UMPa: A Multi-objective, multi-level partitioner for communication minimization

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Introduction

- Problem: distributing communicating tasks, modeled as a graph, among processing units.
  - Balanced load distribution
  - Good communication pattern
- Objective functions from the literature:
  - Total communication volume
  - Maximum communication volume
  - Maximum send volume

Problem: Input and objective

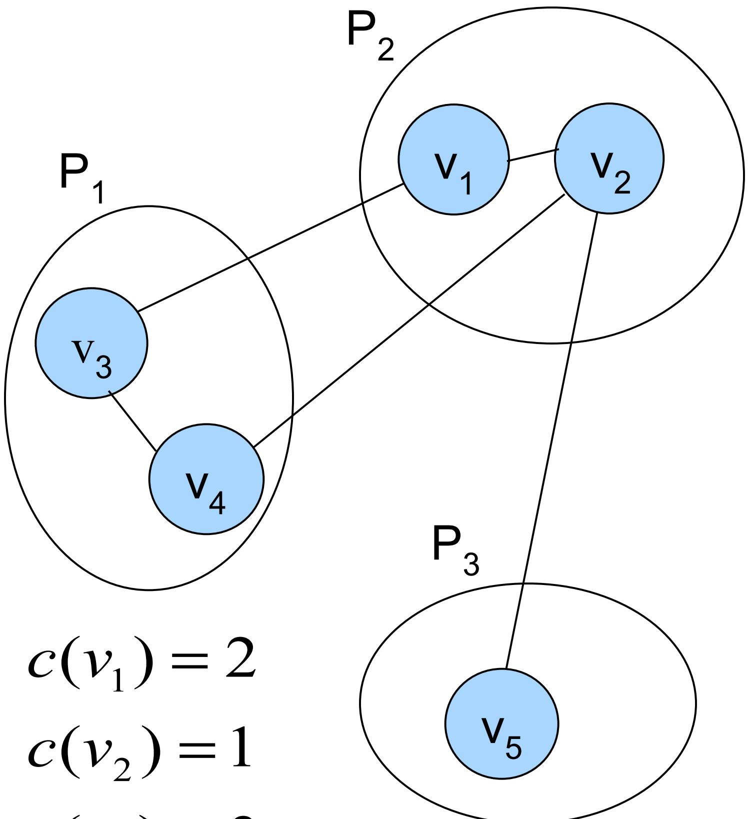
- Input: task graph G=(V,E)
  - V: vertex set representing a set of tasks
  - E: edge set representing task communications
- Objective: Find a partition $\prod = \{P_1, P_2, \ldots, P_K\}$ of the tasks s.t.

$$\max_k \left(\sum_{v \in P_k} c(v) \times f(v) \right)$$

is minimized

  - c(v): volume of each transfer sent by v.
  - f(v): number of parts that requires the data sent by v.

Problem: communication costs



$$c(v_1) = 2$$
$$c(v_2) = 1$$
$$c(v_3) = 3$$
$$c(v_4) = 2$$
$$c(v_5) = 4$$

- The objective function is equivalent to minimizing maximum send volume (maxSV).

Part	**Send Vol.**	Rec. Vol.	Send+rec.
P_1	3 + 2	2 + 1	8
P_2	2 + 1 + 1	3 + 2 + 4	13
P_3	4	1	5
Total	13	13	26

- maxSRV ← max send+receive volume
- totV ← total communication volume

Hypergraph Model

- Hypergraph H = (V, N)
  - A net is a subset of vertices.
  - Each net n has cost $c_H(n)$
- We model the task graph G(V,E) as a hypergraph
  - For each task s in G, let v_s be the corresponding vertex in H.
  - For each task s in G, the net set N contains a net n_s.
    - $n_s = \{v_d : ((s,d) \in E)\} \bigcup \{v_s\}$
    - $c_H(n_s) = c(s)$

