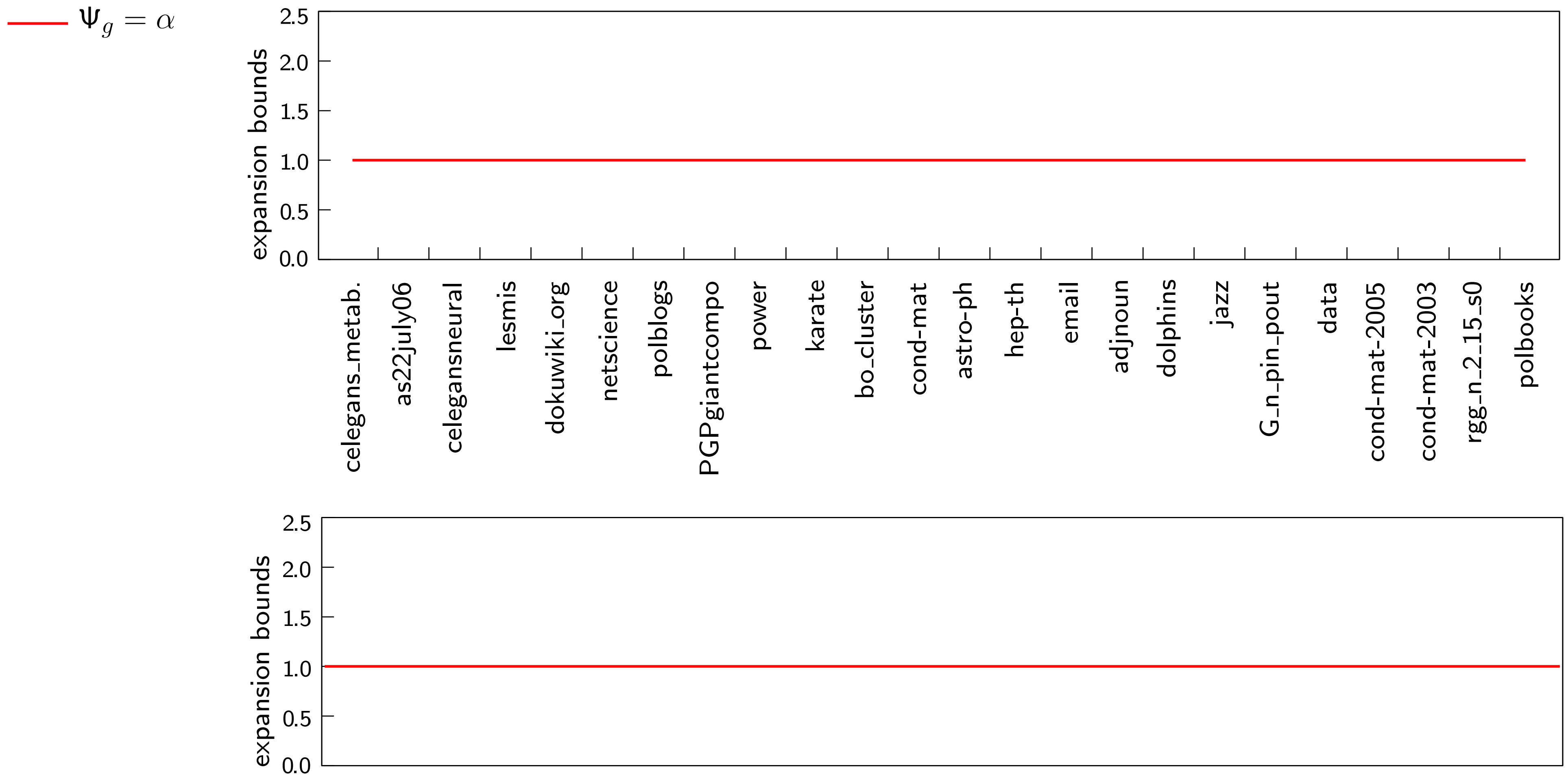

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