CS 4644-DL / 7643-A: LECTURE 7 DANFEI XU

Topics:

- Convolutional Neural Networks: Past and Present
- Convolution Layers

Administrative:

- Assignment due on Sep 17th (with 48hr grace period)
- List of TA expertise up on Piazza
- Proposal template and prompt released.
- Proposal due Sep 24th 11:59pm (**No Grace Period**)
- Start finding a project team if you haven't!

Jacobians

Given a function $f: \mathbb{R}^n \to \mathbb{R}^m$, we have the Jacobian matrix J of shape $m \times n$, where $J_{i,j} = \frac{\partial f_i}{\partial x_i}$ ∂x_j

$$
\mathbf{J} = \left[\begin{array}{ccc} \frac{\partial \mathbf{f}}{\partial x_1} & \cdots & \frac{\partial \mathbf{f}}{\partial x_n} \end{array} \right] = \left[\begin{array}{ccc} \nabla^{\mathrm{T}} f_1 \\ \vdots \\ \nabla^{\mathrm{T}} f_m \end{array} \right] = \left[\begin{array}{ccc} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_n} \end{array} \right]
$$

Figure source: https://en.wikipedia.org/wiki/Jacobian_matrix_and_determinant

Recap: Vector derivatives

Scalar to Scalar

 $x \in \mathbb{R}, y \in \mathbb{R}$

Regular derivative:

Derivative is **Gradient**:

 $x \in \mathbb{R}^N, y \in \mathbb{R}$

Vector to Scalar

Vector to Vector $x \in \mathbb{R}^N, y \in \mathbb{R}^M$

Derivative is **Jacobian**:

 $\frac{\partial y}{\partial x} \in \mathbb{R}^{M \times N}$ $\left(\frac{\partial y}{\partial x}\right)_{n,m} = \frac{\partial y_n}{\partial x_m}$

If x changes by a small amount, how much will y change?

For each element of x, if it changes by a small amount, how much will y change?

For each element of x, if it changes by a small amount, how much will **each element** of y change?

4

Summary (Lecture 5 – here):

- Neural networks, activation functions
- NNs as Universal Function Approximators
- Neurons as biological inspirations to DNNs
- Vector Calculus
- Backpropagation through vectors / matrices

Next: Convolutional Neural Networks

Illustration of LeCun et al. 1998 from CS231n 2017 Lecture 1

A bit of history...

The **Mark I Perceptron** machine was the first implementation of the perceptron algorithm.

The machine was connected to a camera that used 20×20 photocells to produce a 400-pixel image.

 w_0x_0

 w_1x_1

 w_2x_2

cell body

 $f\left(\sum w_i x_i + b\right)$

activation function

 $f(x) = \begin{cases} 1 & \text{if } w \cdot x + b > 0 \\ 0 & \text{otherwise} \end{cases}$ recognized letters of the alphabet

update rule:
 $w_i(t + 1) = w_i(t) + \alpha (d_j - y_j(t)) x_{j,i},$

Frank Rosenblatt, ~1957: Perceptron

age by Rocky Acosta is license

A bit of history...

Widrow and Hoff, ~1960: Adaline/Madaline

Rumelhart et al., 1986: First time back-propagation became popular

A bit of history...

[Hinton and Salakhutdinov 2006]

Reinvigorated research in Deep Learning

Illustration of Hinton and Salakhutdinov 2006 by Lane McIntosh, copyright CS231n 2017

First strong results

Acoustic Modeling using Deep Belief Networks Abdel-rahman Mohamed, George Dahl, Geoffrey Hinton, 2010 Context-Dependent Pre-trained Deep Neural Networks for Large Vocabulary Speech Recognition George Dahl, Dong Yu, Li Deng, Alex Acero, 2012

Imagenet classification with deep convolutional neural networks

Alex Krizhevsky, Ilya Sutskever, Geoffrey E Hinton, 2012

Illustration of Dahl et al. 2012 by Lane McIntosh, copyright CS231n 2017

Figures copyright Alex Krizhevsky, Ilya Sutskever, and Geoffrey Hinton, 2012. Reproduced with permission.