Hypergraph Model

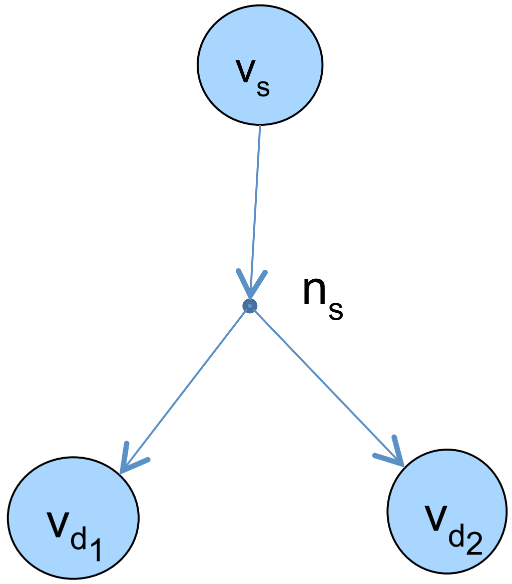
- λ_n: Connectivity of a net **n**, i.e., the number of parts net **n** is connected.

$$totV = \sum_{n \in N} c_H(n) \times (\lambda_n - 1)$$

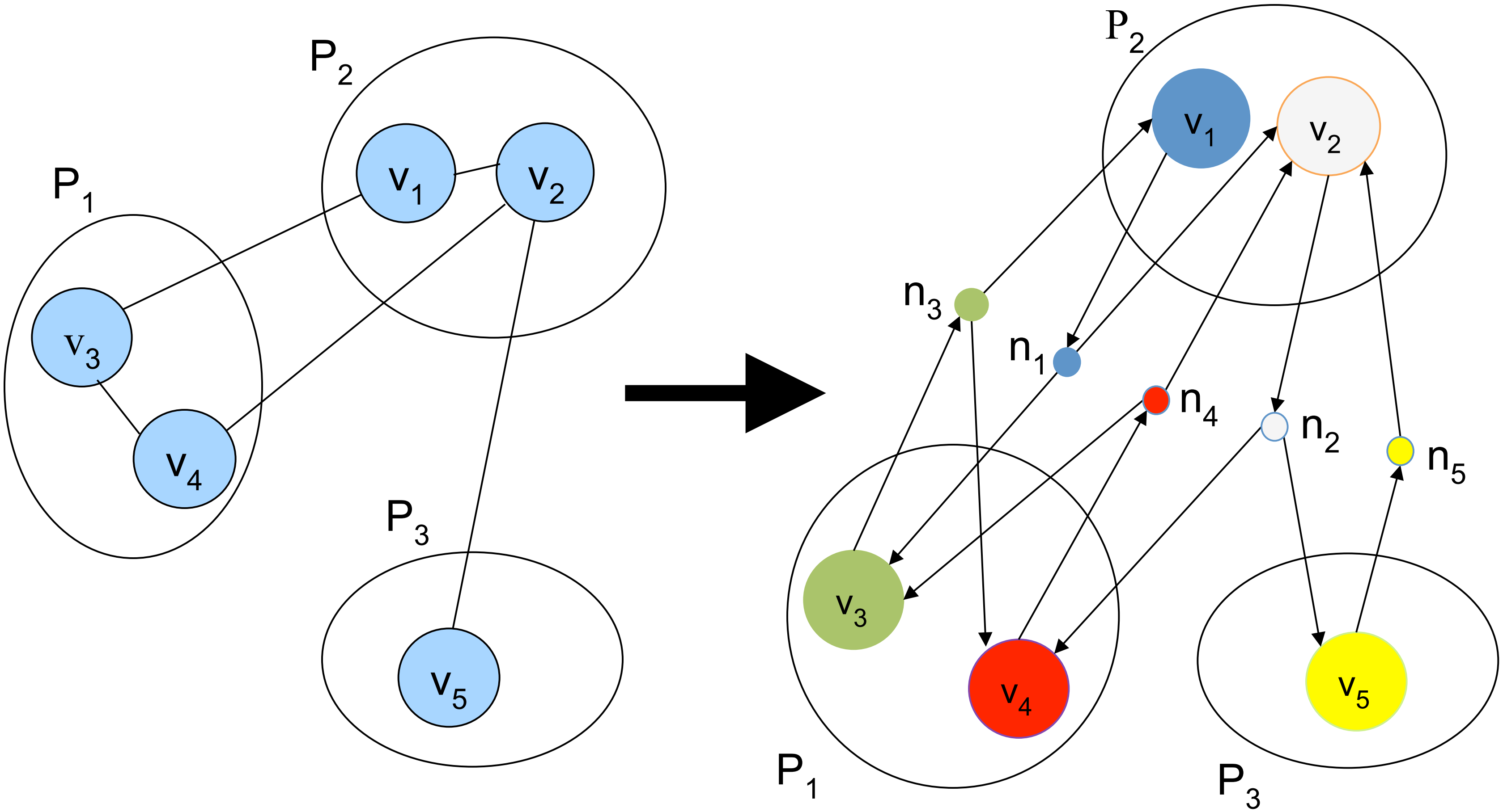
Minimizing the formula, equivalent to minimizing the total communication volume [Ç & Aykanat'99].