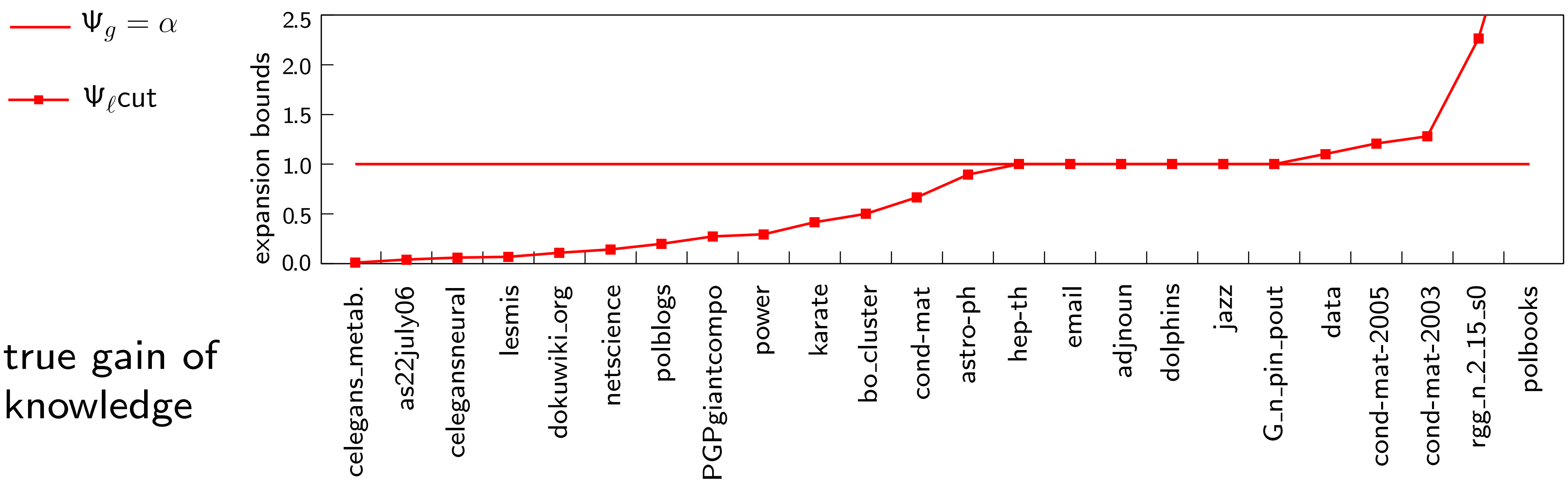
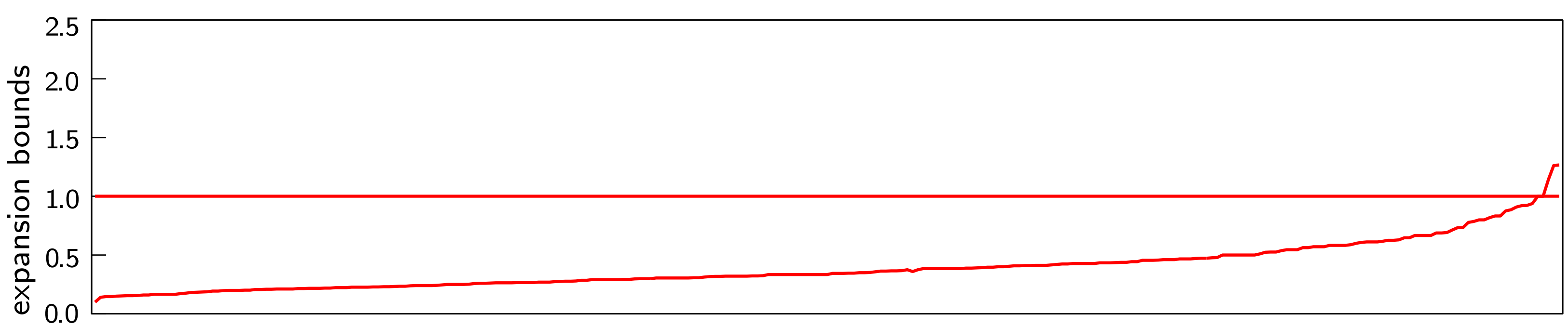


