CS 7643: Deep Learning

Topics:

- Review of Classical Reinforcement Learning
- Value-based Deep RL
- Policy-based Deep RL

Dhruv Batra

Georgia Tech

Types of Learning

- Supervised learning
  - Learning from a “teacher”
  - Training data includes desired outputs
- Unsupervised learning
  - Training data does not include desired outputs
- Reinforcement learning
  - Learning to act under evaluative feedback (rewards)

Supervised Learning

Data: (x, y)
x is data, y is label

Goal: Learn a *function* to map x -> y

Examples: Classification, regression, object detection, semantic segmentation, image captioning, etc.

x y

→ Cat

Classification

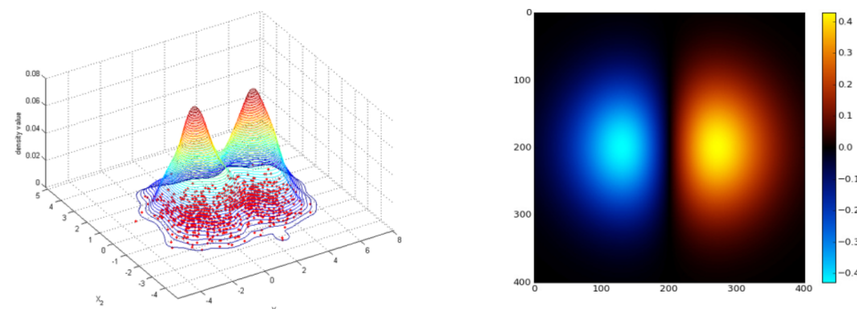
Unsupervised Learning

Data: x
Just data, no labels!

Goal: Learn some underlying hidden *structure* of the data

Examples: Clustering, dimensionality reduction, feature learning, density estimation, etc.

1-d density estimation

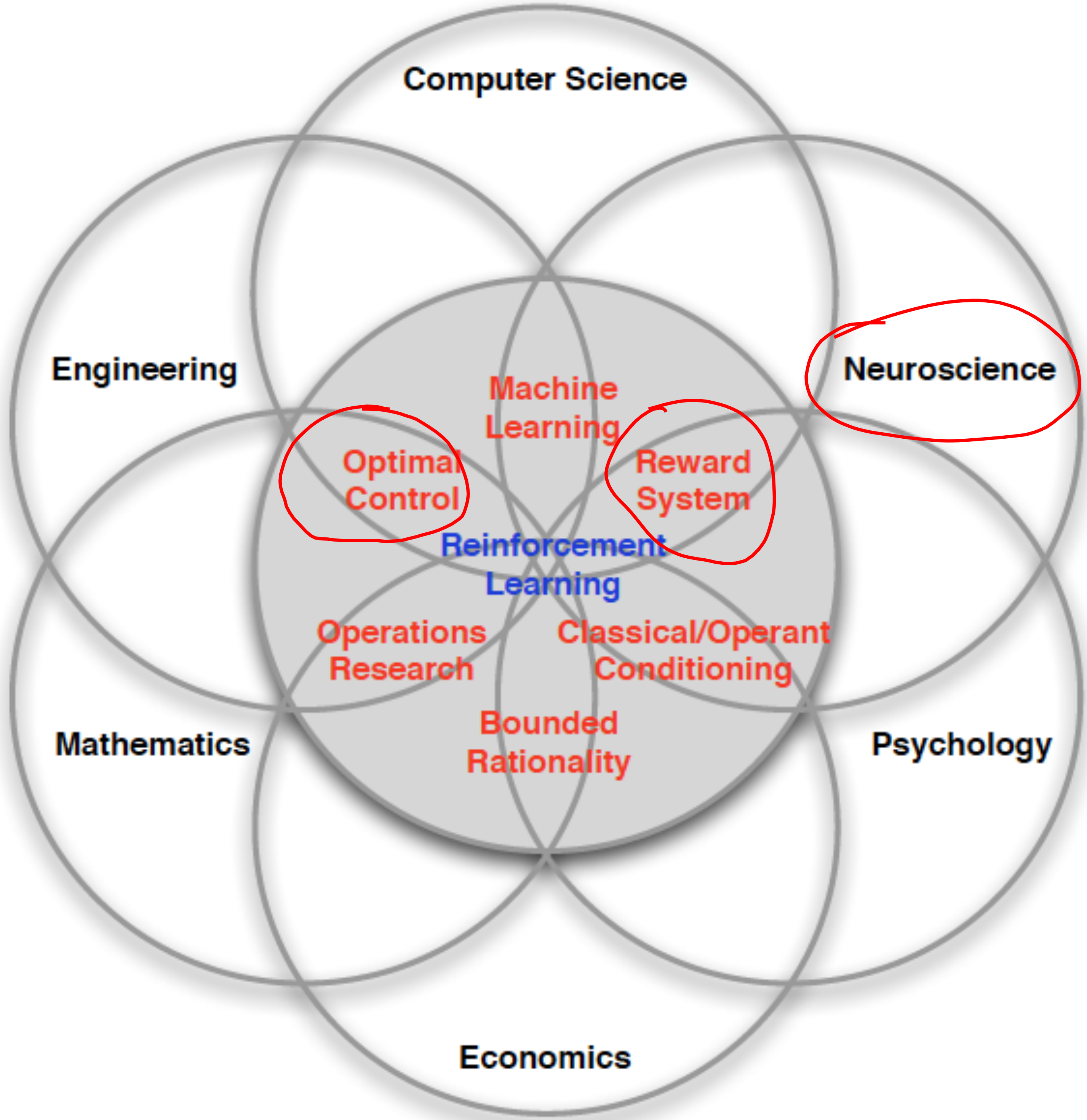


2-d density estimation

What is Reinforcement Learning?

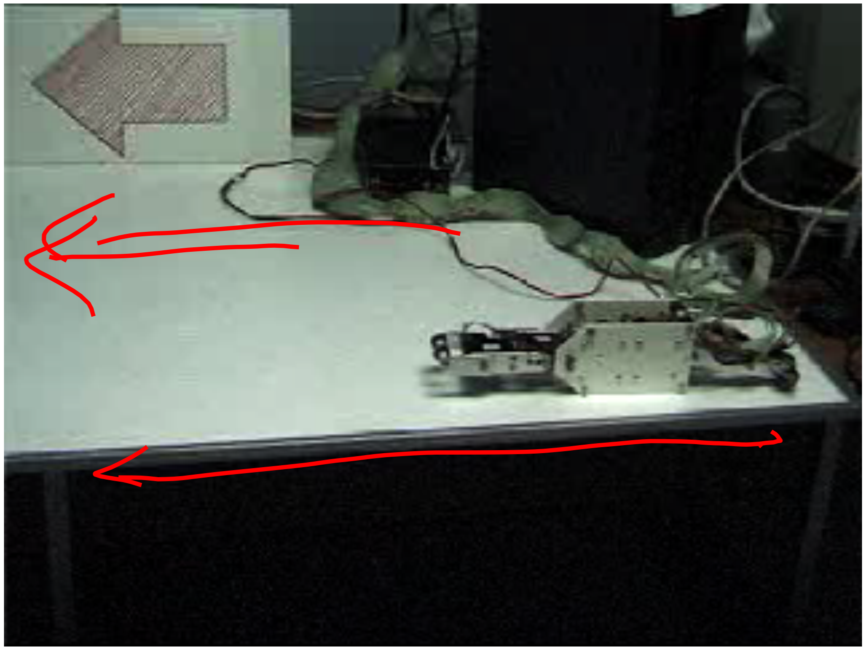
- Agent-oriented learning—learning by interacting with an environment to achieve a goal
  - more realistic and ambitious than other kinds of machine learning
- Learning by trial and error, with only delayed evaluative feedback (reward)
  - the kind of machine learning most like natural learning
  - learning that can tell for itself when it is right or wrong

Slide Credit: Rich Sutton

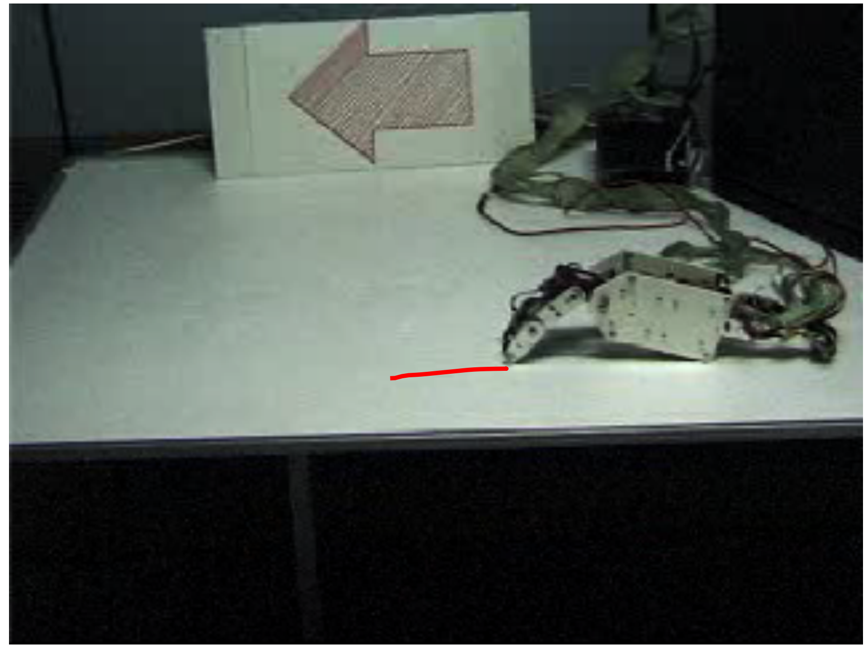

Computer Science
Engineering
Neuroscience
Machine Learning
Optimal Control
Reward System
Reinforcement Learning
Operations Research
Classical/Operant Conditioning
Bounded Rationality
Mathematics
Psychology
Economics