Directed Hypergraph Model

- Directed hypergraph:
  - Flow: from the source pin to the other pins.
  - Source of $n_s = v_s$
- Allows to minimize maxSV and maxSRV (in addition to totV).
- Objective: Partition the vertices into K parts s.t.
  - The load is distributed equally.
    - $W_k < W_{avg}(1+\varepsilon) \text{ for } 1 \le k \le K$
  - maxSV is minimized.
    - $SV(P) = \sum_{v_s \in P} c_H(n_s) \times (\lambda_{n_s} - 1)$
    - $\max SV = \max_k (SV(P_k))$



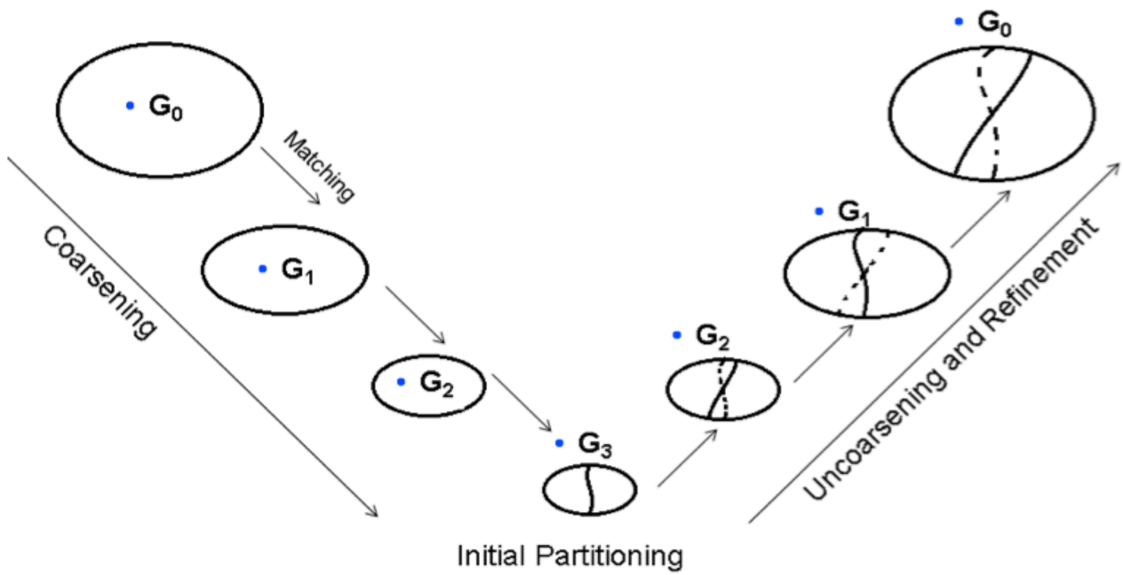
Hypergraph Model: Example



$$\lambda_{n_2} = 3,\ \lambda_{n_1} = \lambda_{n_3} = \lambda_{n_4} = \lambda_{n_5} = 2$$

Multi-level Approach

- Three phases:
  - Coarsening: obtain smaller and similar hypergraphs to the original, until either a minimum vertex count is reached or reduction on vertex number is lower than a threshold.
  - Initial Partitioning: find solution for the smallest hypergraph
  - Uncoarsening: Project the initial solution to the coarser graphs and refine it iteratively until a solution for the original hypergraph obtained.