A bit of history:

Hubel & Wiesel, 1959

RECEPTIVE FIELDS OF SINGLE NEURONES IN THE CAT'S STRIATE CORTEX

1962

RECEPTIVE FIELDS, BINOCULAR INTERACTION AND FUNCTIONAL ARCHITECTURE IN THE CAT'S VISUAL CORTEX

Hierarchical organization

Illustration of hierarchical organization in early visual pathways by Lane McIntosh, copyright CS231n 2017

Simple cells: Response to light orientation

Complex cells: Response to light orientation and movement

Hypercomplex cells: response to movement with an end point

A bit of history:

Neocognitron *[Fukushima 1980]*

"sandwich" architecture (SCSCSC…) simple cells: modifiable parameters complex cells: perform pooling

A bit of history: **Gradient-based learning applied to document recognition** *[LeCun, Bottou, Bengio, Haffner 1998]*

LeNet-5

A bit of history: **ImageNet Classification with Deep Convolutional Neural Networks** *[Krizhevsky, Sutskever, Hinton, 2012]*

Figure copyright Alex Krizhevsky, Ilya Sutskever, and Geoffrey Hinton, 2012. Reproduced with permission.

"AlexNet"

Classification **Retrieval**

Figures copyright Alex Krizhevsky, Ilya Sutskever, and Geoffrey Hinton, 2012. Reproduced with permission.

Detection Segmentation

Figures copyright Shaoqing Ren, Kaiming He, Ross Girschick, Jian Sun, 2015. Reproduced with permission.

[Faster R-CNN: Ren, He, Girshick, Sun 2015]

Figures copyright Clement Farabet, 2012. Reproduced with permission.

[Farabet et al., 2012]

Autonomous Driving: GPUs & specialized chips are fast and compact enough for on-board compute!

https://blogs.nvidia.com/blog/2021/01/27/lidar-sensor-nvidia-drive/

https://www.nvidia.com/en-us/self-driving-cars/

McIntosh.

[Guo et al. 2014]

[Toshev, Szegedy 2014]

Figures copyright Xiaoxiao Guo, Satinder Singh, Honglak Lee, Richard Lewis, and Xiaoshi Wang, 2014. Reproduced with permission.

Slide credit: Stanford CS231n Instructors

Generalized convolution**: spatial convolution**

Choi et al., 2019

Kamnitsas et al., 2015

Generalized convolution**: temporal convolution**

Bai et al., 2018

Generalized convolution**: graph convolution**

Kipf et al., 2017

A white teddy bear sitting in the grass

A man riding a wave on top of a surfboard

A man in a baseball uniform throwing a ball

A cat sitting on a suitcase on the floor

No errors Minor errors Somewhat related **Image**

A woman is holding a cat in her hand

A woman standing on a beach holding a surfboard

[Vinyals et al., 2015] [Karpathy and Fei-Fei, 2015] [Radford,

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https://pixabav.com/en/luggage-a
https://pixabay.com/en/teddy-plus
https://pixabay.com/en/surf-wave
https://pixabav.com/en/woman-fe
https://pixabay.com/en/handstano
https://pixabav.com/en/baseball-r

Captions generated by Justin Johnson

Text-to-Image

[Reed, 2016] [Zhang, 2017] [Johnson, 2018] [Ramesh, 2021] [Frans, 2021] [Saharia, 2022] [Ramesh, 2022]

"An avocado armchair"

Convolutional Neural Networks

The connectivity in linear layers **doesn't always make sense**

Q: How many parameters?

 \bullet M^{*}N (weights) + N (bias)

Hundreds of millions of parameters **for just one layer**

More parameters => More data needed & slower to train / inference

Is this necessary?

Image features are spatially localized!

