NEW BOUNDS FOR THE HARDCORE MODEL ON THE SQUARE LATTICE

PRASAD TETALI

The hardcore model has received much attention in the past couple of decades as a lattice gas model with hard constraints in statistical physics, a multicast model of calls in communication networks, and as a weighted independent set problem in combinatorics, probability and theoretical computer science.

In this model, each independent set I in a graph G is weighted proportionally to $\lambda^{|I|}$, for a positive real parameter λ. For large λ, computing the partition function (namely, the normalizing constant which makes the weighting a probability distribution on a finite graph) on graphs of maximum degree $d \geq 3$, is a well known computationally challenging problem. More concretely, let $\lambda_c(T_d)$ denote the critical value for the so-called uniqueness threshold of the hardcore model on the d-regular tree; recent breakthrough results of Dror Weitz (2006) and Allan Sly (2010) have identified $\lambda_c(T_d)$ as a threshold where the hardness of estimating the above partition function undergoes a *computational* transition.

We focus on the well-studied particular case of the square lattice Z^2, and provide a new lower bound for the uniqueness threshold, in particular taking it well above $\lambda_c(T_4)$. Our technique refines and builds on the tree of self-avoiding walks approach of Dror Weitz, resulting in a new technical sufficient criterion (of potential wider interest) for establishing strong spatial mixing (and hence uniqueness) for the hardcore model. Our results also imply a fully polynomial deterministic approximation algorithm for approximating the partition function and rapid mixing of the associated Glauber dynamics

This is joint work with Ricardo Restrepo, Jinwoo Shin, Eric Vigoda, and Linji Yang.