Multi-level Approach

- Most of the available tools adapt multi-level approach with **recursive bisection method.**
  - A partition is obtained by recursive partitioning into 2 parts.
  - Works fine for total communication.
  - May not be suitable for minimizing maximum send (and/or send+receive) volume.
    - Only the information about 2 parts is available at each step.
    - Send and receive volumes of other parts are unknown.

UMPa

- K-way multi-level partitioner.
- Uses directed hypergraph model.
  - Communication of the net flows from source to target vertices.
- Minimizes maxSV, while breaking ties by favoring reducing maxSRV, then the total volume.
- Currently, only coarsening phase of UMPa is shared memory parallel.
- Ultimate goal: To parallelize UMPa (MPI+OpenMP).

UMPa: Coarsening

- Neighbor vertices (u and v) are clustered by using agglomerative matching in coarsening phase.
  - Similarity of u and v

$$\sum_{n \in nets(u) \cap nets(v)} \frac{c_H(n)}{(pins(n)-1)}$$

```
Data: H = (V, N), rep[]
for each vertex u ∈ V in parallel do
    if CHECKANDLOCK(u) then
        if u is matched before then
            UNLOCK(u)
            continue
        adj_u ← {}
        for each vertex v ∈ neighbors[u] do
            adj_u ← adj_u ∪ {v}
            conn[v] ← sim(v,u)
        v* ← u
        conn* ← 0
        for each vertex v ∈ adj_u do
            v^r ← rep[v]
            if v^r ≠ v then
                conn[v^r] ← conn[v^r] + conn[v]
            if conn[v^r] > conn* then
                if CHECKANDLOCK(v^r) then
                    conn* ← conn[v^r]
                    v* ← v^r
                    UNLOCK(v*)
        rep[u] ← v*
        UNLOCK(v*)
        UNLOCK(u)
```

UMPa: Initial Partitioning

- For the initial partitioning, we used PaToH to obtain k initial parts.
  - Although PaToH is used to minimize total communication volume but not maximum send volume:
    - We do not want a drastic increase in any of the communication metrics. So, minimizing total volume is a good idea both in theory and practice.
    - Using recursive bisection and FM-based improvement are favorable due to the small net sizes and high vertex degrees.

UMPa: Uncoarsening

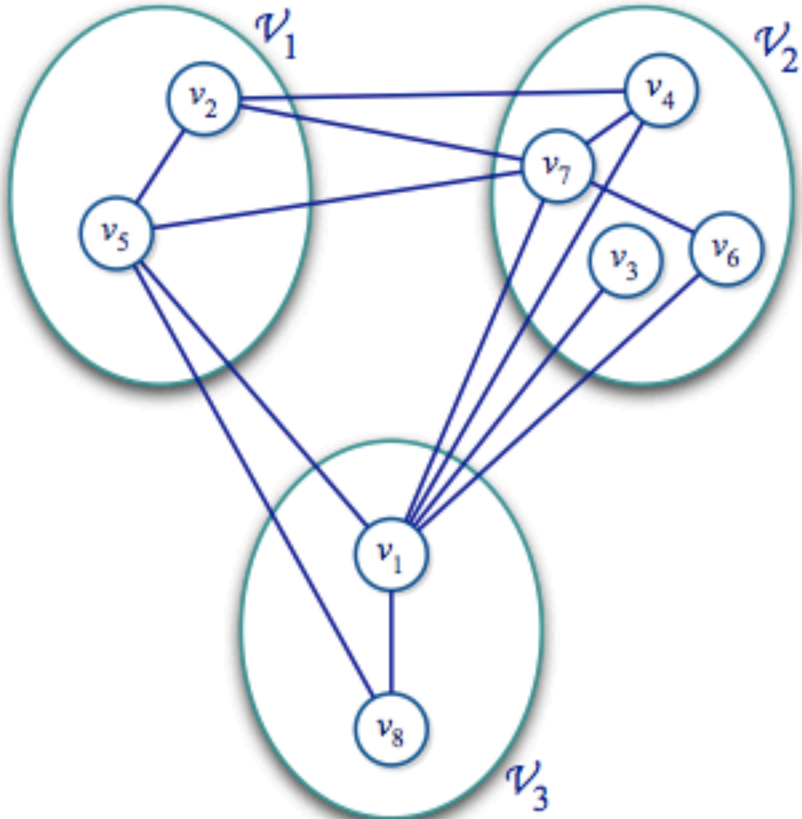
- Solutions for the coarser hypergraphs are iteratively projected to finer ones and refined.
- Refinement method:
  - Traverses the boundary vertices in random order.
    - Random, since FM/KL based heuristics are expensive especially in K-way.
  - Computes move gains for each visited vertex and select the best move.

```
Data: H = (V, N), boundary[], part[], SV[], SRV[]
for each unlocked u ∈ boundary do
    (bestMaxSV, bestMaxSRV, bestTotV) ← (maxSV, maxSRV, totV)
    bestPart ← part[u]
    for each part p other than part[u] do
        if p has enough space for vertex u then
            (SV[], SRV[], moveV) ←calculateComVols(v, p)
            (moveMaxSV, moveMaxSRV) ←
                            calculateMax(moveSV[], moveSRV[])
            MOVESELECT(moveMaxSV, moveMaxSRV, moveV, p,
                            bestMaxSV, bestMaxSRV, bestTotV, bestPart)
    if bestPart ≠ part[u] then
        move u to bestPart and update data structures accordingly
```

