Reminder

- Project 1 due Wednesday
- Intended to be easy intro project
- What if your filtering is slow?
  - Separable filters (Gaussian)
  - Laplacian Pyramid
- What if my results aren't convincing?


BBC TWO

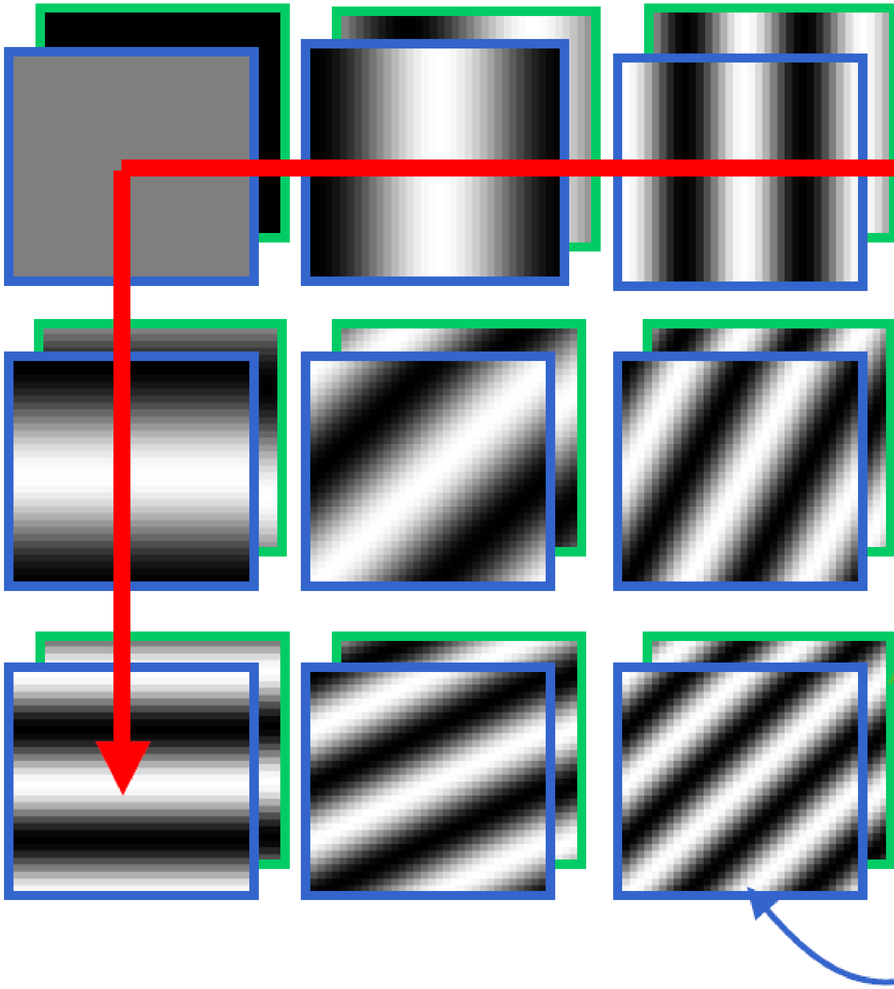
Recap: Fourier domain

2d Fourier Transform

$$\hat{F}(k,\ell) = \frac{1}{\sqrt{MN}} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} F(m,n) e^{-j2\pi\left(k\frac{m}{M}+\ell\frac{n}{N}\right)}$$

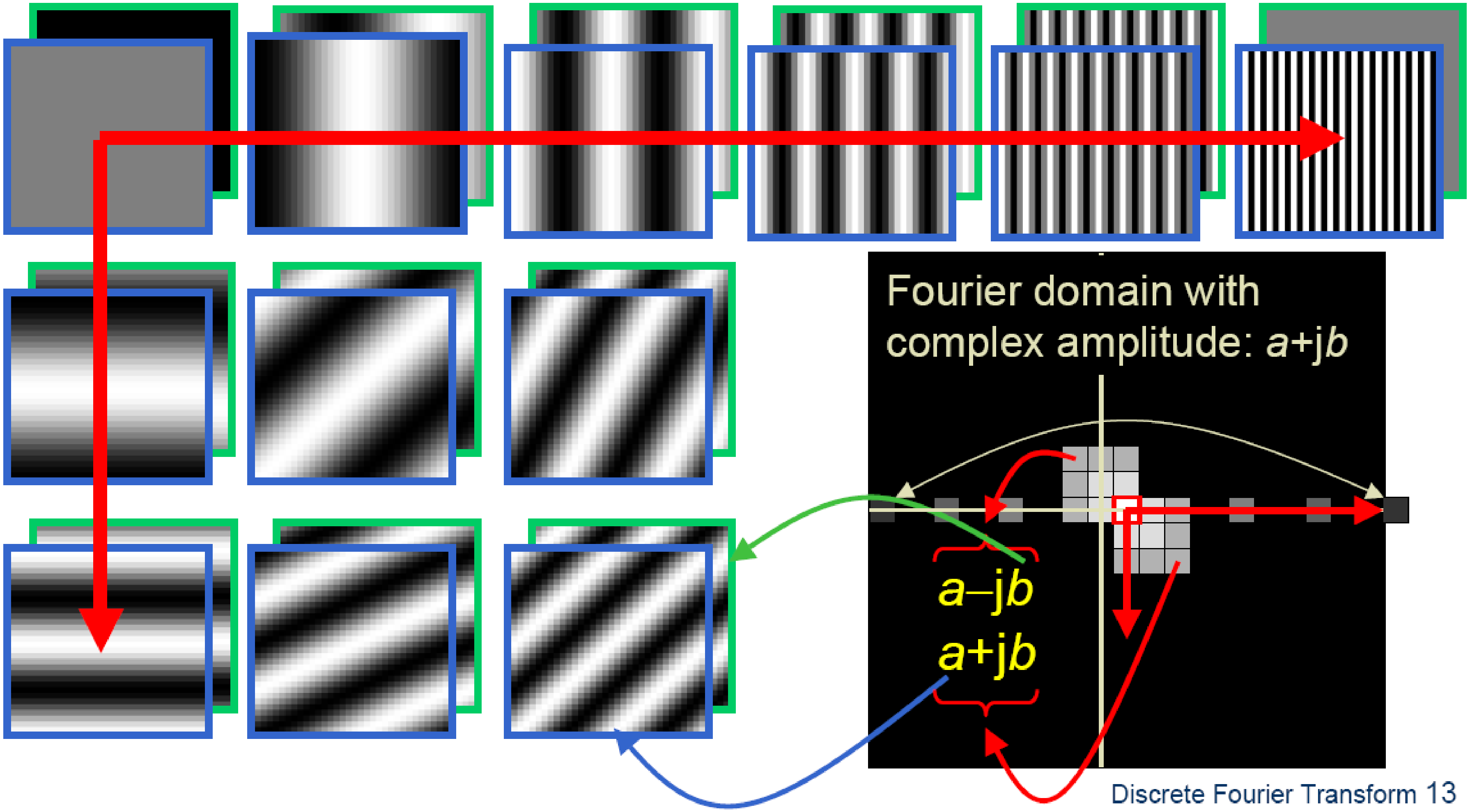
Fourier Bases

Teases away fast vs. slow changes in the image.