Silver 2015

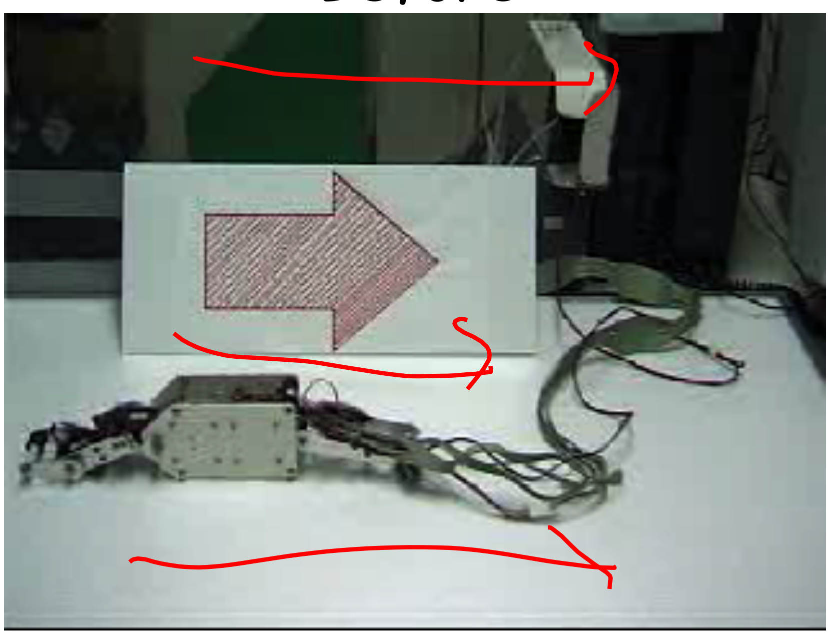
Example: Hajime Kimura's RL Robots



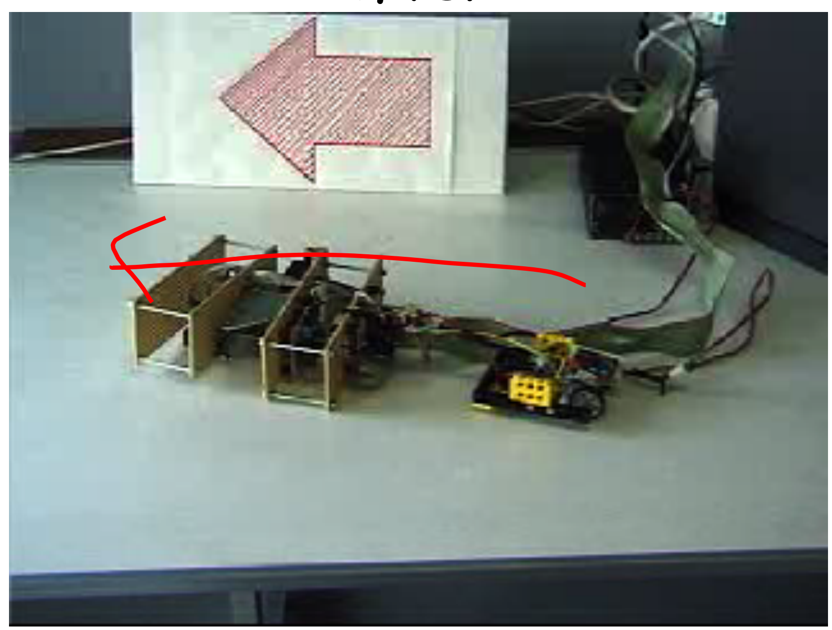
Before



After



Backward



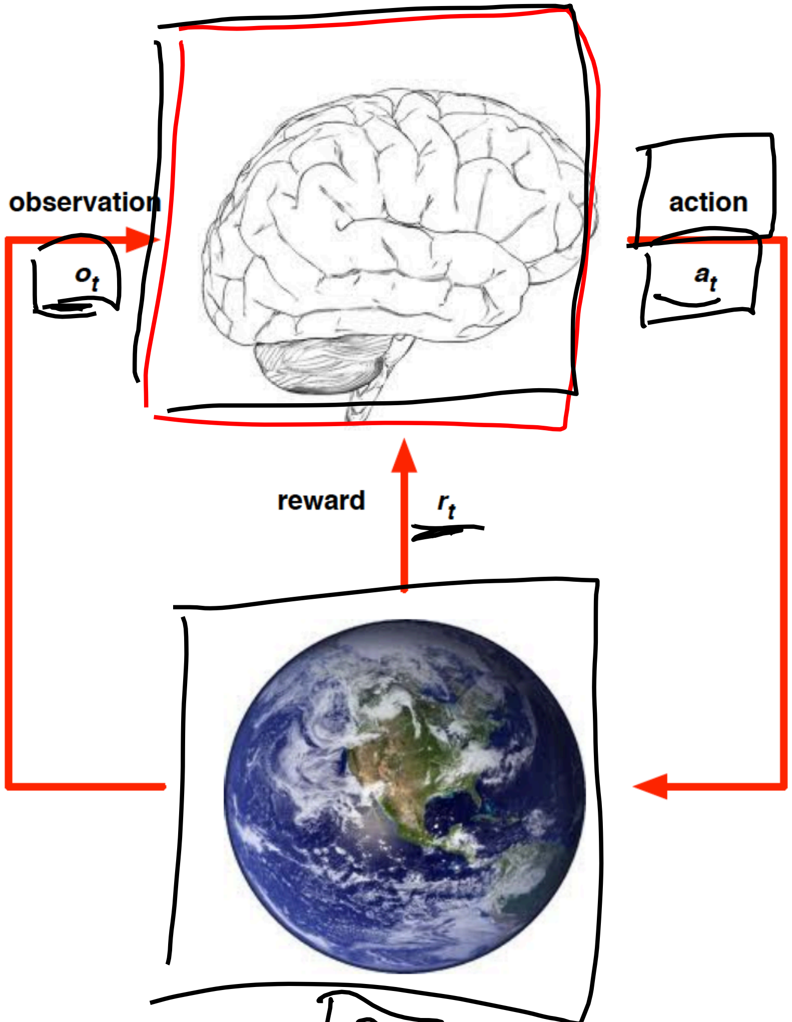
New Robot, Same algorithm

Signature challenges of RL

- Evaluative feedback (reward)
- Sequentiality, delayed consequences
- Need for trial and error, to explore as well as exploit
- Non-stationarity
- The fleeting nature of time and online data

Slide Credit: Rich Sutton

RL API



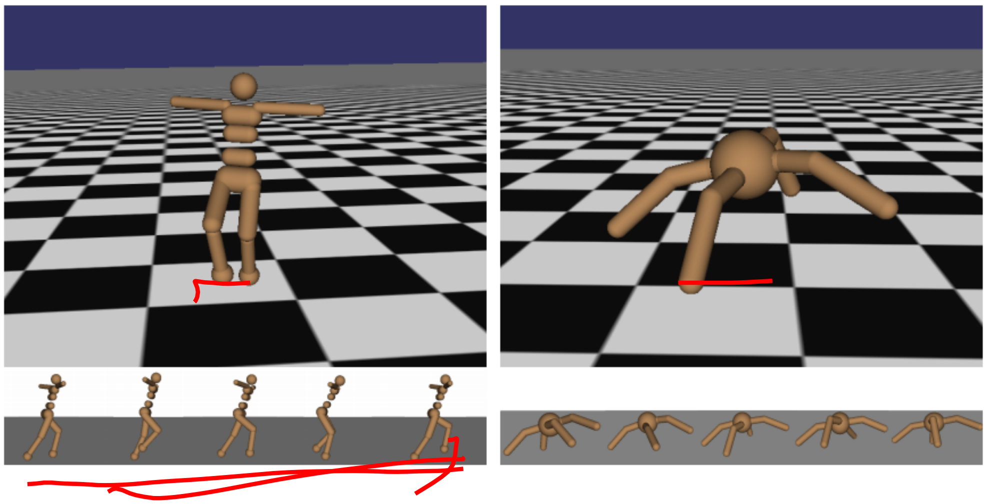
$o_t = s_t$

$o_t = f(s_t)$

- At each step t the agent:
  - Executes action a_t
  - Receives observation o_t
  - Receives scalar reward r_t
- The environment:
  - Receives action a_t
  - Emits observation o_{t+1}
  - Emits scalar reward r_{t+1}

State

Robot Locomotion



Objective: Make the robot move forward

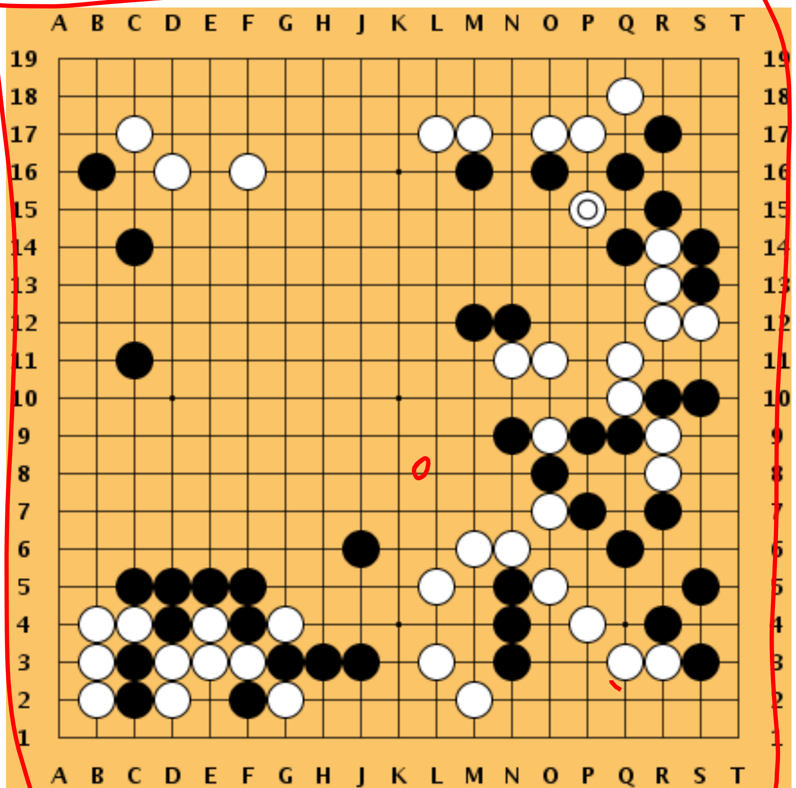
State: Angle and position of the joints
Action: Torques applied on joints
Reward: 1 at each time step upright + forward movement

Atari Games

Objective: Complete the game with the highest score

State: Raw pixel inputs of the game state
Action: Game controls e.g. Left, Right, Up, Down
Reward: Score increase/decrease at each time step

Go



Objective: Win the game!

State: Position of all pieces
Action: Where to put the next piece down
Reward: 1 if win at the end of the game, 0 otherwise

Demo

Markov Decision Process

- Mathematical formulation of the RL problem

$s_t, a_t \rightarrow s_{t+1}$

Defined by: $(\mathcal{S}, \mathcal{A}, \mathcal{R}, \mathbb{P}, \gamma)$

$\mathcal{S}$: set of possible states
$\mathcal{A}$: set of possible actions
$\mathcal{R}$: distribution of reward given (state, action) pair
$\mathbb{P}$: transition probability i.e. distribution over next state given (state, action) pair
γ : discount factor

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- Life is trajectory: $\ldots S_t, A_t, R_{t+1}, S_{t+1}, A_{t+1}, R_{t+2}, S_{t+2}, \ldots$

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- Life is trajectory: $\ldots S_t, A_t, R_{t+1}, S_{t+1}, A_{t+1}, R_{t+2}, S_{t+2}, \ldots$

- **Markov property**: Current state completely characterizes the state of the world

$$p(r, s'|s, a) = Prob\left[R_{t+1} = r, S_{t+1} = s' \mid S_t = s, A_t = a\right]$$

Components of an RL Agent

(Deep)

- Policy -based RL
  - How does an agent behave?
- Value function RL
  - How good is each state and/or state-action pair?
- Model RL
  - Agent's representation of the environment

Policy

- A policy is how the agent acts
- Formally, map from states to actions

e.g.

State	Action
A	→ 2
B	→ 1

Deterministic policy: $a = \pi(s)$

Stochastic policy: $\pi(a|s) = \mathbb{P}[A_t = a | S_t = s]$

The optimal policy π^*

What's a good policy?

The optimal policy **π***

What's a good policy?

Maximizes current reward? Sum of all future reward?

The optimal policy **π***

What's a good policy?

Maximizes current reward? Sum of all future reward?

Discounted future rewards!

$$r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots$$

$(0, 1)$

The optimal policy **π***

What's a good policy?