Move Selection

- Always move a visited vertex to the part with the maximum reduction on maxSV.
  - Tie-breaking is applied for equal reductions.
  - When there is an equality, the vertex move is favored toward the part that minimizes maxSRV, then totV.

Tie-breaking



Vertex	Part	*maxSV*	*maxSRV*	*totV*
v_1	$\mathcal{V}_1$	−1	+1	−2
v_1	$\mathcal{V}_2$	−2	−2	−3
v_2	$\mathcal{V}_2$	0	−1	−1
v_2	$\mathcal{V}_3$	−1	+1	0
v_3	$\mathcal{V}_1$	−1	−1	0
v_3	$\mathcal{V}_3$	−1	−1	−1
v_4	$\mathcal{V}_1$	−1	−1	0
v_4	$\mathcal{V}_3$	−1	+1	+1
v_5	$\mathcal{V}_3$	0	0	−1
v_6	$\mathcal{V}_1$	−1	0	+1
v_6	$\mathcal{V}_3$	−1	0	0
v_7	$\mathcal{V}_1$	−1	+1	0
v_7	$\mathcal{V}_3$	−1	−1	0
v_5	$\mathcal{V}_2$	+2	+2	−1
v_8	$\mathcal{V}_1$	0	0	0
v_8	$\mathcal{V}_2$	+2	+2	+1

- Initially, maxSV=6, maxSRV=9, totV= 12. (v_3, v_4, v_6, v_7)

Experimental Results

- Experiments
  - 123 graphs
  - 10 graph classes
  - For K = 4, 16, 64, 256
  - Each instance is partitioned 10 times.
- Compared with PaToH minimizing total volume.
- The power of tie-breaking is also studied.

Average Performance

K	PaToH + Refinement No tie breaking			UMPa No tie breaking			UMPa With tie breaking		
	$maxSV$	$maxSRV$	$totV$	$maxSV$	$maxSRV$	$totV$	$maxSV$	$maxSRV$	$totV$
4	0.93	1.05	1.06	0.73	0.83	0.93	0.66	0.77	0.84
16	0.93	1.06	1.04	0.84	0.94	1.11	0.73	0.83	0.98
64	0.91	1.04	1.02	0.86	0.98	1.12	0.76	0.87	1.00
256	0.91	1.03	1.01	0.89	1.00	1.10	0.81	0.91	1.02
Avg.	0.92	1.05	1.03	0.83	0.93	1.06	0.74	0.84	0.96

- The geometric mean of the relative results wrt PaToH used to minimize totV.
- Tie-breaking is very useful.
- As K increases the reduction rate decreases, since the total communication is distributed to more parts.