- true gain of knowledge



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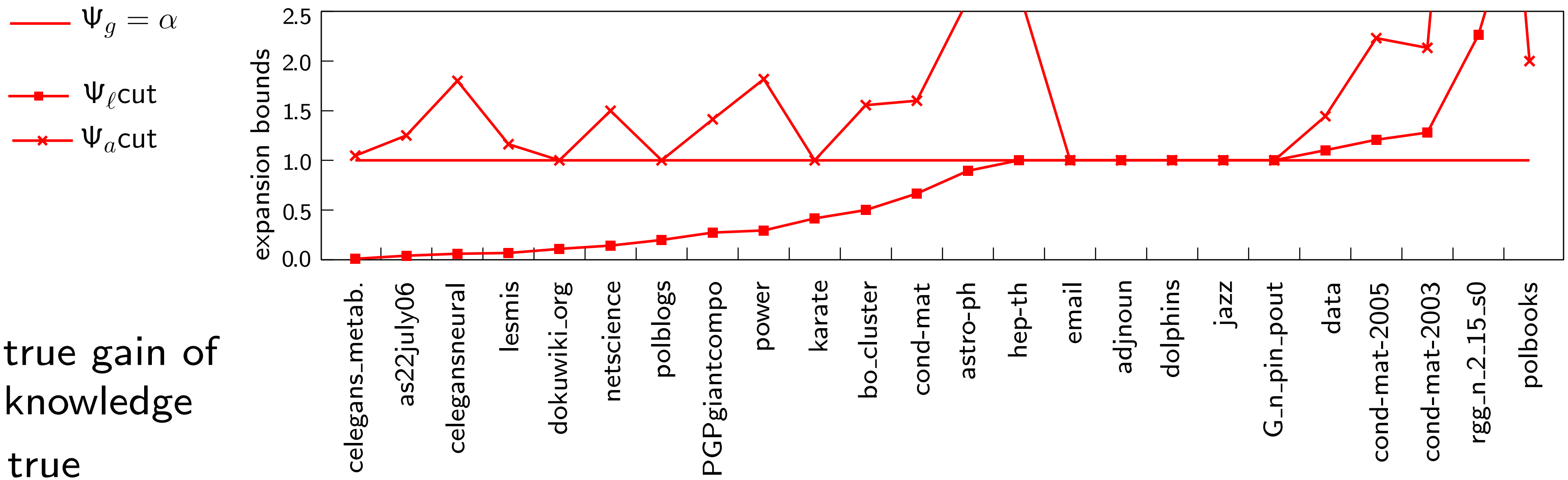

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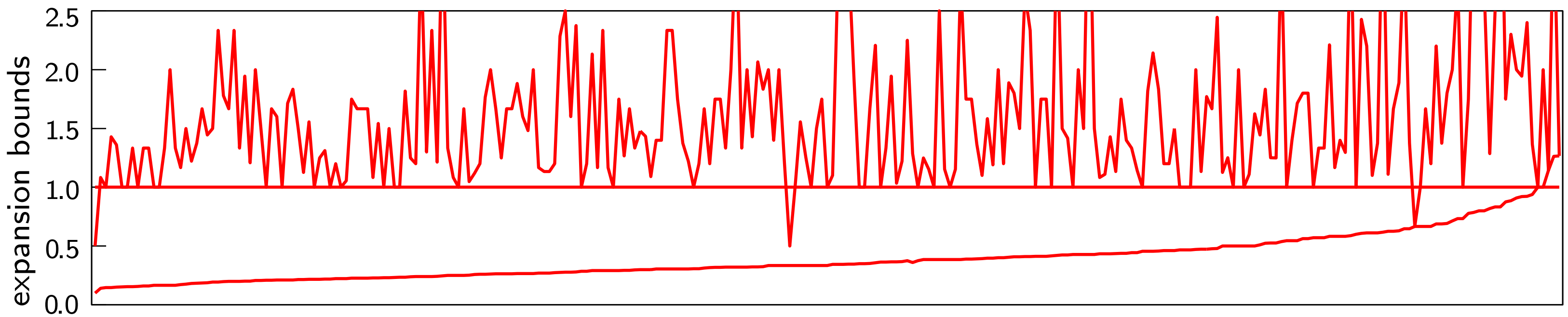