Maximizes current reward? Sum of all future reward?

Discounted future rewards!

Formally: $$\pi^* = \arg\max_{\pi} \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | \pi\right]$$

with $$s_0 \sim p(s_0), a_t \sim \pi(\cdot|s_t), s_{t+1} \sim p(\cdot|s_t, a_t)$$

Value Function

- A value function is a prediction of future reward

- "State Value Function" or simply "Value Function"
  - How good is a state?
  - Am I screwed? Am I winning this game?

- "Action Value Function" or Q-function
  - How good is a state action-pair?
  - Should I do this now?

$Q(s, a)$

Definitions: Value function and Q-value function

Following a policy produces sample trajectories (or paths) s_0, a_0, r_0, s_1, a_1, r_1, …

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How good is a state?
The **value function** at state s, is the expected cumulative reward from state s (and following the policy thereafter):

$$V^{\pi}(s) = \mathbb{E}\left[\sum_{t \geq 0} \gamma^t r_t | s_0 = s, \pi\right]$$

Definitions: Value function and Q-value function

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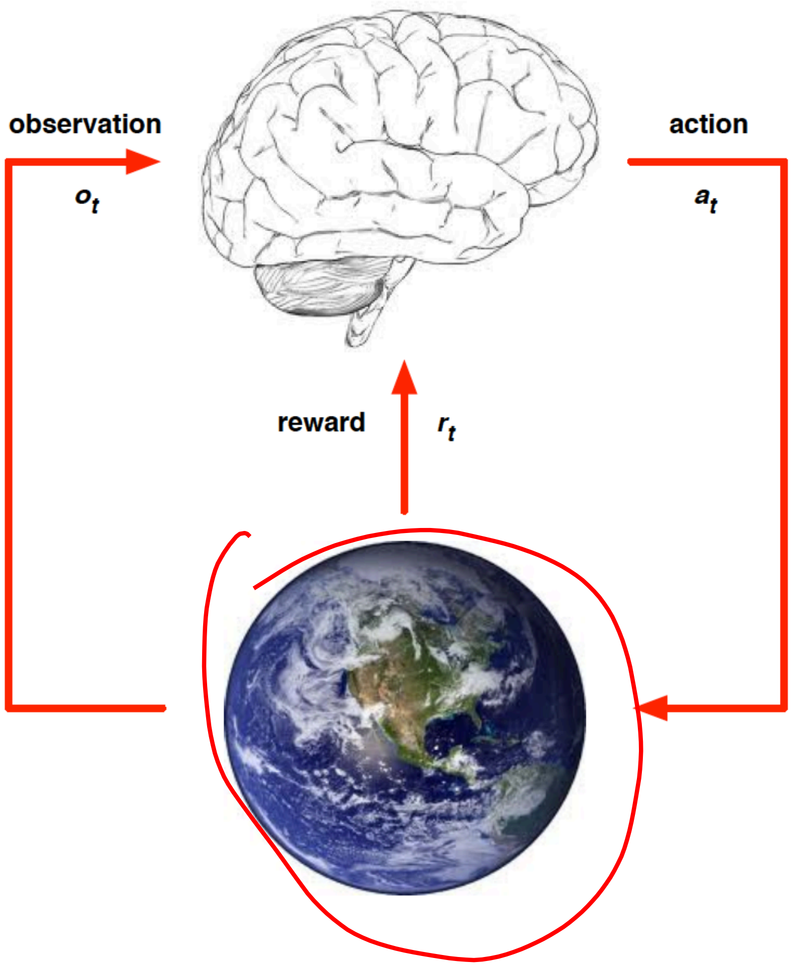
How good is a state-action pair?
The **Q-value function** at state s and action a, is the expected cumulative reward from taking action a in state s (and following the policy thereafter):

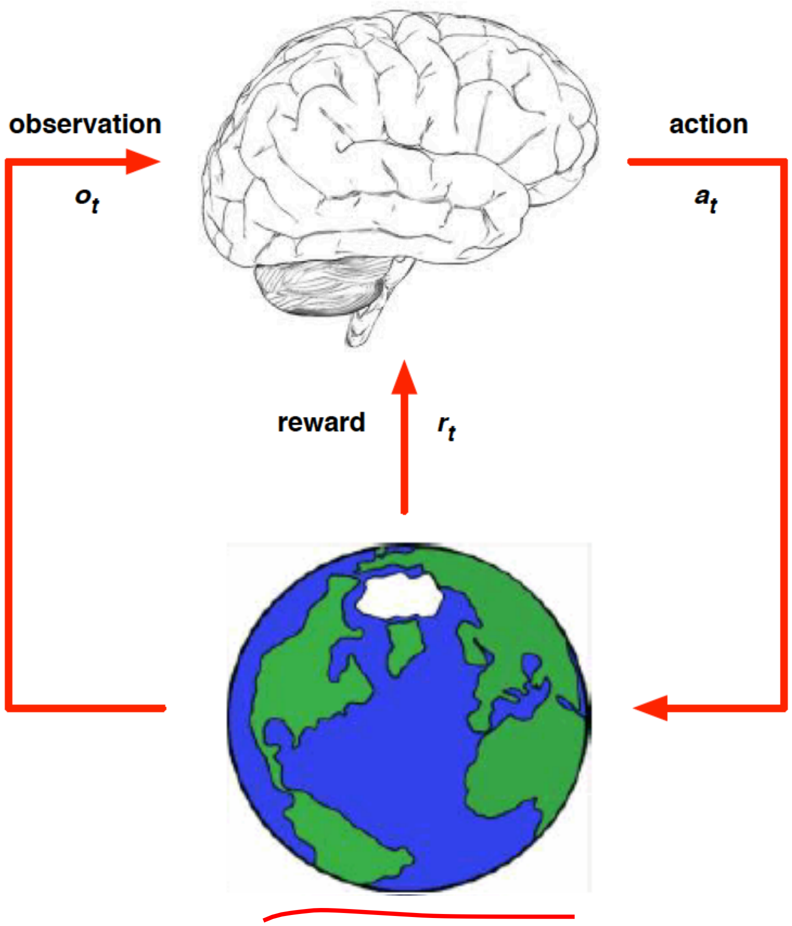
$$Q^{\pi}(s, a) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | s_0 = s, a_0 = a, \pi\right]$$