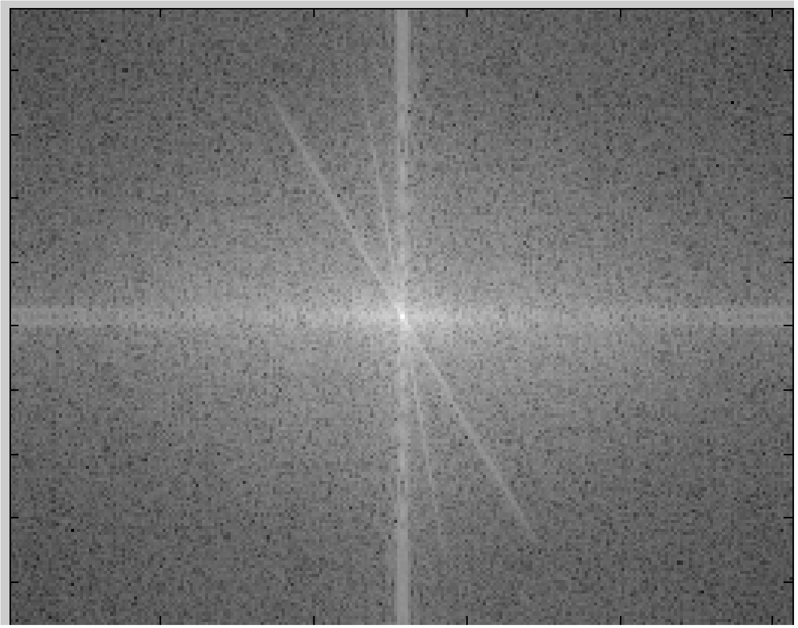
This change of basis is the Fourier Transform

Fourier Bases



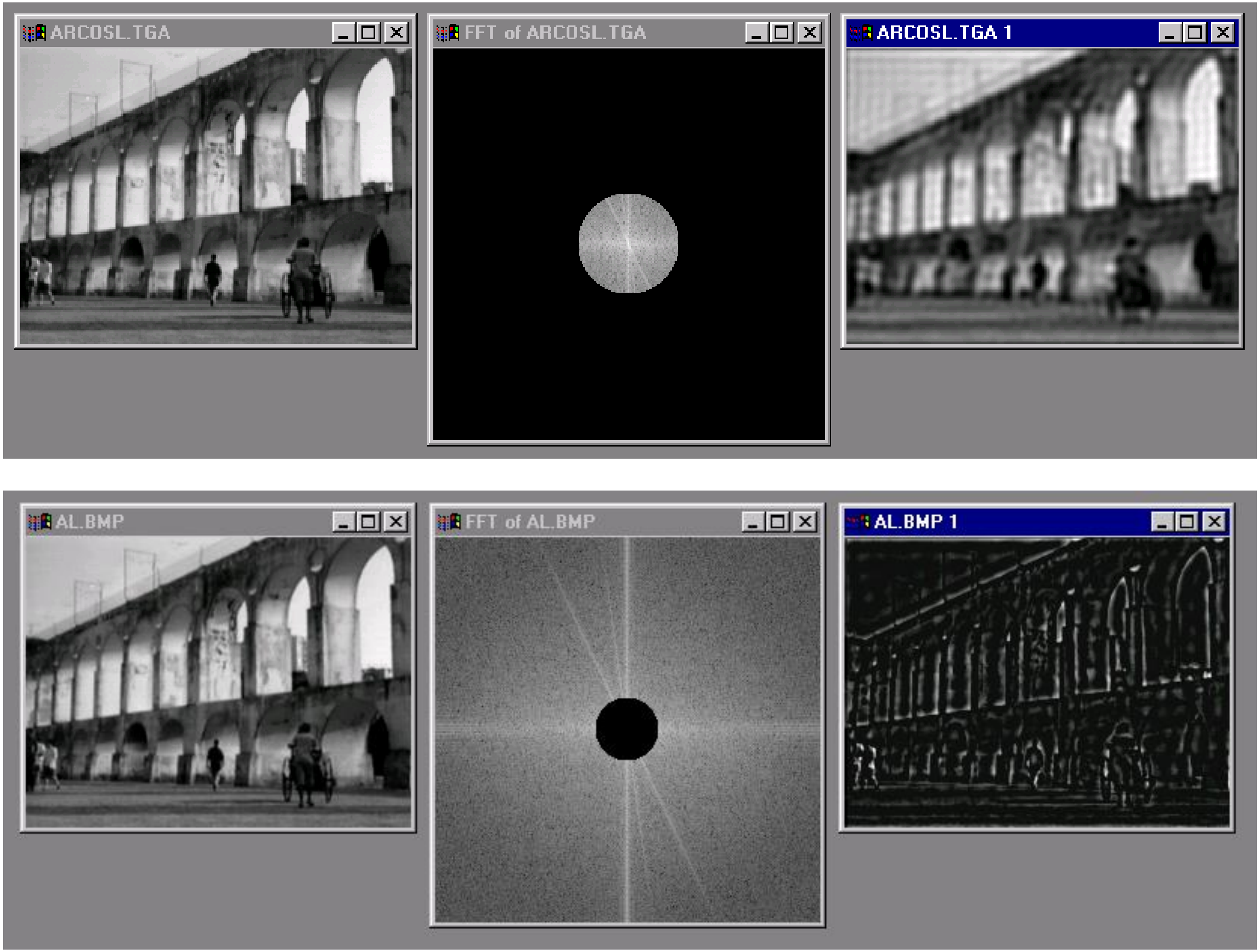
in Matlab, check out: imagesc(log(abs(fftshift(fft2(im)))));

Man-made Scene



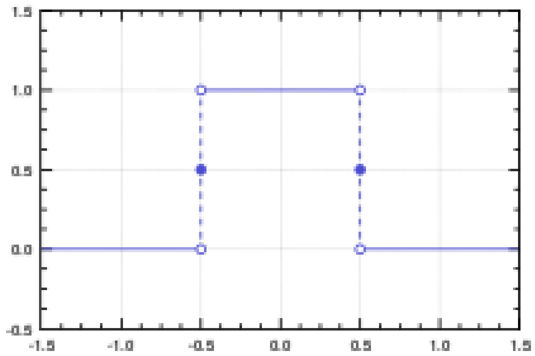
Can change spectrum, then reconstruct

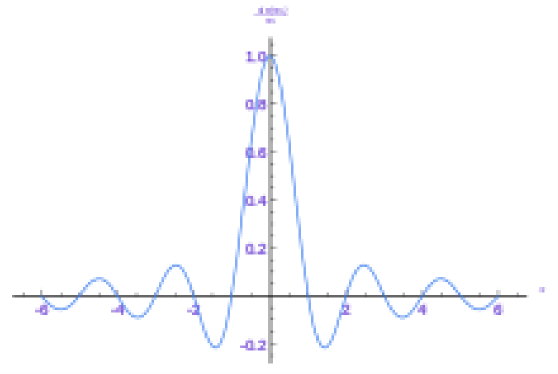
Low and High Pass filtering



Sinc Filter

- What is the spatial representation of the hard cutoff in the frequency domain?



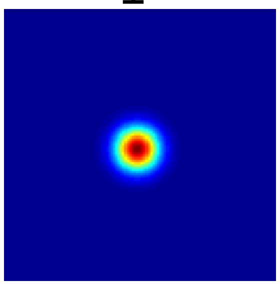


Frequency Domain Spatial Domain

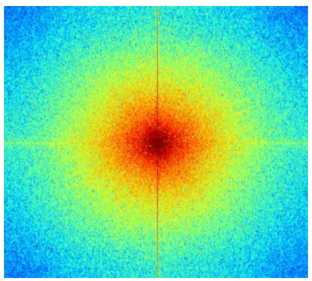
Review

1. Match the spatial domain image to the Fourier magnitude image

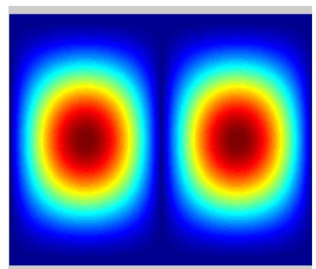
1



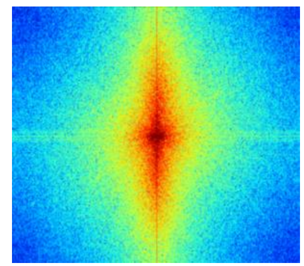
2



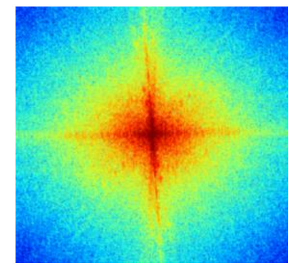
3



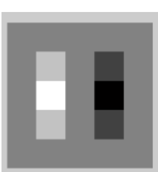
4



5

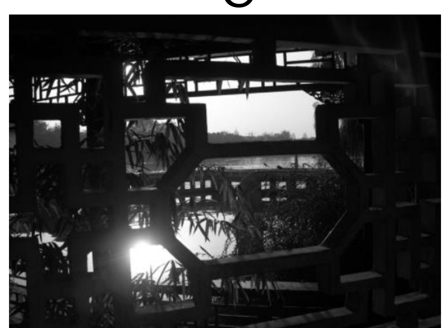


A

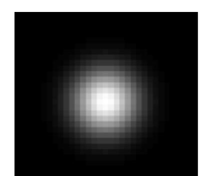


B

C



D



E

The Convolution Theorem

- The Fourier transform of the convolution of two functions is the product of their Fourier transforms