Smaller features repeated across the image

Edges

- Color
- Motifs (corners, etc.)
- No reason to believe one feature tends to appear in a fixed location. Need to search in entire image.

Can we induce a *bias* **in the design of a neural network layer to reflect this?**

Convolution: A 1D Visual Example

Area under $f(x)g(t-x)$ Input function: $f()$ $f(x)$ $0.8\,$ $g(t-x)$ 0.6 $(f+g)(t)$ Kernel/filter function: $g()$ 0.4 0.2 Convolution: $f * g()$ n \cdot -1.5 -1 -0.5 Ω 0.5 1.5 $(f*g)(t):=\int_{-\infty}^{\infty}f(\tau)g(t-\tau)\,d\tau.$ t & t Area under $f(x)g(t-x)$ $f(x)$ $g(t-x)$ 0.5 $(f+g)(t)$ O -1.5 -0.5 0.5 1.5 $\overline{2}$ 2.5 -1 0 τ&t

From https://en.wikipedia.org/wiki/Convolution

 $\overline{2}$

3.

Convolution

$$
(f*g)(t):=\int_{-\infty}^{\infty}f(\tau)g(t-\tau)\,d\tau.
$$

1-D Convolution is defined as the **integral** of the **product** of two functions after one is reflected about the y-axis and shifted.

Intuitively: given function f and filter g . How similar is $g(-x)$ with the part of $f(x)$ that it's operating on.

Cross-correlation is convolution without the y-axis reflection.

For ConvNets, we don't flip filters to improve efficiency, so we are really using **Cross-Correlation Nets**!

From https://en.wikipedia.org/wiki/Convolution

Convolution in Computer Vision (non-Deep)

Convolution with Gaussian Filter (Gaussian Blur) Convolution with Sobel Filter (Edge Detection)

Locality of Features

$$
\mathbf{G}_x = \begin{bmatrix} +1 & 0 & -1 \\ +2 & 0 & -2 \\ +1 & 0 & -1 \end{bmatrix} * \mathbf{A}
$$

$$
\mathbf{G}_y = \begin{bmatrix} +1 & +2 & +1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} * \mathbf{A}
$$

Intuition for pattern recognition and learning using convolution

- g(): filter / pattern template
- f(): signal / observed data
- f*g(): how well data matches with the template

- For Convolution Layers in NN:
- g() as the **weights** to learn
- f() as the **input** to the layer
- f*g() as the **output** of the layer (result of convolution)
- Discrete instead of continuous convolution (sum instead of integral)
- g() and f() may be N-dimensional, where $N \ge 1$

From https://en.wikipedia.org/wiki/Convolution

Fully Connected Layer

32x32x3 image -> stretch to 3072 x 1

2-D Convolution Layer

32x32x3 image -> preserve spatial structure

2-D Convolution Layer

32x32x3 image

5x5x3 filter

Convolve the filter with the image i.e. "slide over the image spatially, computing dot products at each location"

2-D Convolution Layer

Filters always extend the full depth of the input volume

5x5x**3** filter

Convolve the filter with the image i.e. "slide over the image spatially, computing dot products"

activation map

consider a second, green filter

For example, if we had 6 5x5 filters, we'll get 6 separate activation maps:

We stack these up to get a "new image" of size 28x28x6!

Preview: ConvNet is a sequence of Convolution Layers, interspersed with activation functions

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Preview *isomate the preview and Fergus 2013 Visualization of VGG-16 by Lane McIntosh. VGG-16* architecture from [Simonyan and Zisserman 2014]. architecture from [Simonyan and Zisserman 2014].

Conv2D in PyTorch

Applies a 2D convolution over an input signal composed of several input planes.