Model



Model

- Model predicts what the world will do next

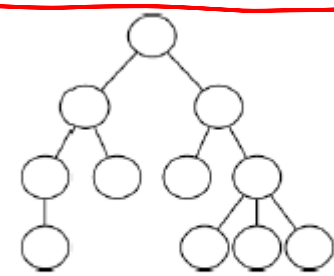


Model is learnt from experience

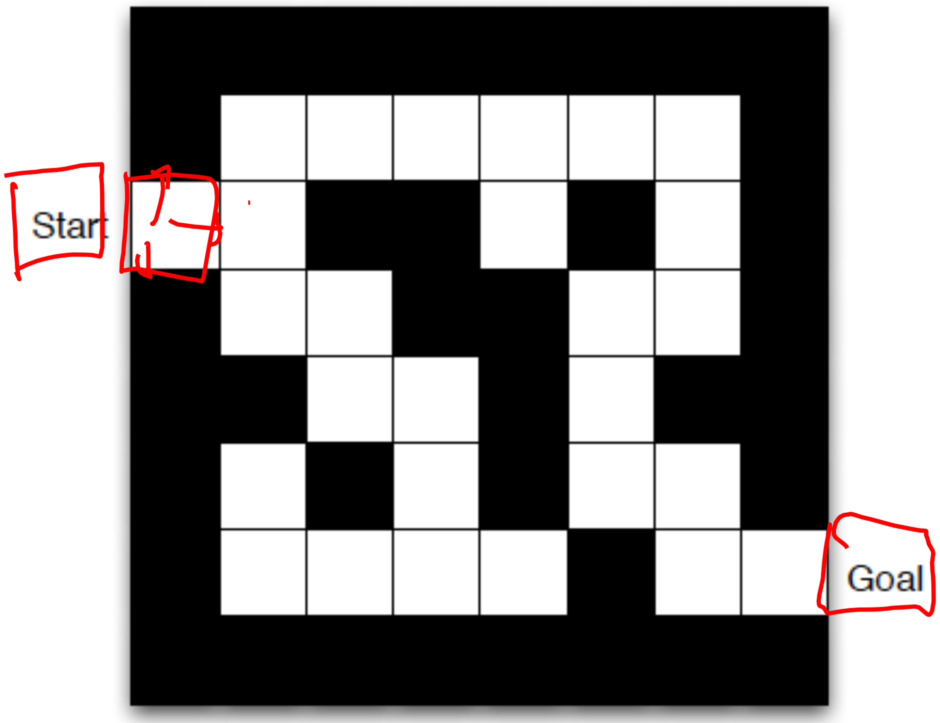
Acts as proxy for environment

Planner interacts with model

e.g. using lookahead search

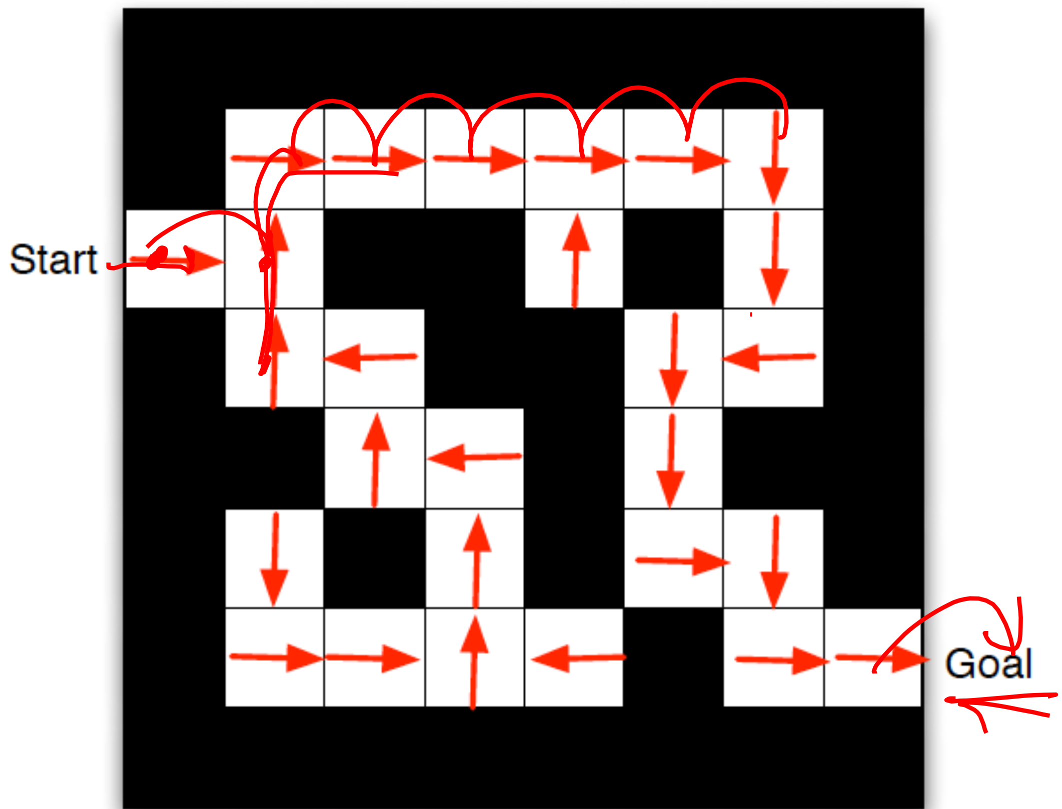


Maze Example



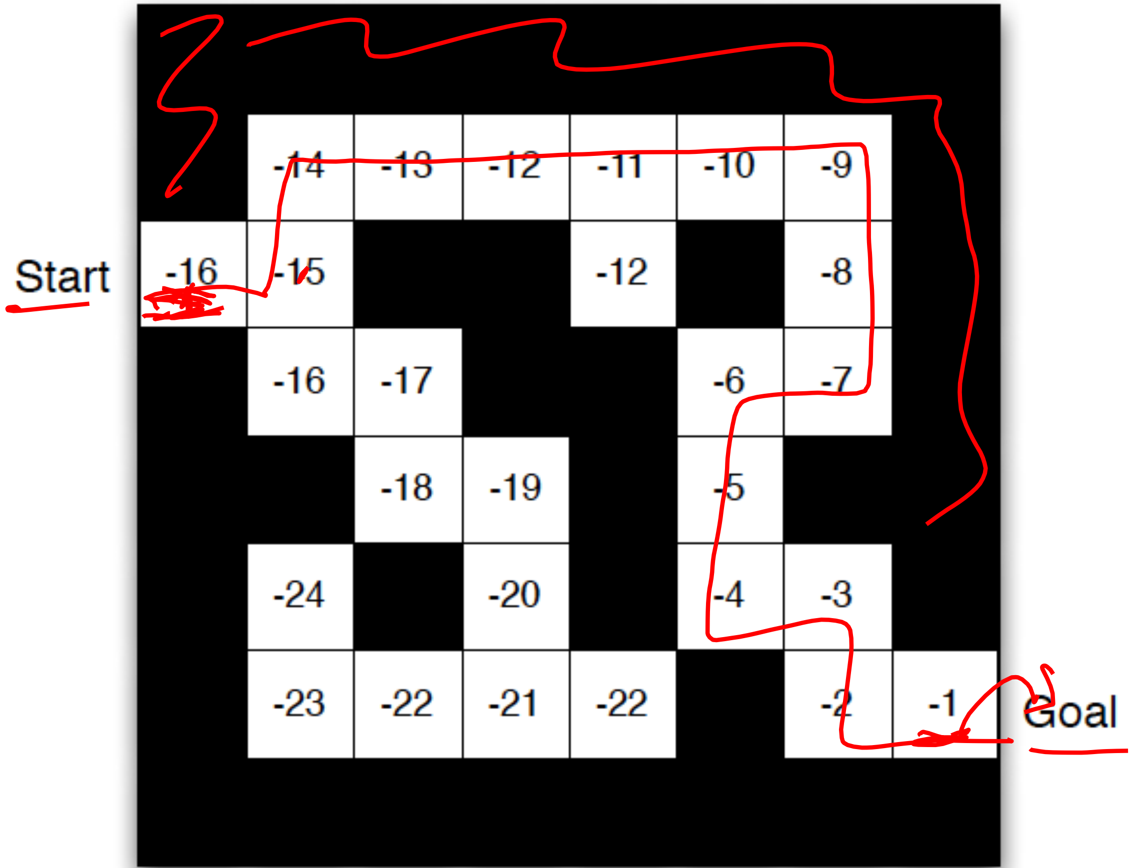
- Rewards: -1 per time-step
- Actions: N, E, S, W
- States: Agent's location

Maze Example: Policy



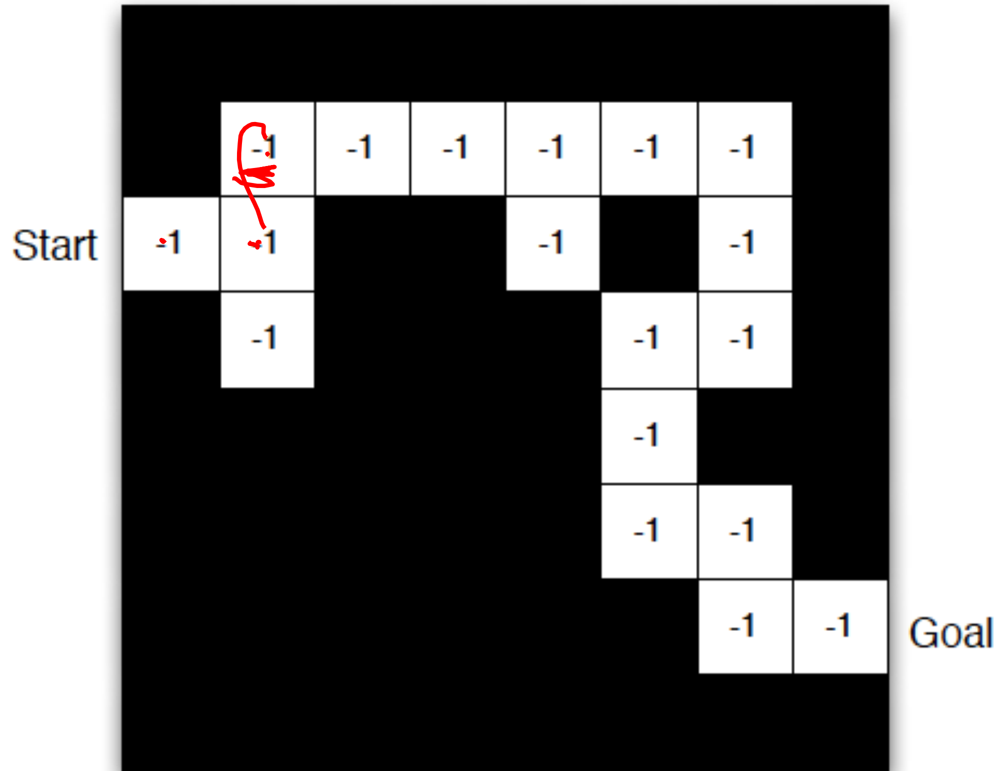
- Arrows represent policy $\pi(s)$ for each state s

Maze Example: Value



- Numbers represent value $v_\pi(s)$ of each state s

Maze Example: Model



- Agent may have an internal model of the environment
- Dynamics: how actions change the state
- Rewards: how much reward from each state
- The model may be imperfect

- Grid layout represents transition model $\mathcal{P}^a_{ss'}$
- Numbers represent immediate reward $\mathcal{R}^a_s$ from each state s (same for all a)

Components of an RL Agent

- Value function
  - How good is each state and/or state-action pair?
- Policy
  - How does an agent behave?
- Model
  - Agent's representation of the environment

Approaches to RL

- Value-based RL
  - Estimate the optimal action-value function $Q^*(s, a)$
- Policy-based RL
  - Search directly for the optimal policy π^*
- Model
  - Build a model of the world
    - State transition, reward probabilities
  - Plan (e.g. by look-ahead) using model

Deep RL

- Value-based RL
  - Use neural nets to represent Q function $Q(s, a; \theta)$

$$Q(s, a; \theta^*) \approx Q^*(s, a)$$

- Policy-based RL
  - Use neural nets to represent policy π_θ

$$\pi_{\theta^*} \approx \pi^*$$

- Model
  - Use neural nets to represent and learn the model

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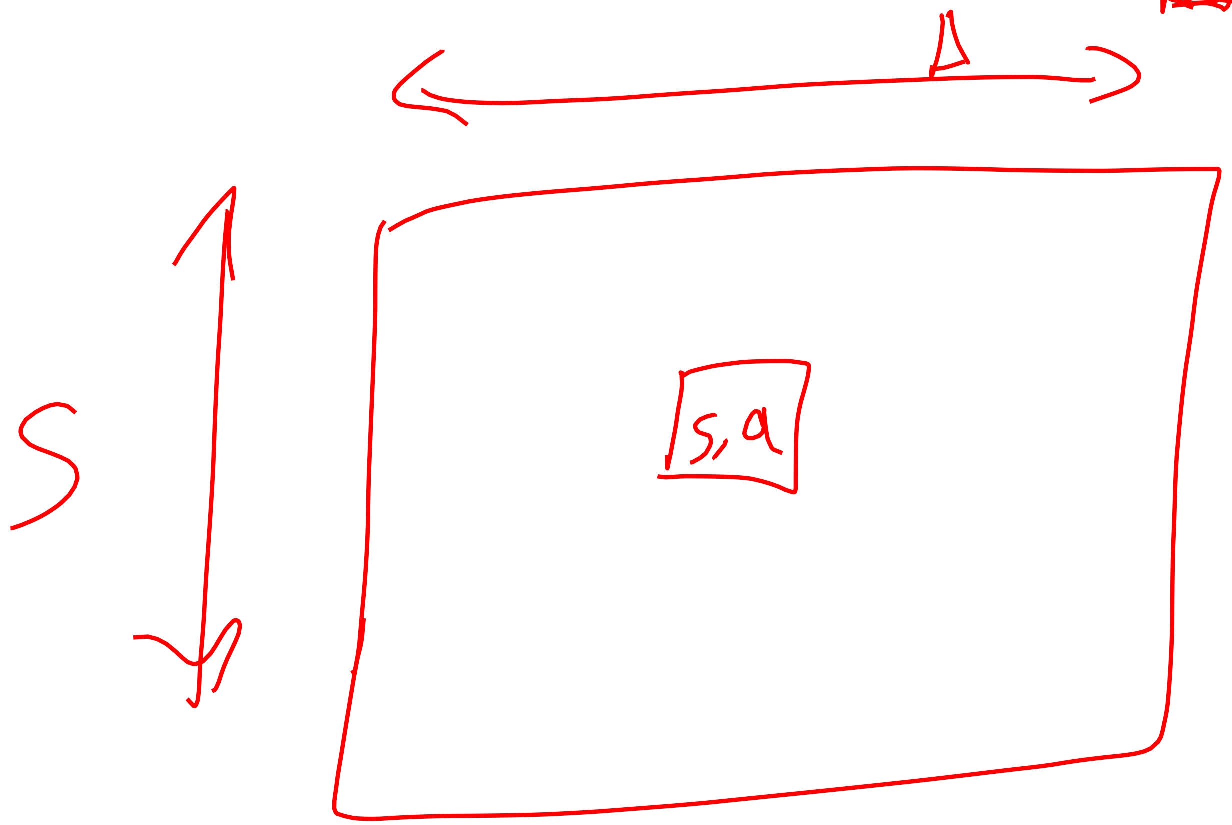

Optimal Value Function

- Optimal Q-function is the maximum achievable value

$$Q^*(s,a) = \max_{\pi} Q^{\pi}(s,a) = Q^{\pi^*}(s,a)$$

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- Optimal Q-function is the maximum achievable value

$$Q^*(s,a) = \max_\pi Q^\pi(s,a) = Q^{\pi^*}(s,a)$$

- Once we have it, we can act optimally

$$\pi^*(s) = \operatorname*{argmax}_a Q^*(s,a)$$

Optimal Value Function

$$Q^*(s,a) = \max_{\pi} Q^{\pi}(s,a) = Q^{\pi^*}(s,a)$$

$$\pi^*(s) = \operatorname{argmax}_{a} Q^*(s,a)$$

- Optimal value maximizes over all future decisions

$$Q^*(s,a) = r_{t+1} + \gamma \max_{a_{t+1}} r_{t+2} + \gamma^2 \max_{a_{t+2}} r_{t+3} + \ldots$$

$$= r_{t+1} + \gamma \max_{a_{t+1}} Q^*(s_{t+1}, a_{t+1})$$

Optimal Value Function

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$Q_{i+1}(s,a) \leftarrow Q_i(s,a)$

- Formally, Q* satisfies Bellman Equations

$$Q^*(s,a) = \mathbb{E}_{s'}\left[r + \gamma \max_{a'} Q^*(s',a') \mid s,a \right]$$

Solving for the optimal policy

Value iteration algorithm: Use Bellman equation as an iterative update

$$Q_{i+1}(s,a) = \mathbb{E}\left[r + \gamma \max_{a'} Q_i(s',a')|s,a\right]$$

Q_i will converge to Q^* as i -> infinity

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What's the problem with this?
Not scalable. Must compute Q(s,a) for every state-action pair. If state is e.g. current game state pixels, computationally infeasible to compute for entire state space!