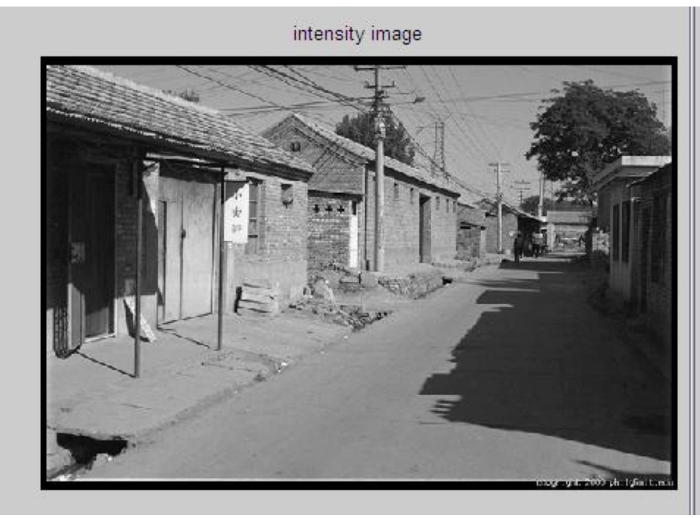
$$\mathrm{F}[g * h] = \mathrm{F}[g]\,\mathrm{F}[h]$$

- **Convolution** in spatial domain is equivalent to **multiplication** in frequency domain!

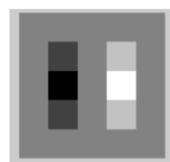
$$g * h = \mathrm{F}^{-1}[\mathrm{F}[g]\,\mathrm{F}[h]]$$

Filtering in spatial domain

1	0	-1
2	0	-2
1	0	-1



*



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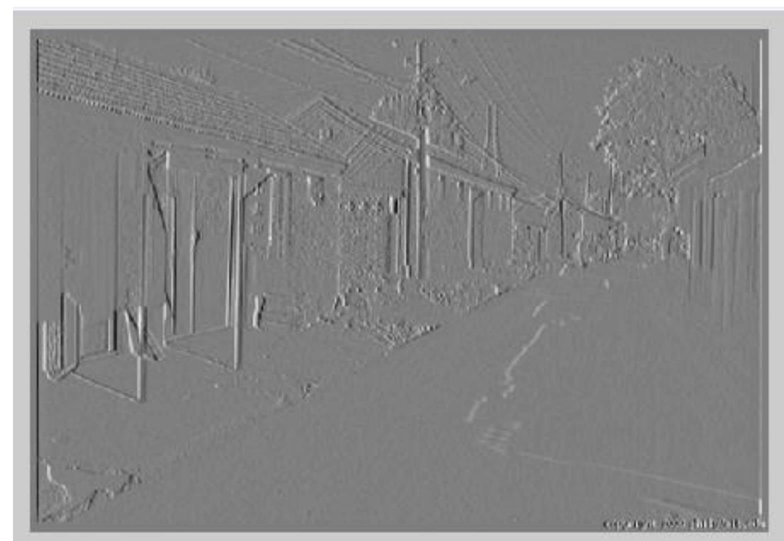
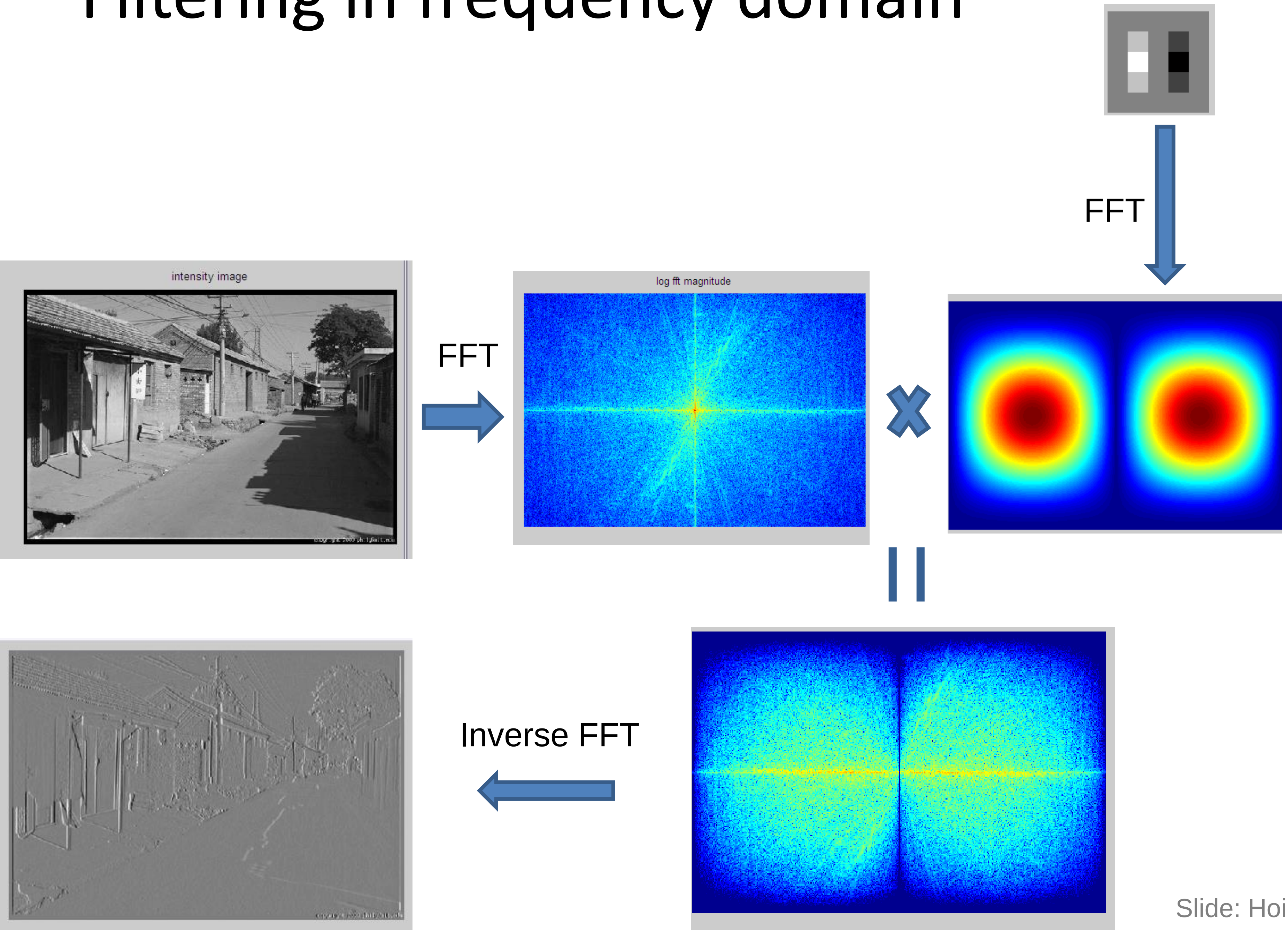