In the simplest case, the output value of the layer with input size $(N, C_{\rm in}, H, W)$ and output $(N, C_{\text{out}}, H_{\text{out}}, W_{\text{out}})$ can be precisely described as:

$$
\operatorname{out}(N_i, C_{\operatorname{out}_j}) = \operatorname{bias}(C_{\operatorname{out}_j}) + \sum_{k=0}^{C_{\operatorname{in}}-1} \operatorname{weight}(C_{\operatorname{out}_j}, k) \star \operatorname{input}(N_i, k)
$$

where \star is the valid 2D cross-correlation operator, N is a batch size, C denotes a number of channels, H is a height of input planes in pixels, and W is width in pixels.

7x7 input (spatially) assume 3x3 filter

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The # of grid that the filter shifts is called **stride.**

E.g., here we have stride $= 1$

7x7 input (spatially) assume 3x3 filter **with stride = 1**

7x7 input (spatially) assume $3x3$ filter with stride = 1

=> 5x5 output

7x7 input (spatially) assume $3x3$ filter with stride = 1

=> 5x5 output

But what about the features at the border?

In practice: Common to zero pad the border

e.g. input 7x7 **3x3** filter, applied with **stride 1 pad with 1 pixel** border => what is the output?

In practice: Common to zero pad the border

e.g. input 7x7 **3x3** filter, applied with **stride 1 pad with 1 pixel** border => what is the output?

7x7 output!

in general, common to see CONV layers with stride 1, filters of size FxF, and zero-padding with (F-1)/2. (will preserve size spatially) e.g. $F = 3 \Rightarrow$ zero pad with 1 $F = 5 \Rightarrow$ zero pad with 2 $F = 7 \Rightarrow$ zero pad with 3

In practice: Common to zero pad the border

e.g. input 7x7 **3x3** filter, applied with **stride 1 pad with 1 pixel** border => what is the output?

7x7 output!

- $N =$ input dimension
- $P =$ padding size
- $F =$ filter size

Output size = $(N - F + 2P)$ / stride + 1 $= (7 - 3 + 2 * 1)/1 + 1 = 7$

7x7 input (spatially) assume 3x3 filter applied **with stride 2**

7x7 input (spatially) assume 3x3 filter applied **with stride 2**

7x7 input (spatially) assume 3x3 filter applied **with stride 2 => 3x3 output!**

7x7 input (spatially) assume 3x3 filter applied **with stride 3?**

7x7 input (spatially) assume 3x3 filter applied **with stride 3?** N

Output size: **(N - F) / stride + 1**

e.g. N = 7, F = 3: stride 1 => (7 - 3)/1 + 1 = 5 stride 2 => (7 - 3)/2 + 1 = 3 stride 3 => (7 - 3)/3 + 1 = 2.33 : \

We use floor division to calculate output size: $(7 - 3)$ // 3 + 1 = 2

Remember back to…

E.g. 32x32 input convolved repeatedly with 5x5 filters shrinks volumes spatially! (32 -> 28 -> 24 ...). Shrinking too fast is not good, doesn't work well.

Remember back to…

With padding, we can keep the same spatial feature dimension throughout the convolution layers.

Examples time:

Input volume: **32x32x3** Conv layer: 10 5x5 filters with stride 1, pad 2

Output volume size: ? $(N - F + 2P) /$ stride + 1

Examples time:

Input volume: **32x32x3** Conv layer: 10 5x5 filters with stride 1, pad 2

Output volume size: $(32+2^*2-5)/1+1 = 32$ spatially, so **32x32x10**

Examples time:

Input volume: **32x32x3** Conv layer: 10 5x5 filters with stride 1, pad 2

Number of parameters in this layer?

Examples time:

Input volume: **32x32x3** Conv layer: 10 5x5 filters with stride 1, pad 2

Number of parameters in this layer? each filter has $5*5*3 + 1 = 76$ params (+1 for bias) => 76*10 = **760**

Convolution layer: summary

Let's assume input is W_1 x H₁ x C Conv layer needs 4 hyperparameters:

- Number of filters **K**
- The filter size **F**
- The stride **S**
- The zero padding **P**

This will produce an output of W_2 x H₂ x K where:

- $W_2 = (W_1 F + 2P)/S + 1$
- $H_2 = (H_1 F + 2P)/S + 1$

Number of parameters: F²CK and K biases

Convolution layer: summary

Let's assume input is W_1 x H₁ x C Conv layer needs 4 hyperparameters:

- Number of filters **K**
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This will produce an output of W_2 x H₂ x K where:

- $W_2 = (W_1 F + 2P)/S + 1$
- H₂ = (H₁ F + 2P)/S + 1

Number of parameters: F²CK and K biases

Common settings:

K = (powers of 2, e.g. 32, 64, 128, 512)

$$
-
$$
 F = 3, S = 1, P = 1

$$
-
$$
 F = 5, S = 1, P = 2

$$
- F = 5, S = 2, P = ? (whatever fits)
$$

-
$$
F = 1
$$
, $S = 1$, $P = 0$

Example: CONV layer in PyTorch

Conv layer needs 4 hyperparameters:

- Number of filters **K**
- The filter size **F**
- The stride **S**
- The zero padding **P**

Conv2d

CLASS torch.nn.Conv2d(in_channels, out_channels, kernel_size, stride=1, padding=0, [SOURCE] dilation=1, groups=1, bias=True)

Applies a 2D convolution over an input signal composed of several input planes.

In the simplest case, the output value of the layer with input size (N, C_{in}, H, W) and output $(N, C_{\rm out}, H_{\rm out}, W_{\rm out})$ can be precisely described as:

$$
\operatorname{out}(N_i, C_{\operatorname{out}_j}) = \operatorname{bias}(C_{\operatorname{out}_j}) + \sum_{k=0}^{C_{\text{in}}-1} \operatorname{weight}(C_{\operatorname{out}_j}, k) \star \operatorname{input}(N_i, k)
$$

where \star is the valid 2D cross-correlation operator, N is a batch size, C denotes a number of channels, H is a height of input planes in pixels, and W is width in pixels.

- stride controls the stride for the cross-correlation, a single number or a tuple.
- · padding controls the amount of implicit zero-paddings on both sides for padding number of points for each dimension.
- · dilation controls the spacing between the kernel points; also known as the à trous algorithm. It is harder to describe, but this link has a nice visualization of what dilation does.
- · groups controls the connections between inputs and outputs. in_channels and out_channels must both be divisible by groups. For example,
	- o At groups=1, all inputs are convolved to all outputs.
	- o At groups=2, the operation becomes equivalent to having two conv layers side by side, each seeing half the input channels, and producing half the output channels, and both subsequently concatenated.
	- \circ At groups= in_channels , each input channel is convolved with its own set of filters, of size: $\left| \frac{C_{\text{out}}}{C_{\text{in}}} \right|$.

The parameters kernel_size, stride, padding, dilation can either be:

- a single int in which case the same value is used for the height and width dimension
- a tuple of two ints in which case, the first int is used for the height dimension, and the second int for the width dimension

PyTorch is

Conv features can grow really big really quickly…

Solution 1: 1x1 Convolution

Solution 1: 1x1 Convolution

Solution 2: Pooling (downsampling)

- makes the representations spatially smaller
- saves computation (GPU mem & speed), allows go deeper
- operates over each activation map independently:

MAX POOLING

Single depth slice

y

x

max pool with 2x2 filters and stride 2 6 8

- Intuitively, only forward the most important features in the region.
- Also improve spatial invariance (output is agnostic to where the max value comes from)

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AVG POOLING

Single depth slice

y

x

average pool with 2x2 filters and stride 2.

Pooling layer: summary

Let's assume input is W_1 x H₁ x C Pooling layer needs 2 hyperparameters:

- The spatial extent **F** (e.g., 2)
- The stride **S** (e.g., 2)

This will produce an output of W_2 x H₂ x C where:

- $W_2 = (W_1 F)/S + 1$
- $H_2 = (H_1 F)/S + 1$

Number of parameters? Ω

A canonical (shallow) convolutional neural net

Next Time:

• Convolutional Neural Nets!