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Solution: use a function approximator to estimate Q(s,a). E.g. a neural network!

Demo

- http://cs.stanford.edu/people/karpathy/reinforcejs/gridworld_td.html

Deep RL

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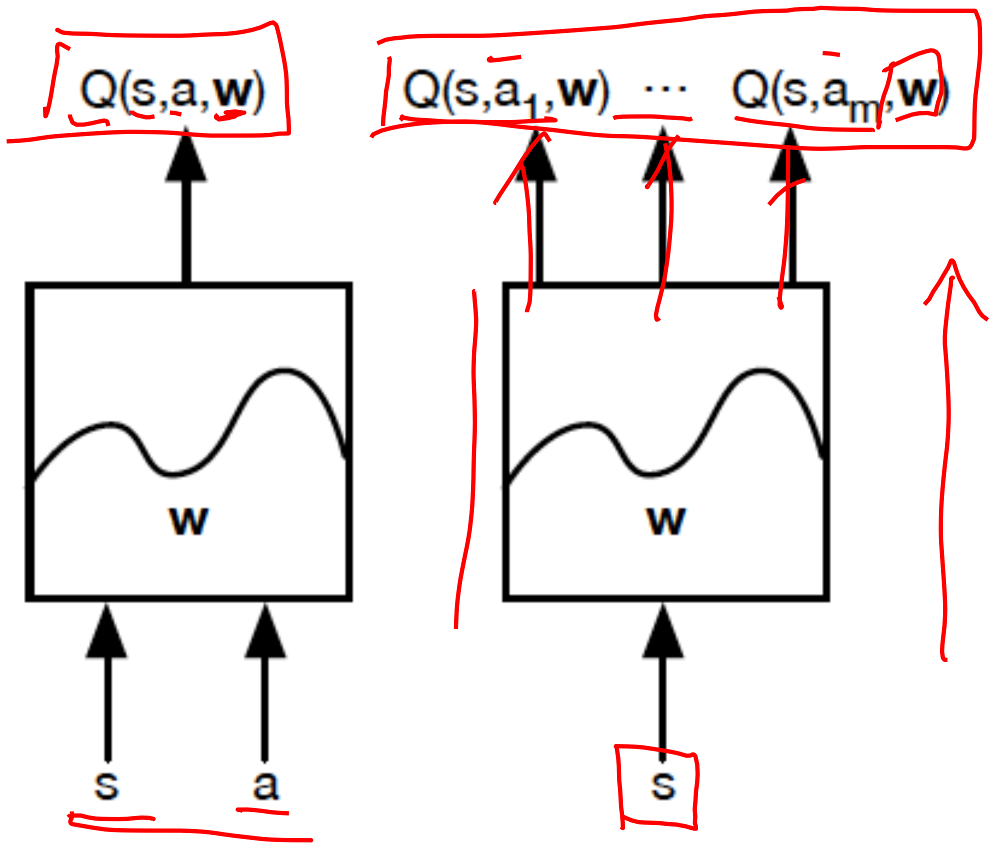
- Model
  - Use neural nets to represent and learn the model

Q-Networks

$$Q(s, a, \mathbf{w}) \approx Q^*(s, a)$$



Slide Credit: David Silver

[Mnih et al. NIPS Workshop 2013; Nature 2015]

Case Study: Playing Atari Games

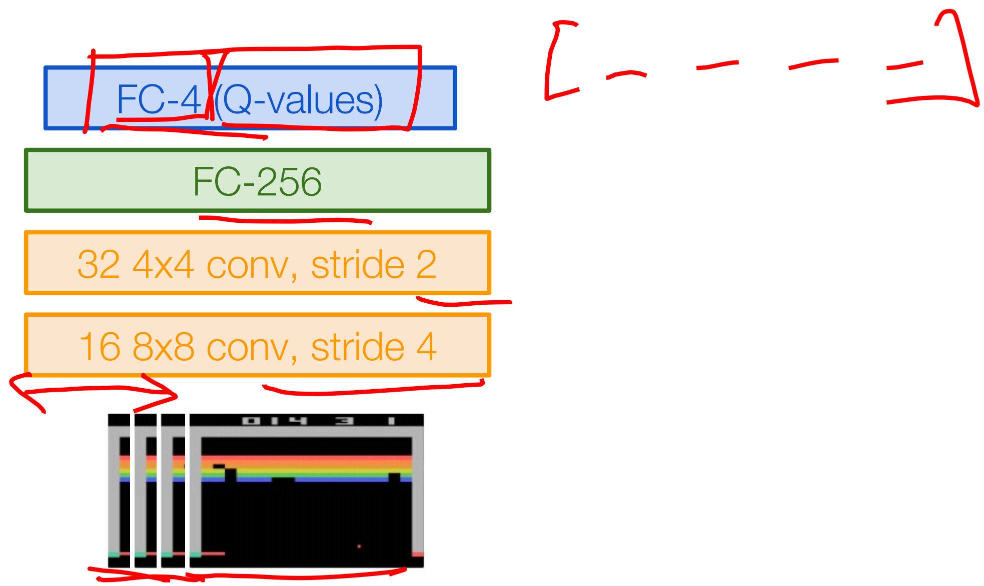
Objective: Complete the game with the highest score

State: Raw pixel inputs of the game state
Action: Game controls e.g. Left, Right, Up, Down
Reward: Score increase/decrease at each time step

[Mnih et al. NIPS Workshop 2013; Nature 2015]

Q-network Architecture

$Q(s, a; \theta)$:
neural network
with weights θ



Current state s_t: 84x84x4 stack of last 4 frames
(after RGB->grayscale conversion, downsampling, and cropping)

[Mnih et al. NIPS Workshop 2013; Nature 2015]

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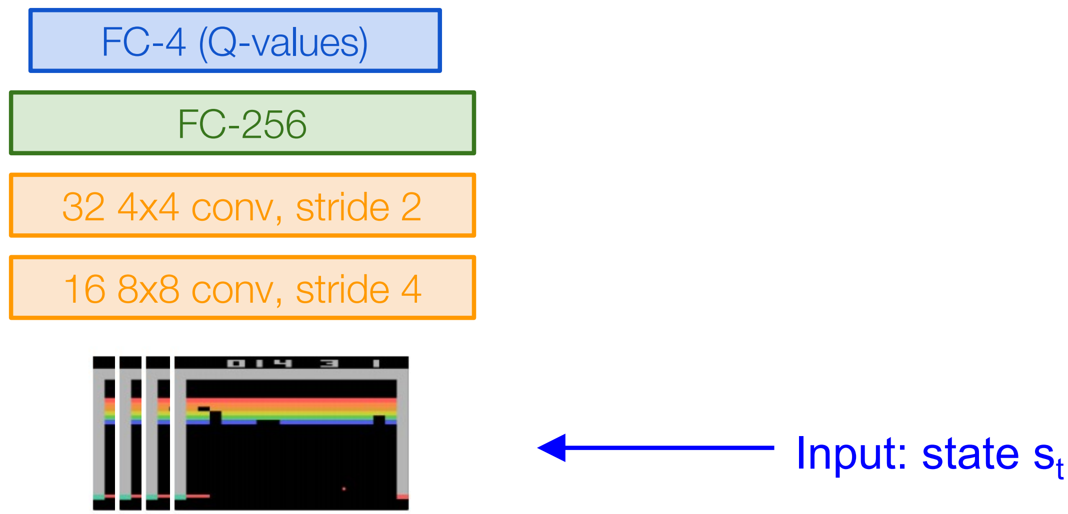

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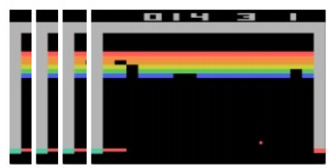
FC-4 (Q-values)

FC-256

32 4x4 conv, stride 2

16 8x8 conv, stride 4

← Familiar conv layers, FC layer



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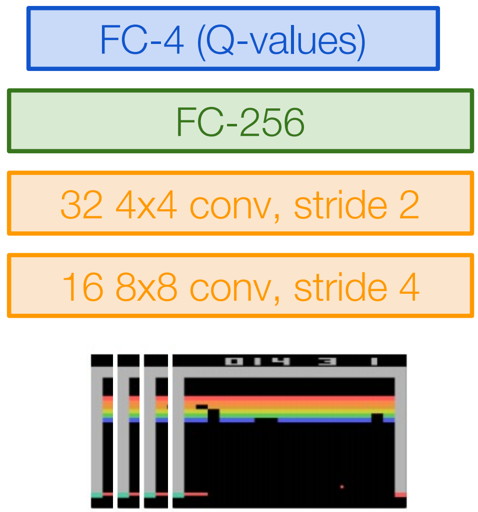

Last FC layer has 4-d output (if 4 actions), corresponding to $Q(s_t, a_1)$, $Q(s_t, a_2)$, $Q(s_t, a_3)$, $Q(s_t,a_4)$

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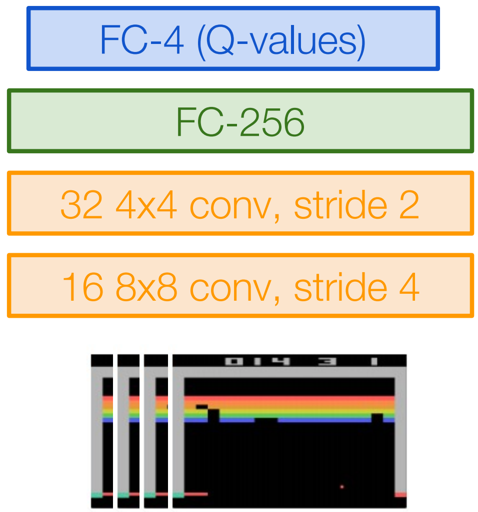

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Number of actions between 4-18 depending on Atari game

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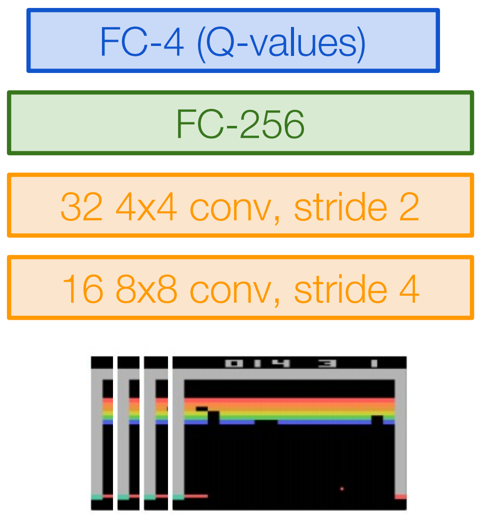
[Mnih et al. NIPS Workshop 2013; Nature 2015]

Q-network Architecture

$Q(s, a; \theta)$: neural network with weights θ

A single feedforward pass to compute Q-values for all actions from the current state => efficient!