Filtering in frequency domain

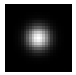


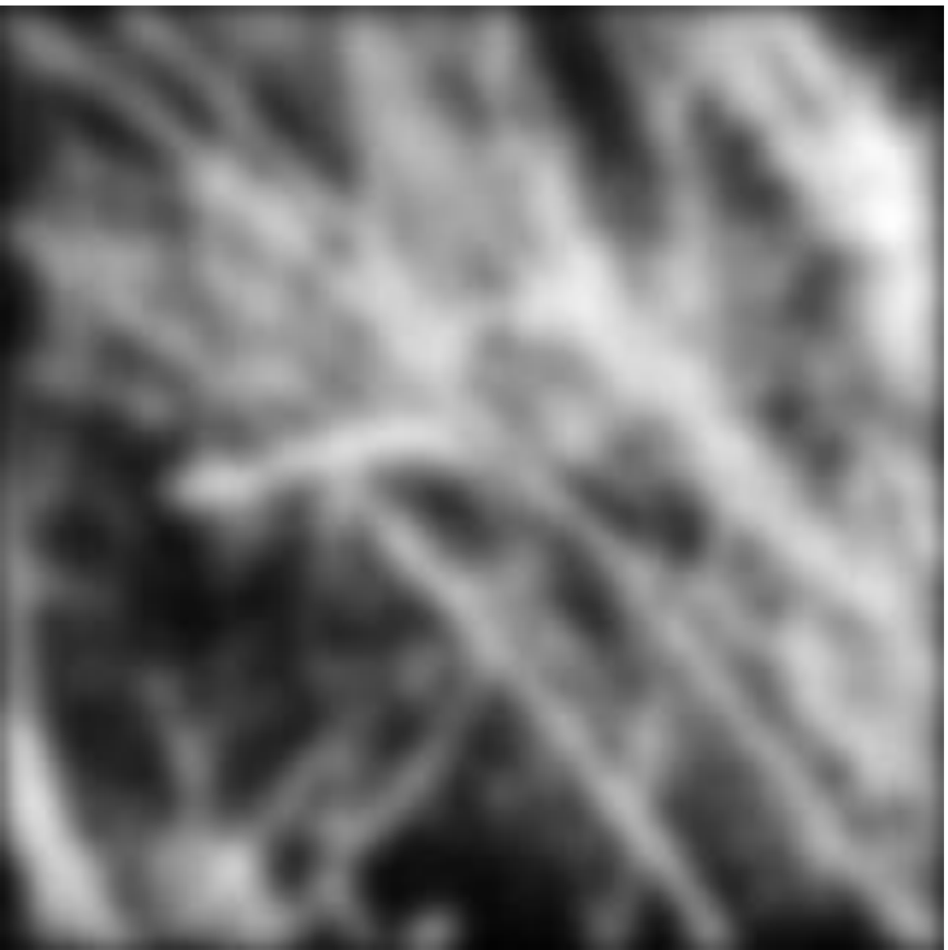
Slide: Hoiem

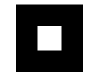
Fourier Matlab demo

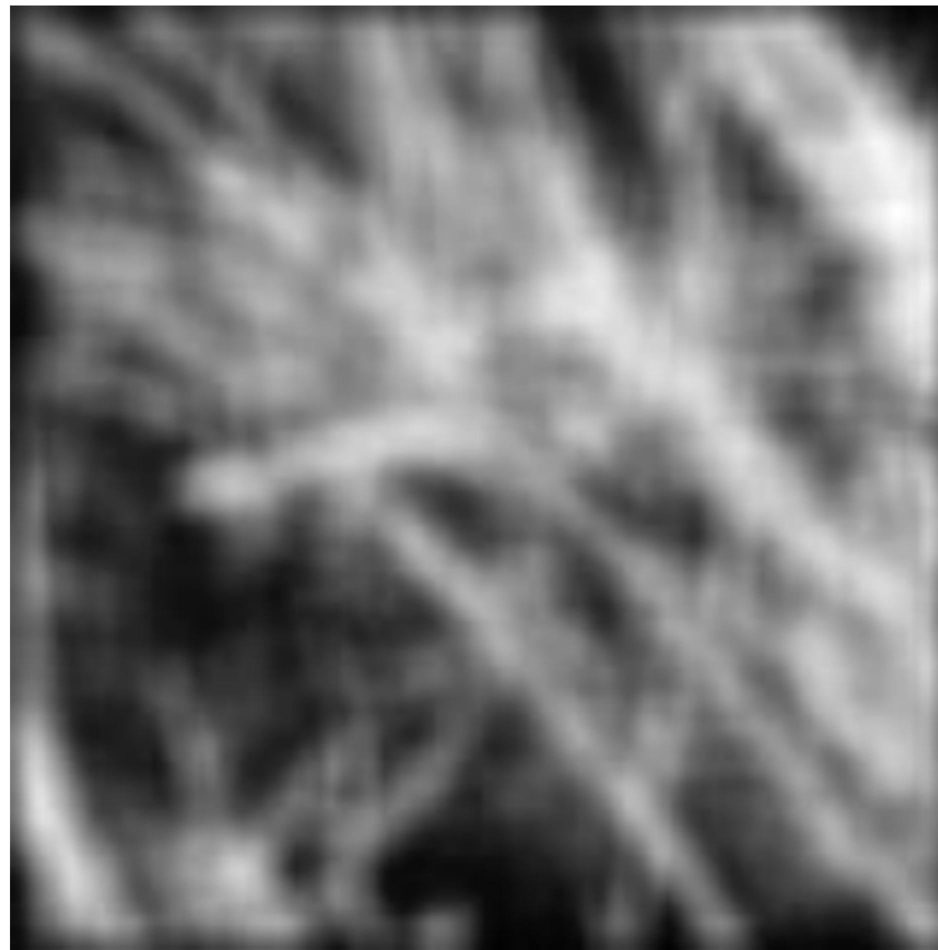
Filtering

Why does the Gaussian give a nice smooth image, but the square filter give edgy artifacts?