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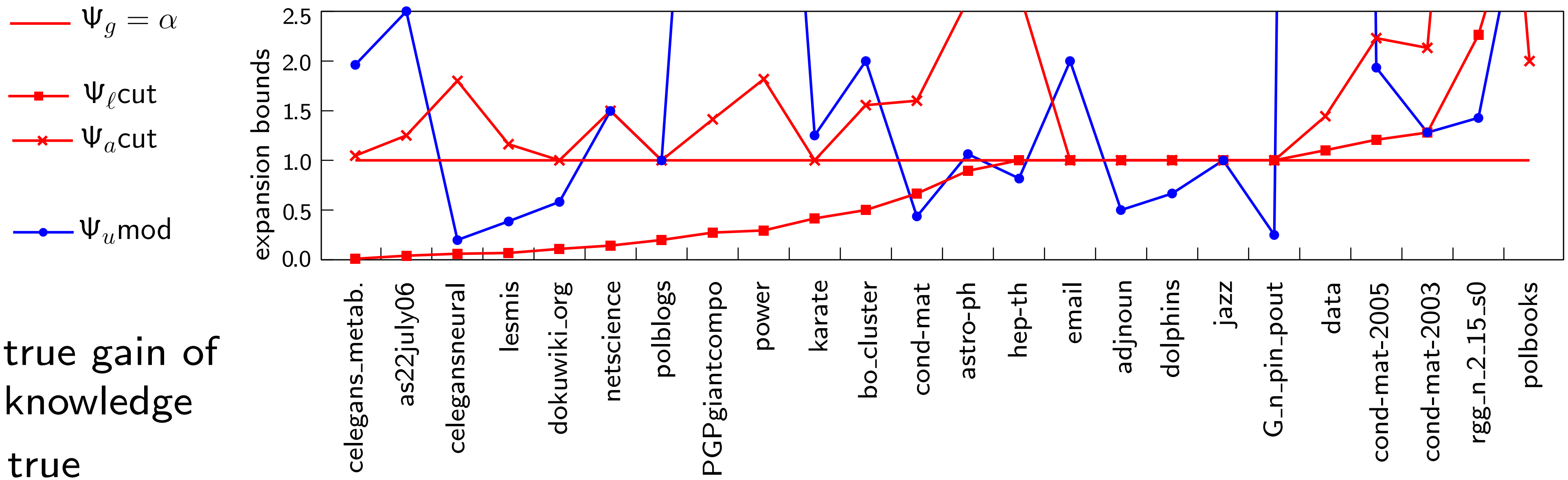

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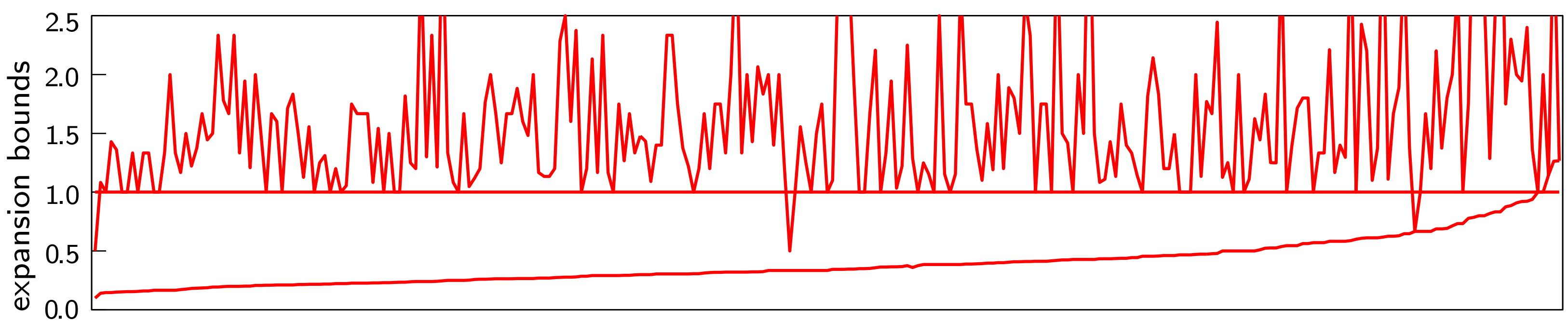