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Deep Q-learning

Remember: want to find a Q-function that satisfies the Bellman Equation:

$$Q^*(s, a) = \mathbb{E}[r + \gamma \max_{a'} Q^*(s', a') \mid s, a]$$

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Forward Pass

Loss function: $L_i(\theta_i) = \mathbb{E}\left[(y_i - Q(s,a;\theta_i))^2\right]$

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Backward Pass

Gradient update (with respect to Q-function parameters θ):

$$\nabla_{\theta_i} L_i(\theta_i) = \mathbb{E}\left[\left(r + \gamma \max_{a'} Q(s',a';\theta_{i-1}) - Q(s,a;\theta_i)\right)\nabla_{\theta_i} Q(s,a;\theta_i)\right]$$

Deep Q-learning

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Forward Pass

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where $y_i = \mathbb{E}[r + \gamma \max_{a'} Q^*(s',a') \mid s,a]$

Iteratively try to make the Q-value close to the target value (y_i) it should have, if Q-function corresponds to optimal Q* (and optimal policy **π***)

Backward Pass

Gradient update (with respect to Q-function parameters θ):

$$\nabla_{\theta_i} L_i(\theta_i) = \mathbb{E}\left[r + \gamma \max_{a'} Q(s',a';\theta_{i-1}) - Q(s,a;\theta_i))\nabla_{\theta_i} Q(s,a;\theta_i)\right]$$

[Mnih et al. NIPS Workshop 2013; Nature 2015]

Training the Q-network: Experience Replay

Learning from batches of consecutive samples is problematic:

- Samples are correlated => inefficient learning
- Current Q-network parameters determines next training samples (e.g. if maximizing action is to move left, training samples will be dominated by samples from left-hand size) => can lead to bad feedback loops

[Mnih et al. NIPS Workshop 2013; Nature 2015]

Training the Q-network: Experience Replay

Learning from batches of consecutive samples is problematic:
- Samples are correlated => inefficient learning
- Current Q-network parameters determines next training samples (e.g. if maximizing action is to move left, training samples will be dominated by samples from left-hand size) => can lead to bad feedback loops

Address these problems using **experience replay**
- Continually update a **replay memory** table of transitions (s_t, a_t, r_t, s_{t+1}) as game (experience) episodes are played
- Train Q-network on random minibatches of transitions from the replay memory, instead of consecutive samples

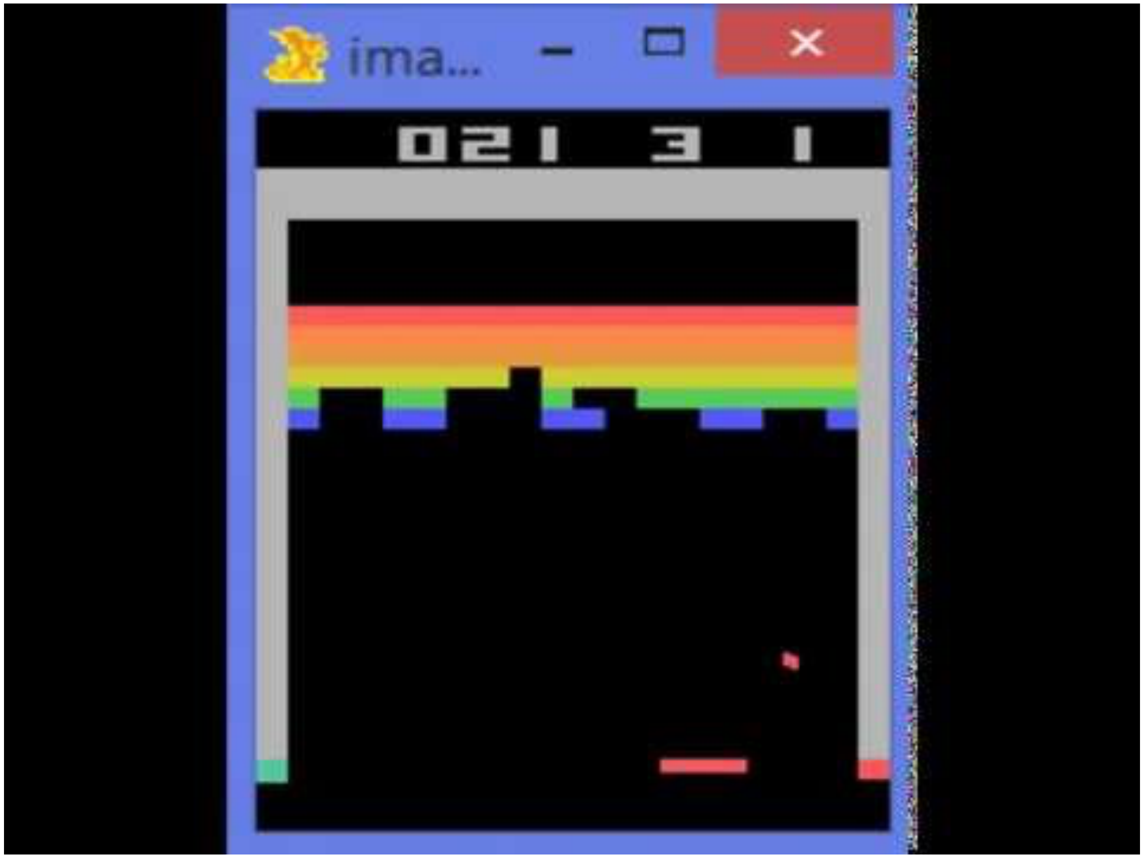
Experience Replay

To remove correlations, build data-set from agent's own experience

s_1, a_1, r_2, s_2
s_2, a_2, r_3, s_3
s_3, a_3, r_4, s_4
$\dots$
$s_t, a_t, r_{t+1}, s_{t+1}$

$s_t, a_t, r_{t+1}, s_{t+1} \rightarrow$ (added to data-set)

(sampled from data-set) $\rightarrow s, a, r, s'$



https://www.youtube.com/watch?v=V1eYniJ0Rnk

Video by Károly Zsolnai-Fehér. Reproduced with permission.

Deep RL

- Value-based RL
  - Use neural nets to represent Q function $Q(s, a; \theta)$

$$Q(s, a; \theta^*) \approx Q^*(s, a)$$

- Policy-based RL
  - Use neural nets to represent policy π_θ

$$\pi_{\theta^*} \approx \pi^*$$

- Model
  - Use neural nets to represent and learn the model

Policy Gradients

Formally, let's define a class of parameterized policies: $\Pi = \{\pi_\theta, \theta \in \mathbb{R}^m\}$

For each policy, define its value:

$$J(\theta) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | \pi_\theta\right]$$

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$$\frac{\partial J}{\partial \theta}$$

We want to find the optimal policy $\theta^* = \arg\max_\theta J(\theta)$

How can we do this?

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Gradient ascent on policy parameters!

REINFORCE algorithm

Mathematically, we can write:

$$J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)}\left[r(\tau)\right]$$
$$= \int_{\tau} r(\tau) p(\tau;\theta) \mathrm{d}\tau$$

Where r(τ) is the reward of a trajectory $\tau = (s_0, a_0, r_0, s_1, \ldots)$

REINFORCE algorithm

Expected reward:
$$J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)}\left[r(\tau)\right]$$
$$= \int_{\tau} r(\tau)p(\tau;\theta)\mathrm{d}\tau$$

REINFORCE algorithm

Expected reward: $$J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)} \left[r(\tau) \right]$$
$$= \int_{\tau} r(\tau) p(\tau;\theta) \mathrm{d}\tau$$

Now let's differentiate this: $$\nabla_{\theta} J(\theta) = \int_{\tau} r(\tau) \nabla_{\theta} p(\tau;\theta) \mathrm{d}\tau$$

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Intractable! Expectation of gradient is problematic when p depends on θ

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If we inject this back:

$$\nabla_\theta J(\theta) = \int_\tau \left(r(\tau) \nabla_\theta \log p(\tau;\theta) \right) p(\tau;\theta) \mathrm{d}\tau$$
$$= \mathbb{E}_{\tau \sim p(\tau;\theta)} [r(\tau) \nabla_\theta \log p(\tau;\theta)]$$

Can estimate with Monte Carlo sampling

REINFORCE algorithm

Can we compute those quantities without knowing the transition probabilities?

We have: $p(\tau;\theta) = \prod_{t\geq 0} p(s_{t+1}|s_t, a_t)\pi_\theta(a_t|s_t)$

REINFORCE algorithm

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Doesn't depend on transition probabilities!

REINFORCE algorithm

$$\nabla_\theta J(\theta) = \int_\tau \left(r(\tau)\nabla_\theta \log p(\tau;\theta)\right) p(\tau;\theta)\mathrm{d}\tau$$
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Doesn't depend on transition probabilities!

Therefore when sampling a trajectory τ, we can estimate J(θ) with

$$\nabla_\theta J(\theta) \approx \sum_{t\geq 0} r(\tau)\nabla_\theta \log \pi_\theta(a_t|s_t)$$

1 sample

Intuition

Gradient estimator: $$\nabla_\theta J(\theta) \approx \sum_{t \geq 0} r(\tau) \nabla_\theta \log \pi_\theta(a_t|s_t)$$

Interpretation:

- If r(τ) is high, push up the probabilities of the actions seen
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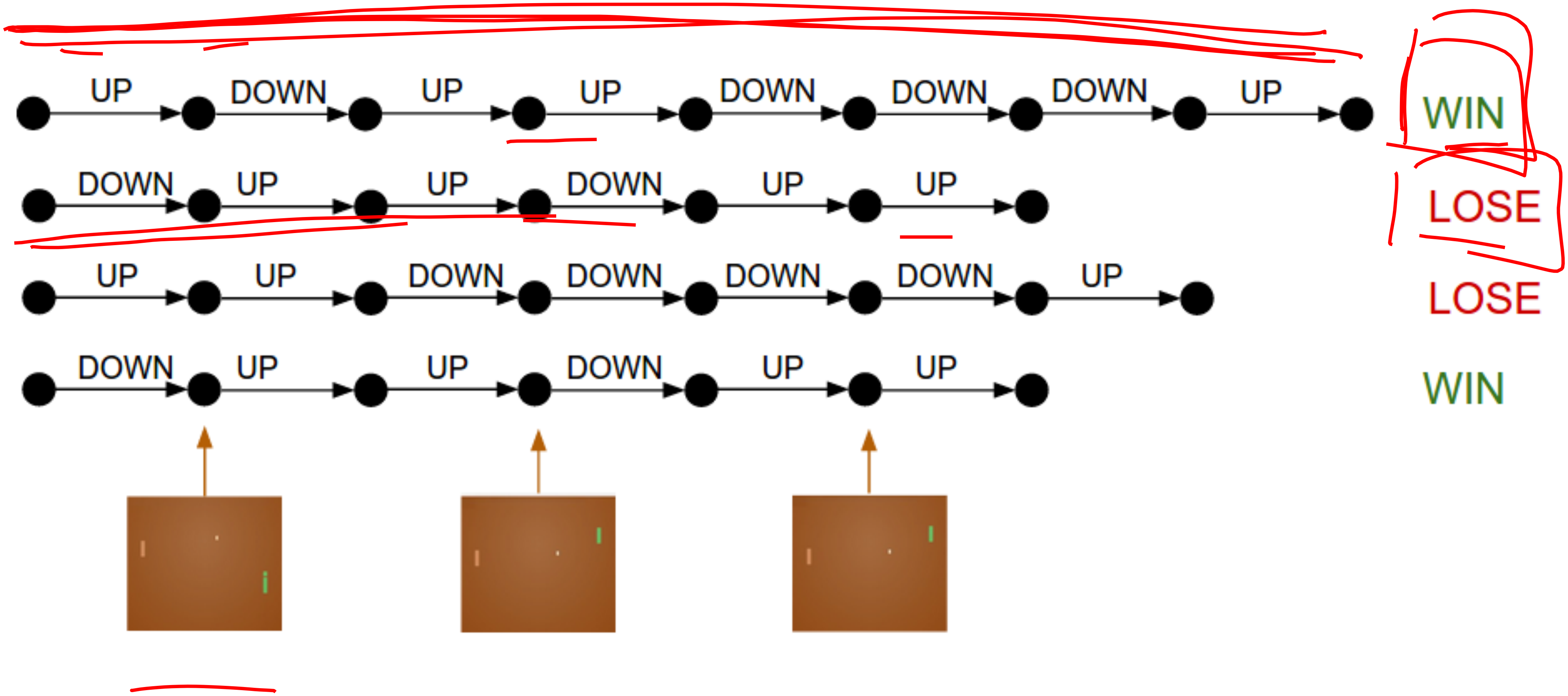
UP
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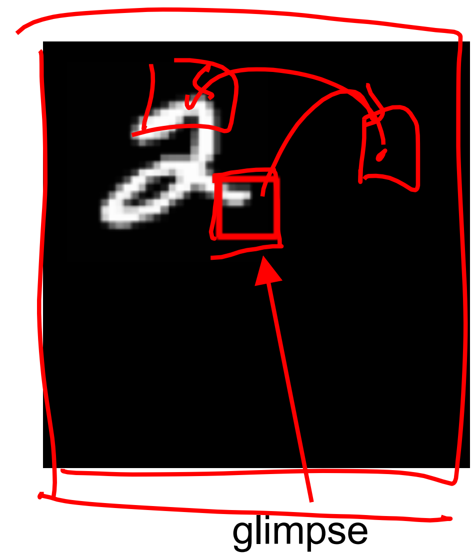
REINFORCE in action: Recurrent Attention Model (RAM)

Objective: Image Classification

Take a sequence of “glimpses” selectively focusing on regions of the image, to predict class

- Inspiration from human perception and eye movements
- Saves computational resources => scalability
- Able to ignore clutter / irrelevant parts of image

State: Glimpses seen so far
Action: (x,y) coordinates (center of glimpse) of where to look next in image
Reward: 1 at the final timestep if image correctly classified, 0 otherwise



[Mnih et al. 2014]

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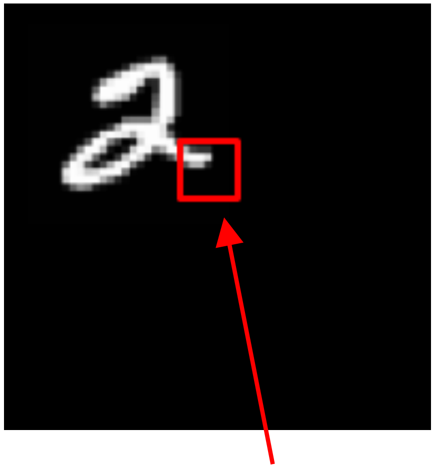

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Glimpsing is a non-differentiable operation => learn policy for how to take glimpse actions using REINFORCE
Given state of glimpses seen so far, use RNN to model the state and output next action

[Mnih et al. 2014]

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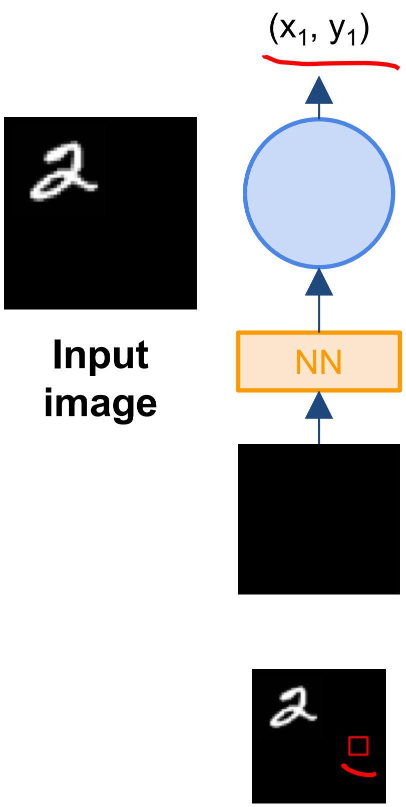

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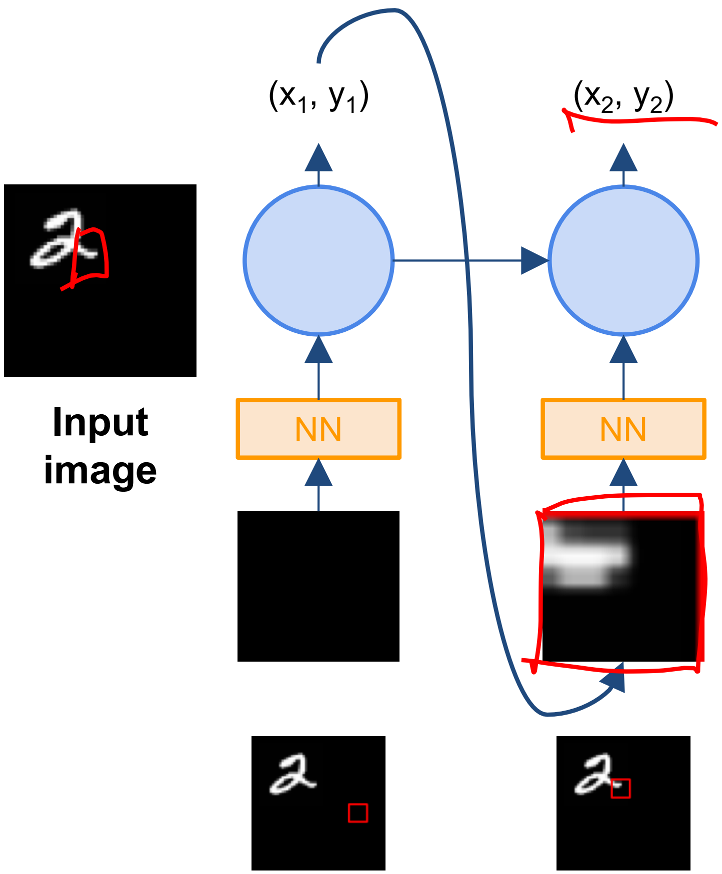

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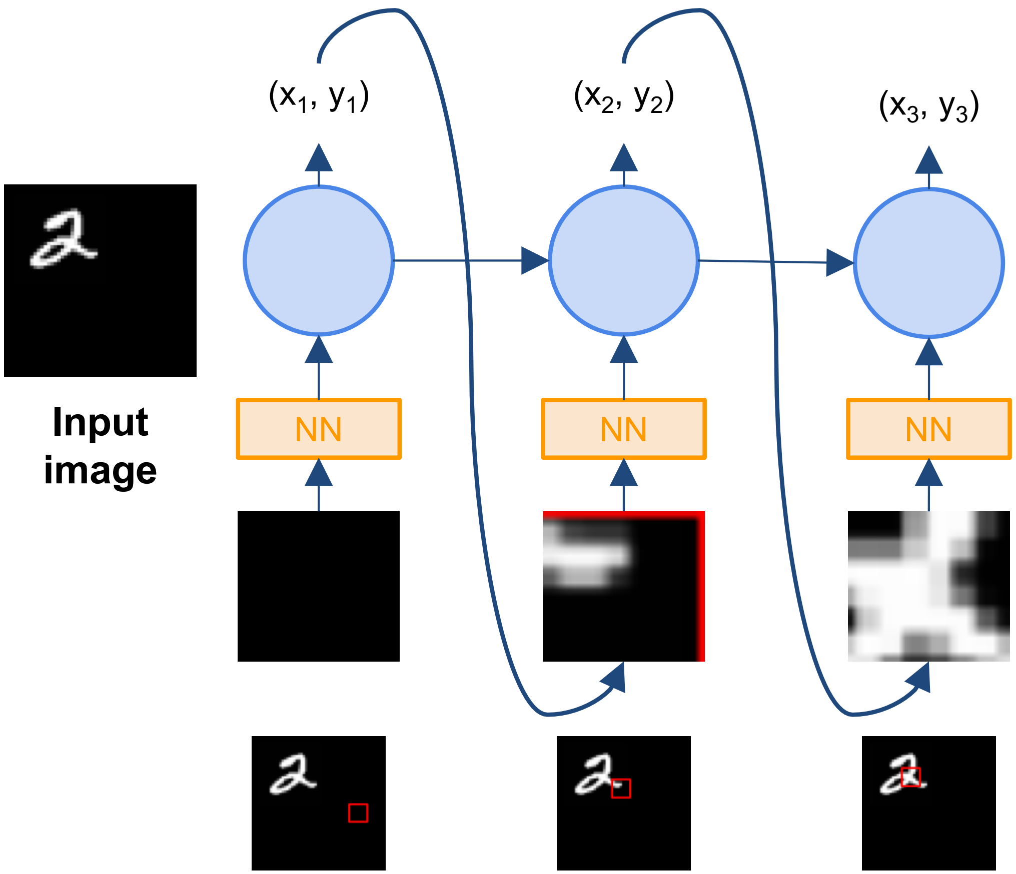