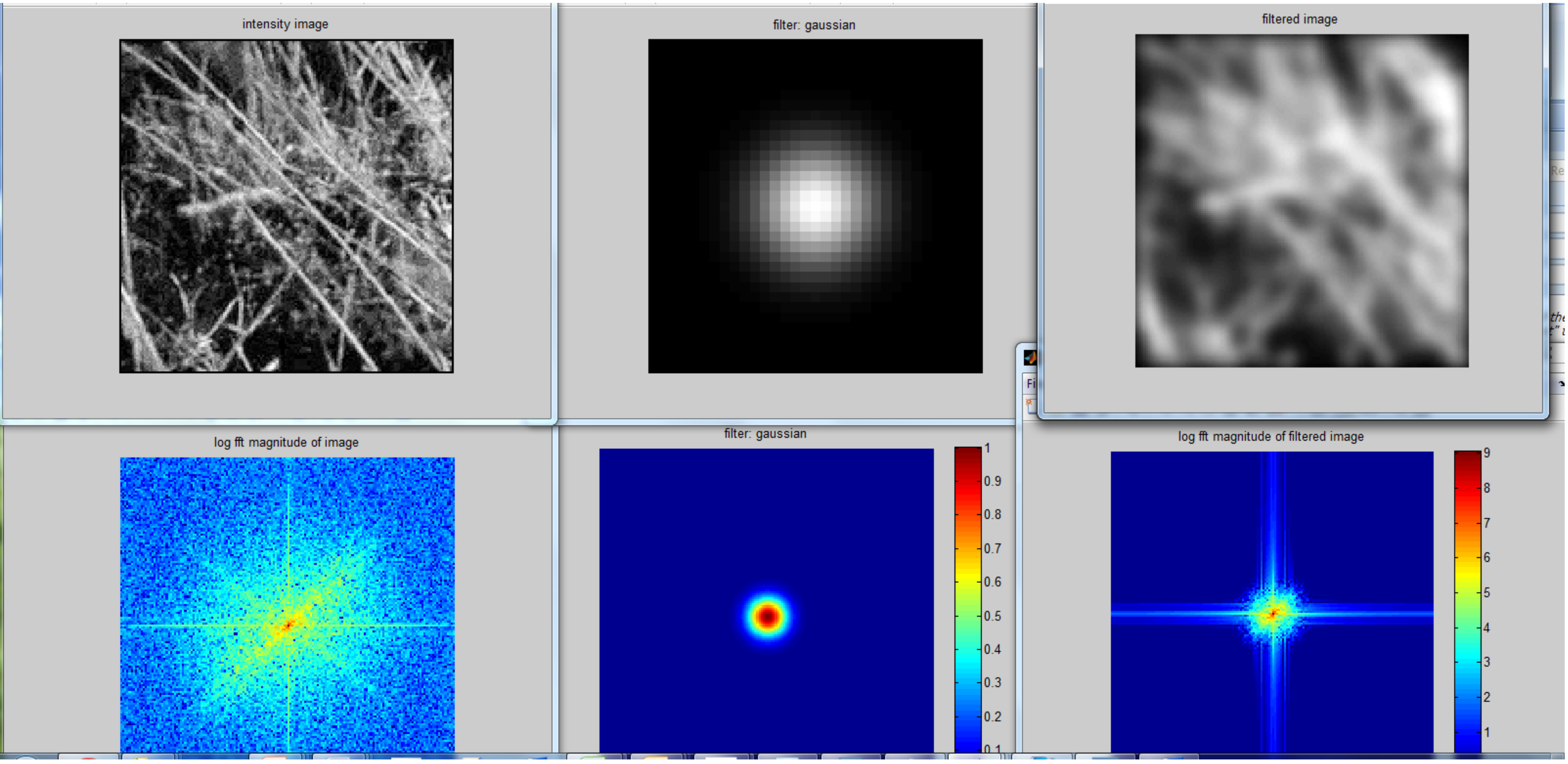
Gaussian 



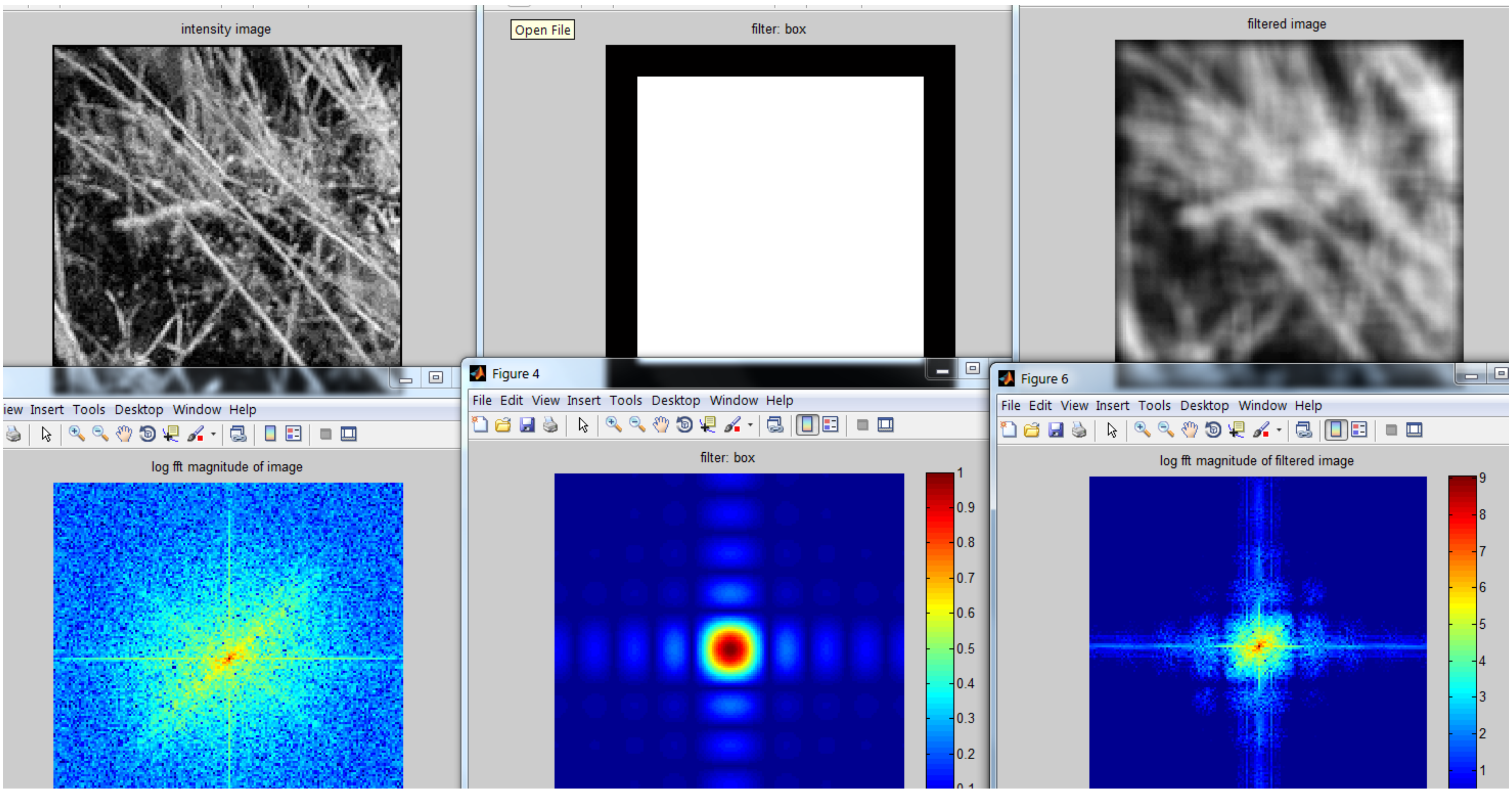
Box filter 



Gaussian


intensity image
filter: gaussian
filtered image
log fft magnitude of image
filter: gaussian
log fft magnitude of filtered image

Box Filter



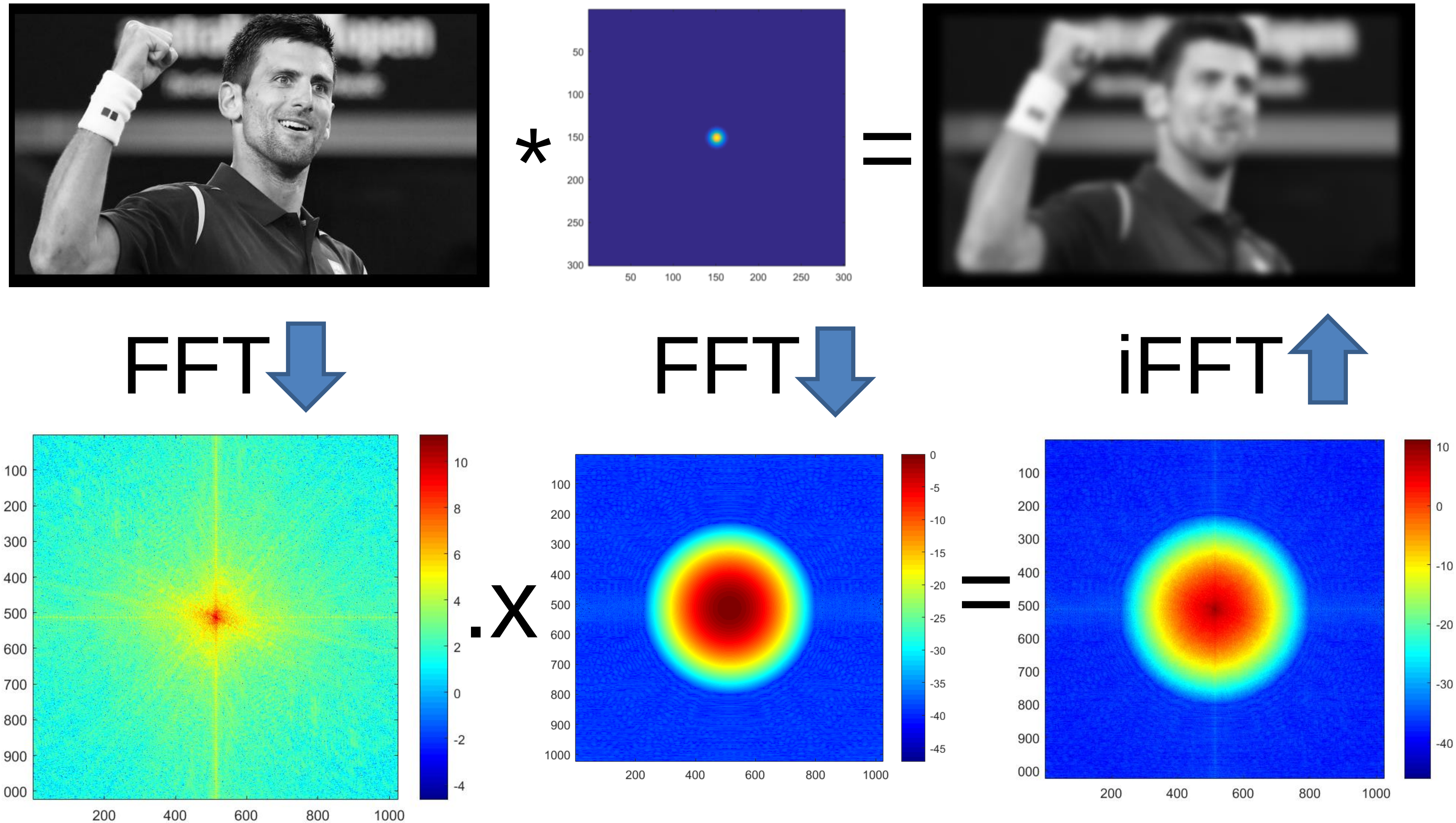
intensity image
Open File
filter: box
filtered image
Figure 4
Figure 6
File Edit View Insert Tools Desktop Window Help
iew Insert Tools Desktop Window Help
log fft magnitude of image
filter: box
log fft magnitude of filtered image