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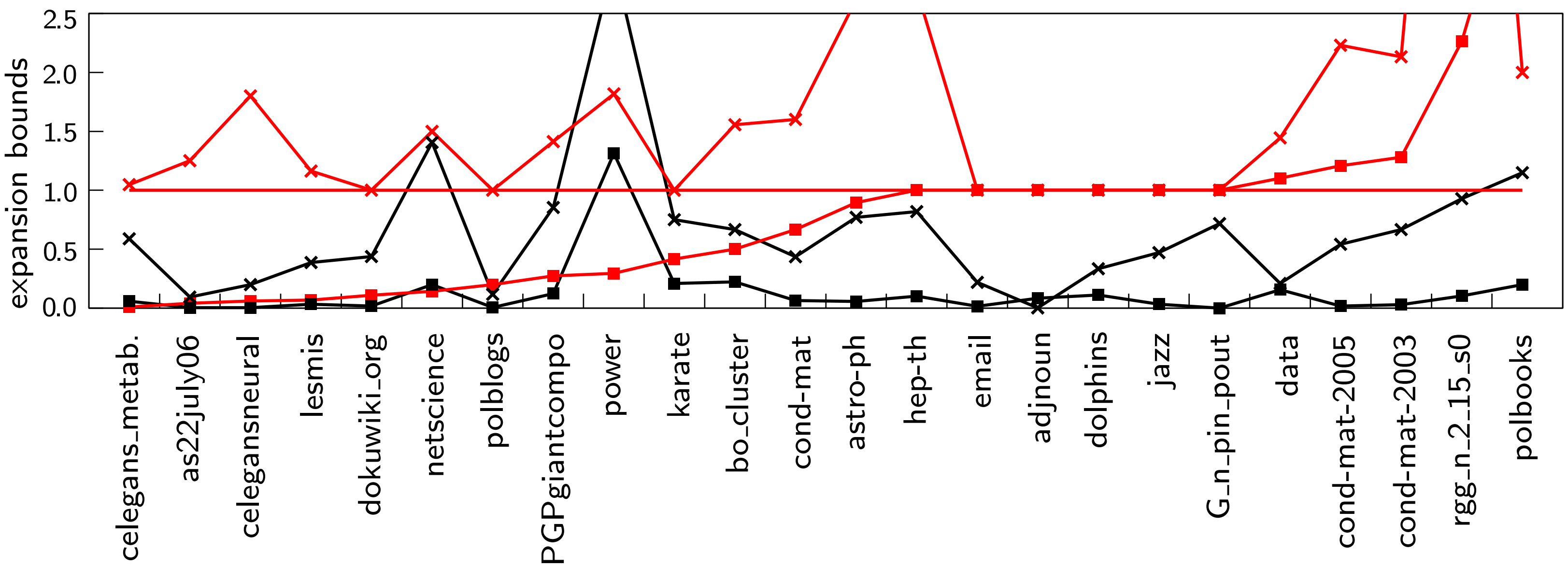

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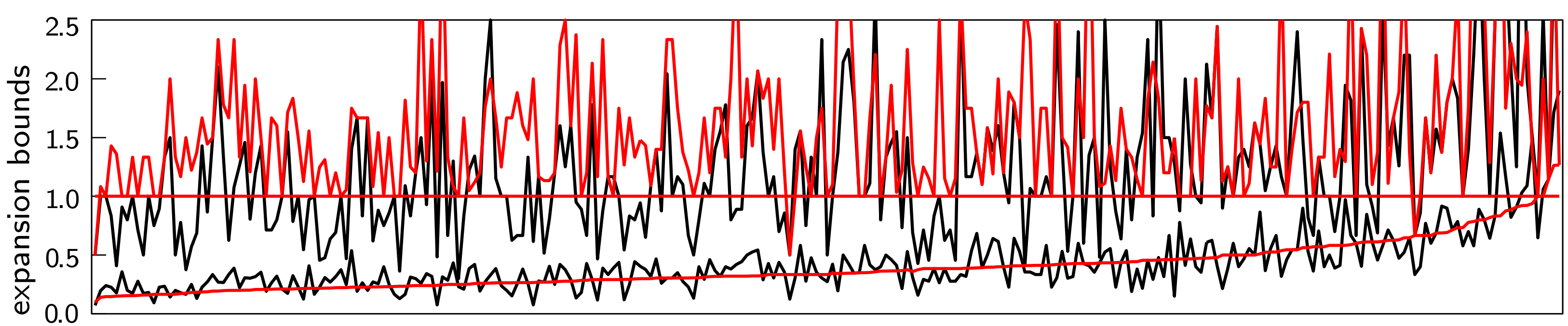

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Conclusion

Analysis of the Hierarchical Cut-Clustering Algorithm of Flake et al. [2004]

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  $\rightarrow$ complete hierarchy
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