Medical Center

Experiment Results: K=16

Graph	$maxSV$	$maxSRV$	$totV$	Time
coPapersDBLP	0.862	0.845	1.252	1.591
as-22july06	0.760	0.787	1.016	4.286
road_central	0.558	0.577	0.716	0.247
smallworld	0.907	0.909	0.928	6.236
delaunay_n14	0.966	1.004	1.019	4.632
delaunay_n17	0.917	0.928	1.033	2.330
hugetrace-00010	0.980	0.981	1.107	0.462
hugetric-00020	0.964	0.964	1.075	0.484
venturiLevel3	0.924	0.925	1.072	0.584
adaptive	0.944	0.945	1.062	0.543
rgg_n_2_15_s0	0.815	0.867	0.982	3.029
rgg_n_2_21_s0	0.919	0.949	1.030	0.440
tn2010	0.838	1.062	4.214	1.222
ut2010	0.219	0.253	0.328	1.907
af_shell9	0.987	0.986	1.065	0.583
audikw1	0.787	0.826	1.094	0.479
asia.osm	0.476	0.496	0.790	0.190
belgium.osm	0.855	0.865	0.990	0.408
memplus	0.696	0.522	1.267	3.130
t60k	0.958	0.958	1.055	3.414

- Best and worst improvements for each graph class normalized w.r.t. PaToH.
- 78% (75%, 67%) improvement on maxSV for ut2010.

Experiment Results: K=256

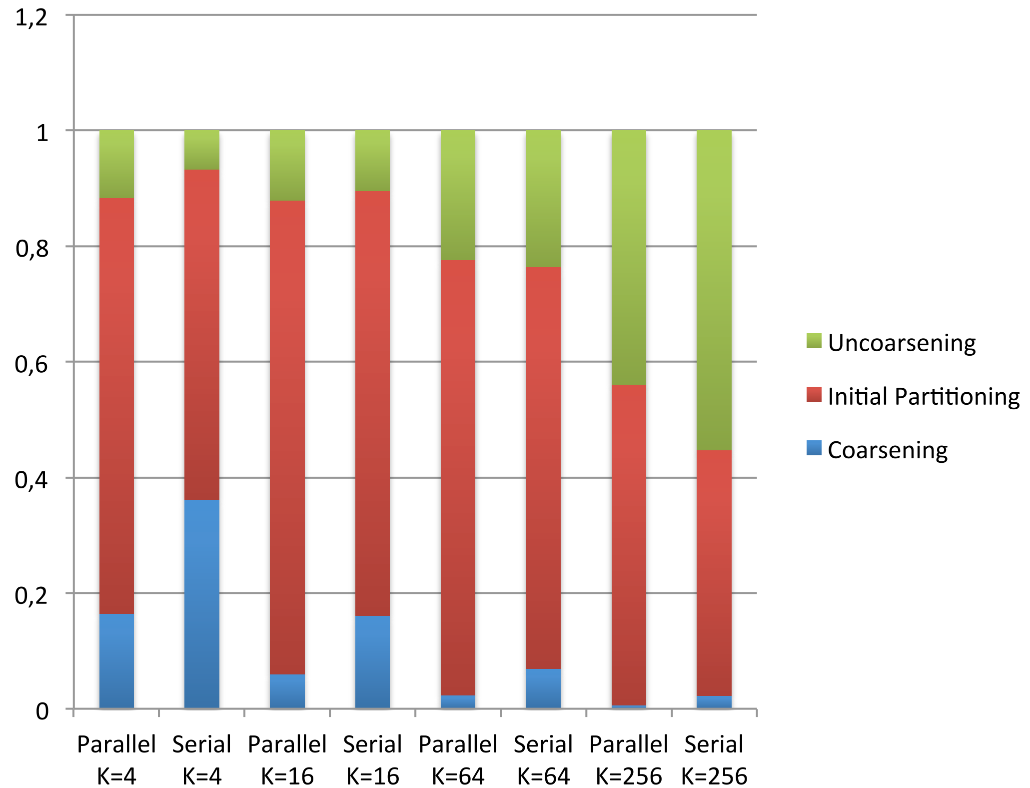
Graph	$maxSV$	$maxSRV$	$totV$	Time
coPapersCiteseer	0.694	0.693	1.005	2.937
coPapersDBLP	0.730	0.690	0.972	7.216
as-22july06	0.397	0.637	1.181	12.495
smallworld	0.839	0.843	0.899	7.965
delaunay_n20	0.927	0.943	1.024	3.829
delaunay_n21	0.948	0.963	1.033	3.066
hugetrace-00000	1.020	1.021	1.075	2.228
hugetric-00010	0.950	0.951	1.065	1.814
adaptive	0.976	0.978	1.063	2.322
venturiLevel3	0.993	0.996	1.057	2.642
rgg_n_2_22_s0	0.906	0.941	1.010	1.647
rgg_n_2_23_s0	0.862	0.891	1.009	1.286
ri2010	0.866	0.965	0.994	12.028
tx2010	0.586	0.816	0.951	1.836
af_shell10	0.986	0.987	1.056	1.758
audikw1	0.895	0.917	1.018	2.551
asia.osm	0.917	0.925	0.989	0.260
great-britain.osm	0.788	0.804	0.997	0.505
finan512	0.965	1.040	1.022	10.073
memplus	0.509	0.541	1.264	16.837