Is convolution invertible?

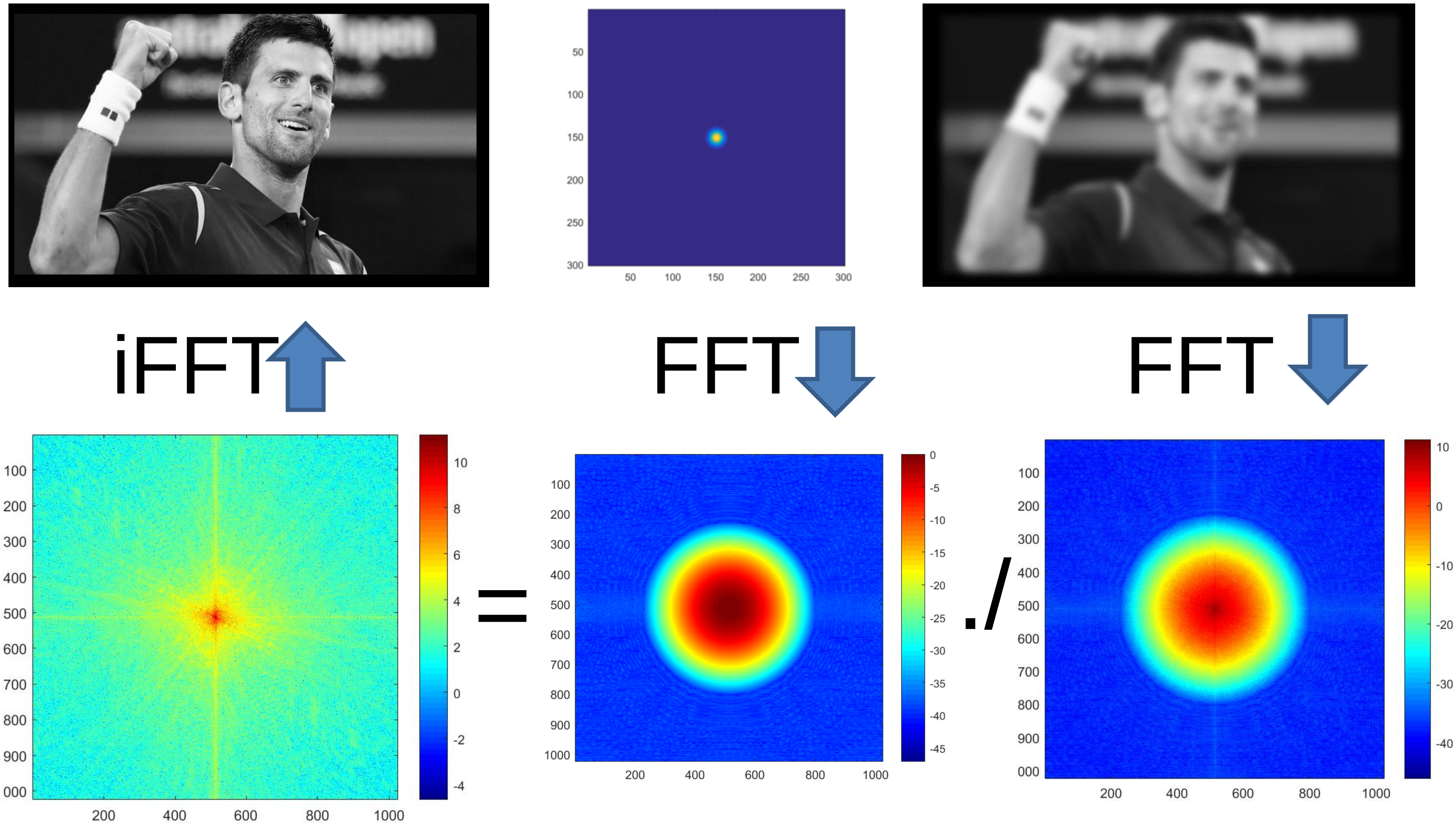
- If convolution is just multiplication in the Fourier domain, isn’t deconvolution just division?
- Sometimes, it clearly is invertible (e.g. a convolution with an identity filter)
- In one case, it clearly isn’t invertible (e.g. convolution with an all zero filter)
- What about for common filters like a Gaussian?