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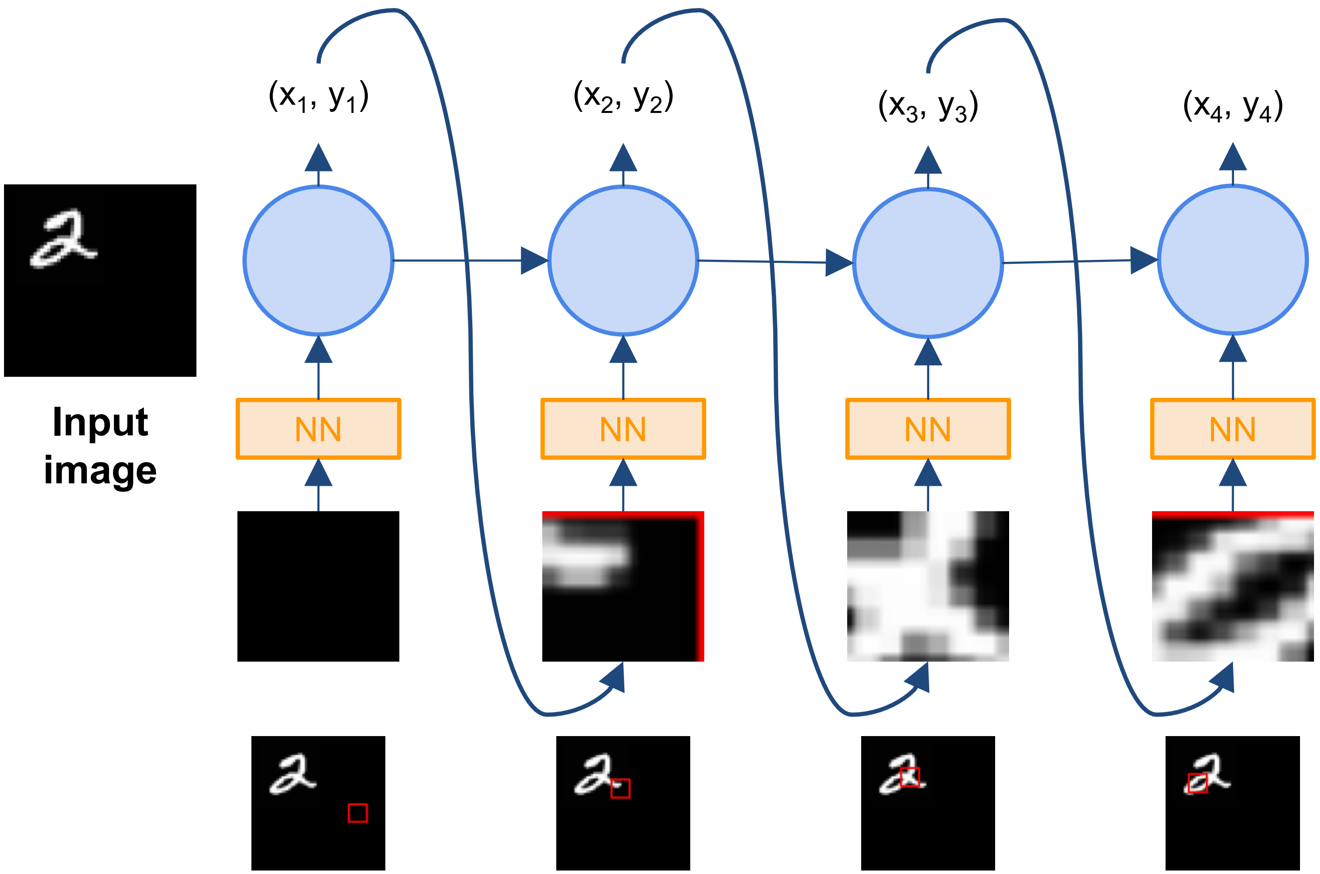

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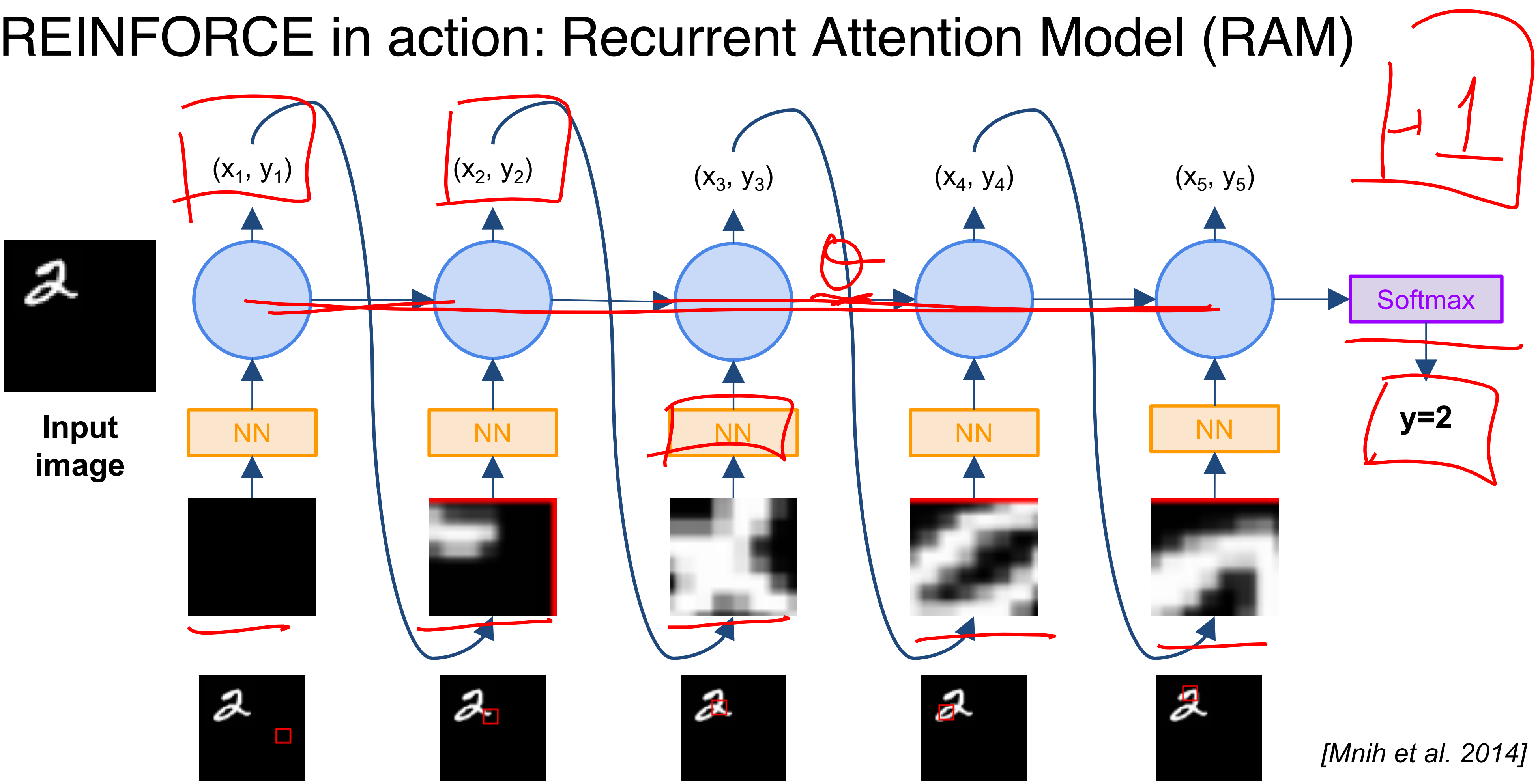

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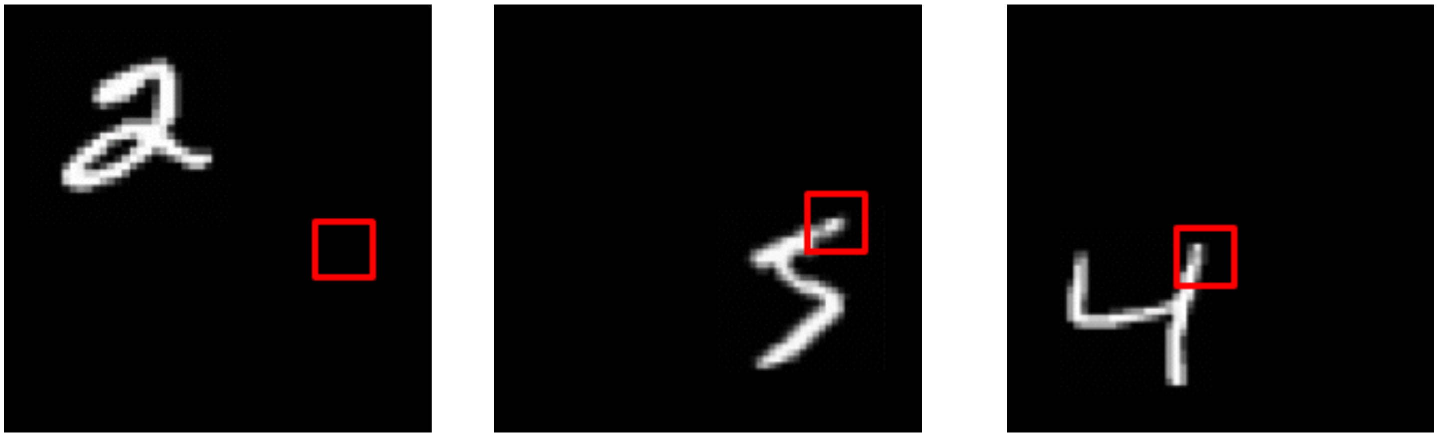

Has also been used in many other tasks including fine-grained image recognition, image captioning, and visual question-answering!

[Mnih et al. 2014]

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First idea: Push up probabilities of an action seen, only by the cumulative future reward from that state

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Second idea: Use discount factor γ to ignore delayed effects

$$\nabla_\theta J(\theta) \approx \sum_{t\geq 0} \left(\sum_{t'\geq t} \gamma^{t'-t} r_{t'} \right) \nabla_\theta \log \pi_\theta(a_t|s_t)$$

Variance reduction: Baseline

Problem: The raw value of a trajectory isn't necessarily meaningful. For example, if rewards are all positive, you keep pushing up probabilities of actions.

What is important then? Whether a reward is better or worse than what you expect to get

Idea: Introduce a baseline function dependent on the state.
Concretely, estimator is now:

$$\nabla_\theta J(\theta) \approx \sum_{t \geq 0} \left(\sum_{t' \geq t} \gamma^{t'-t} r_{t'} - b(s_t) \right) \nabla_\theta \log \pi_\theta(a_t | s_t)$$

How to choose the baseline?

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A simple baseline: constant moving average of rewards experienced so far from all trajectories

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A simple baseline: constant moving average of rewards experienced so far from all trajectories

Variance reduction techniques seen so far are typically used in "Vanilla REINFORCE"

How to choose the baseline?

A better baseline: Want to push up the probability of an action from a state, if this action was better than the **expected value of what we should get from that state**.

Q: What does this remind you of?

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Using this, we get the estimator: $\nabla_\theta J(\theta) \approx \sum_{t \geq 0} (Q^{\pi_\theta}(s_t, a_t) - V^{\pi_\theta}(s_t)) \nabla_\theta \log \pi_\theta(a_t|s_t)$

Slide Credit: Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231n

Actor-Critic Algorithm

Initialize policy parameters θ, critic parameters ϕ

For iteration=1, 2 … **do**

Sample m trajectories under the current policy

$\Delta\theta \leftarrow 0$

For i=1, …, m **do**

For t=1, ... , T **do**

$$A_t = \sum_{t' \geq t} \gamma^{t'-t} r_t^i - V_\phi(s_t^i)$$

$$\Delta\theta \leftarrow \Delta\theta + A_t \nabla_\theta \log(a_t^i | s_t^i)$$

$$\Delta\phi \leftarrow \sum_i \sum_t \nabla_\phi ||A_t^i||^2$$

$$\theta \leftarrow \alpha\Delta\theta$$

$$\phi \leftarrow \beta\Delta\phi$$

End for

Summary

- **Policy gradients**: very general but suffer from high variance so requires a lot of samples. **Challenge**: sample-efficiency
- **Q-learning**: does not always work but when it works, usually more sample-efficient. **Challenge**: exploration

- Guarantees:
  - **Policy Gradients**: Converges to a local minima of $J(\theta)$, often good enough!
  - **Q-learning**: Zero guarantees since you are approximating Bellman equation with a complicated function approximator