- 50% improvement on maxSV and maxSRV for memplus although total volume increases by 26%.

Execution Time

- K-way partitioners are costly due to the complexity of the refinement heuristic for maxSV.
- UMPa gets slower when the number of parts is large

K	4	16	64	256	Avg.
Relative time	1.02	1.29	2.01	5.76	1.98



Conclusion & Future Work

- Proposed a directed hypergraph model to minimize maxSV, maxSRV and totV.
- We developed a multi-level, K-way partitioner, UMPa.
- Employed a tie-breaking scheme to handle multiple communication metrics.
- Currently, UMPa is parallel (shared memory) at coarsening phase.
- Parallelizing & speeding up UMPa and the proposed refinement approach.

Thanks

- For more information visit
  - http://bmi.osu.edu/hpc

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for each $n \in \mathsf{nets}[u]$ **do**
  if $s(n) = u$ **then**
    $\mathsf{sendGain}[\mathsf{part}[u]] \leftarrow \mathsf{sendGain}[\mathsf{part}[u]] + (\lambda_n - 1)\mathsf{c}[n]$
    if $\Lambda(n, \mathsf{part}[u]) > 1$ **then**
      $receiveGain \leftarrow receiveGain - \mathsf{c}[n]$
      $uToPartU \leftarrow uToPartU + \mathsf{c}[n]$
  else if $\Lambda(n, \mathsf{part}[u]) = 1$ **then**
    $\mathsf{sendGain}[\mathsf{part}[s(n)]] \leftarrow \mathsf{sendGain}[\mathsf{part}[s(n)]] + \mathsf{c}[n]$
    $receiveGain \leftarrow receiveGain + \mathsf{c}[n]$

```
for each part p other than part[u] do
    if p has enough space for vertex u then
        receiveLoss ← 0
        sendLoss[] ← 0
        sendLoss[p] ← sendGain[part[u]] + uToPartU
        for each n ∈ nets[u] do
            if s(n) = u then
                if Λ(n,p) > 0 then
                    sendLoss[p] ← sendLoss[p] − c[n]
                    receiveLoss ← receiveLoss − c[n]
            else if Λ(n,p) = 0 then
                sendLoss[part[s(n)]] ← sendLoss[part[s(n)]] + c[n]
                receiveLoss ← receiveLoss + c[n]
        (moveSV, moveSRV) ← (−∞, −∞)
        for each part q do
            Δ_S ← sendLoss[q] − sendGain[q]
            Δ_R ← 0
            if q = part[u] then
                Δ_R ← receiveGain
            else if q = p then
                Δ_R ← receiveLoss
            moveSV ← max(moveSV, SV[q] + Δ_S)
            moveSRV ← max(moveSRV, SV[q] + Δ_S + RV[q] + Δ_R)
        moveV ← totV + receiveLoss − receiveGain
        MoveSelect(moveSV, moveSRV, moveV, p,
                   bestMaxSV, bestMaxSRV, bestTotV, bestPart)
```