Let’s experiment on Novak

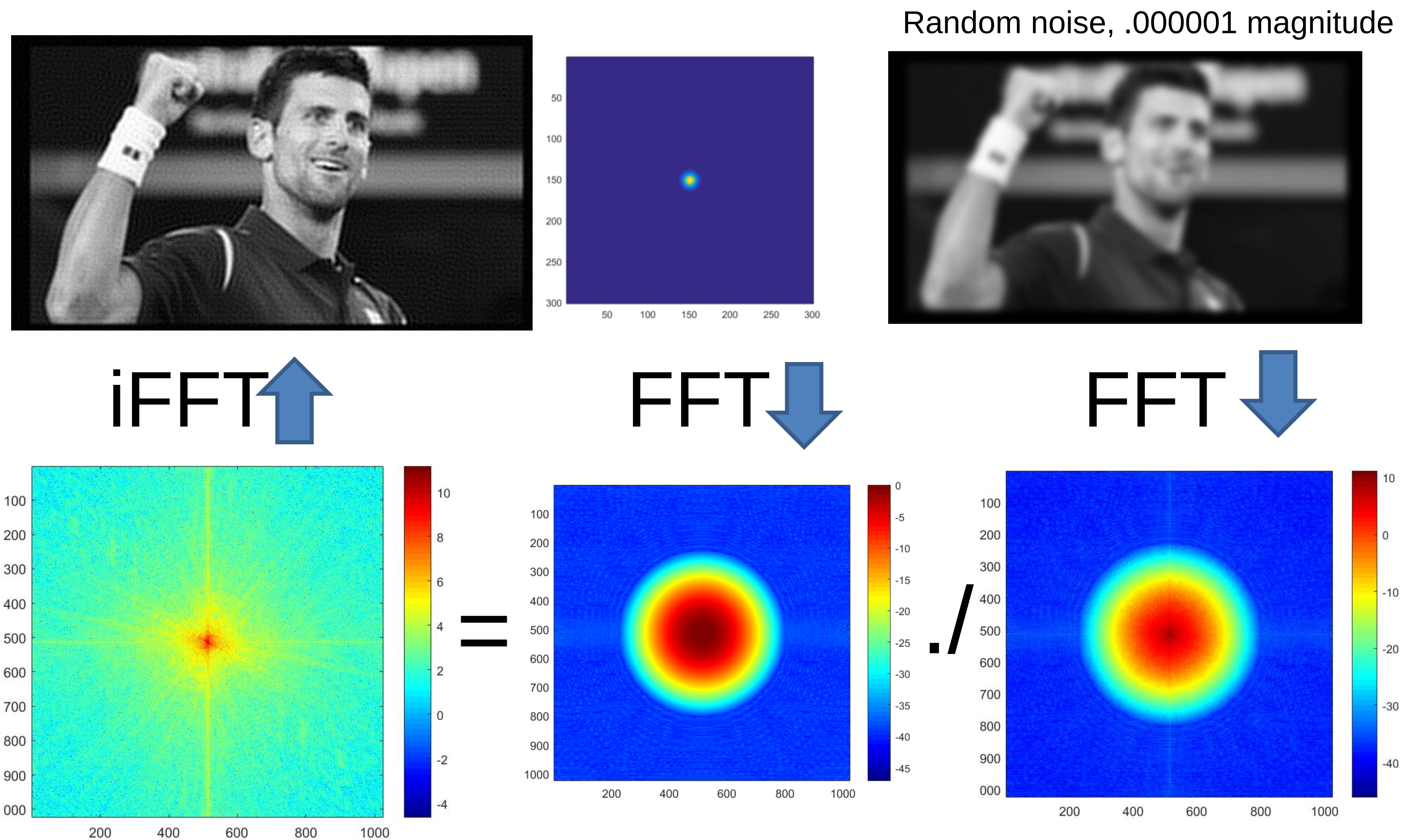
Convolution



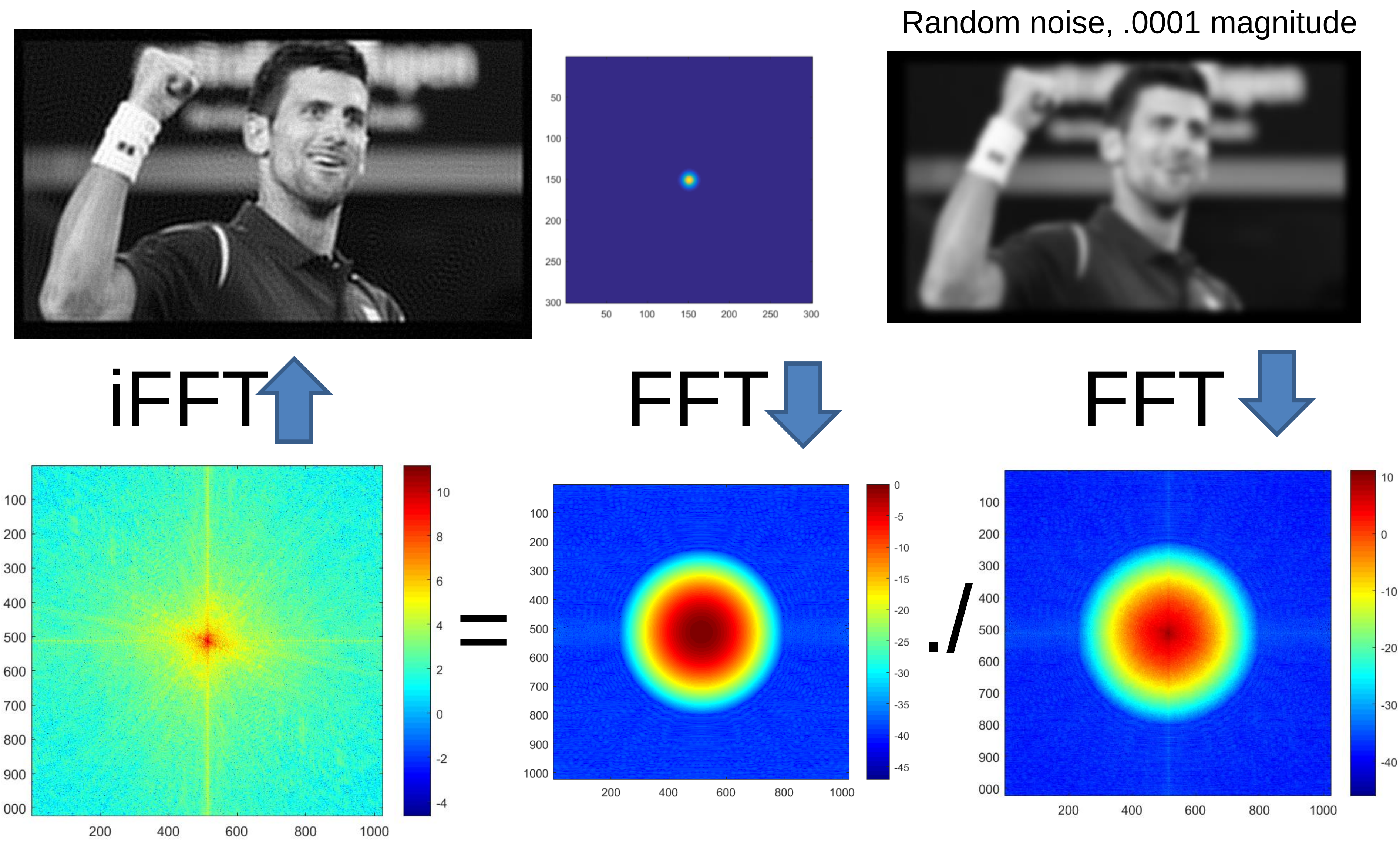
Deconvolution?



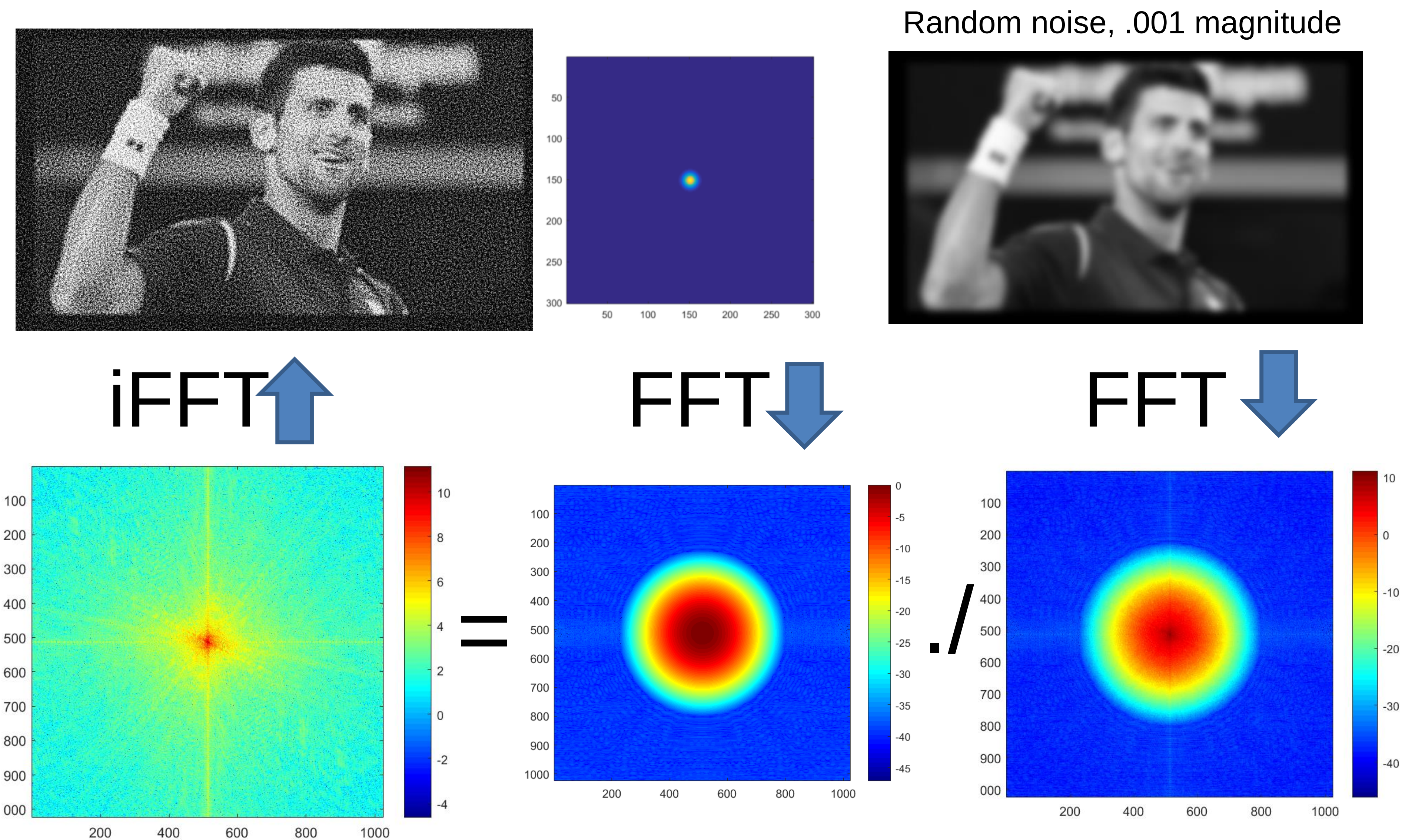
But under more realistic conditions



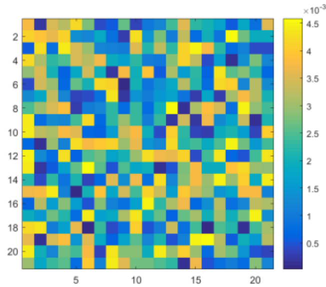
But under more realistic conditions



But under more realistic conditions



With a random filter…

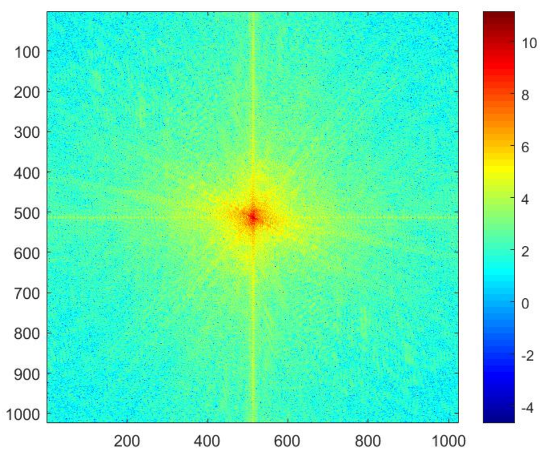


Random noise, .001 magnitude

iFFT

FFT

FFT



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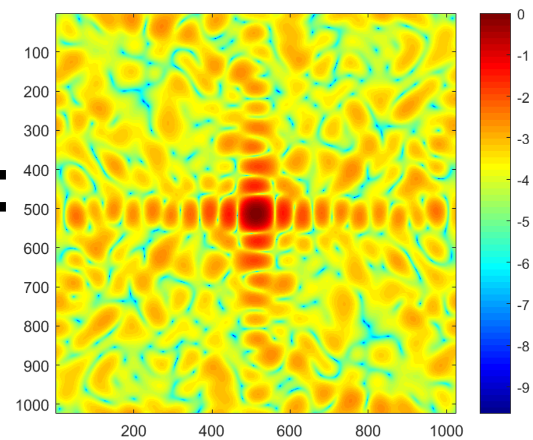
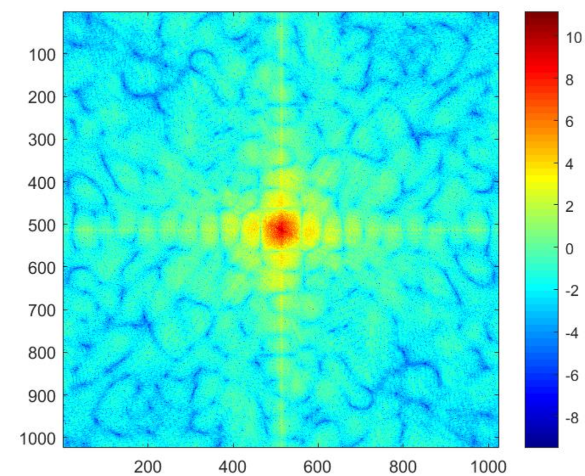


./



Deconvolution is hard

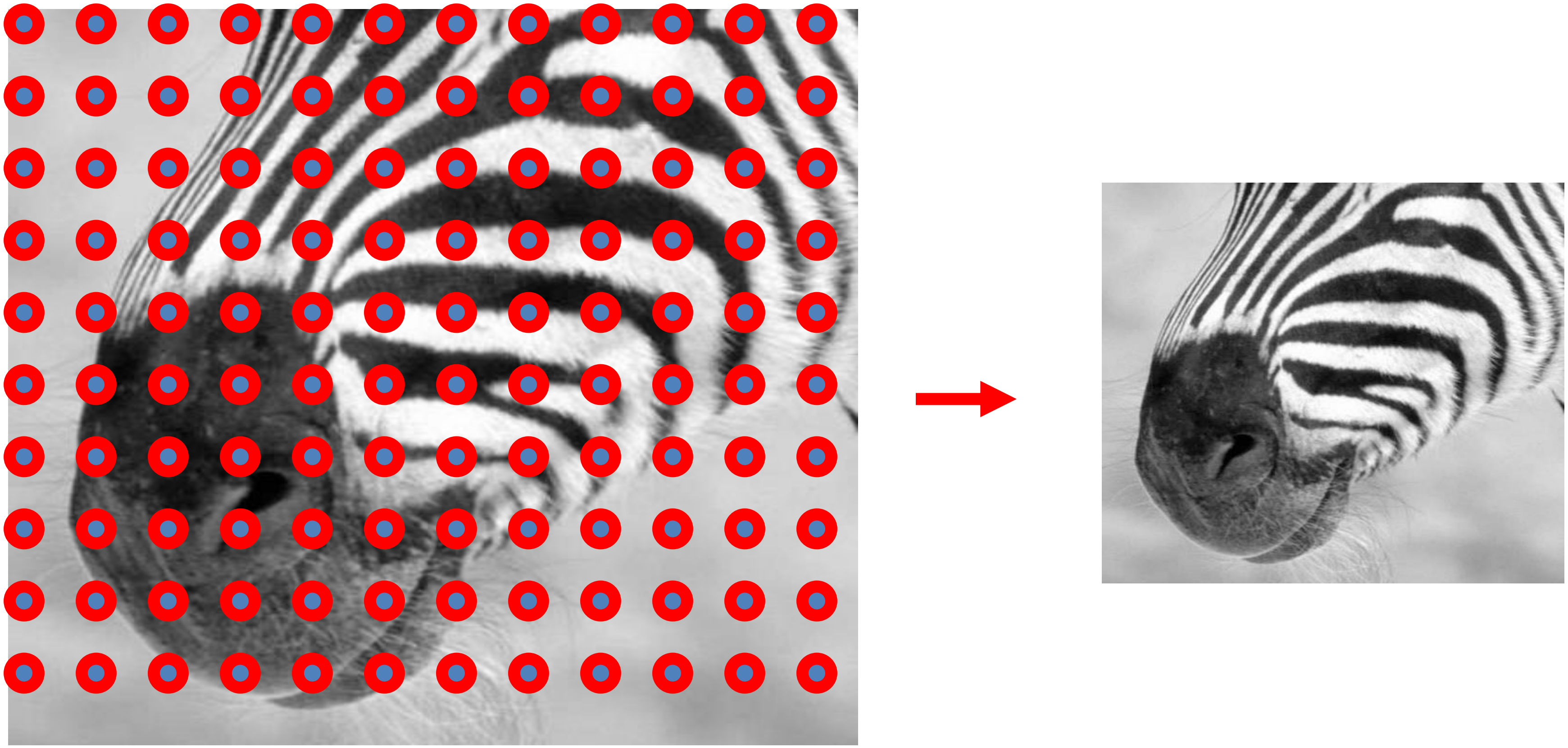
- Active research area.
- Even if you know the filter (non-blind deconvolution), it is still very hard and requires strong *regularization.*
- If you don’t know the filter (blind deconvolution) it is harder still.

Sampling

Why does a lower resolution image still make sense to us? What do we lose?

Image: http://www.flickr.com/photos/igorms/136916757/

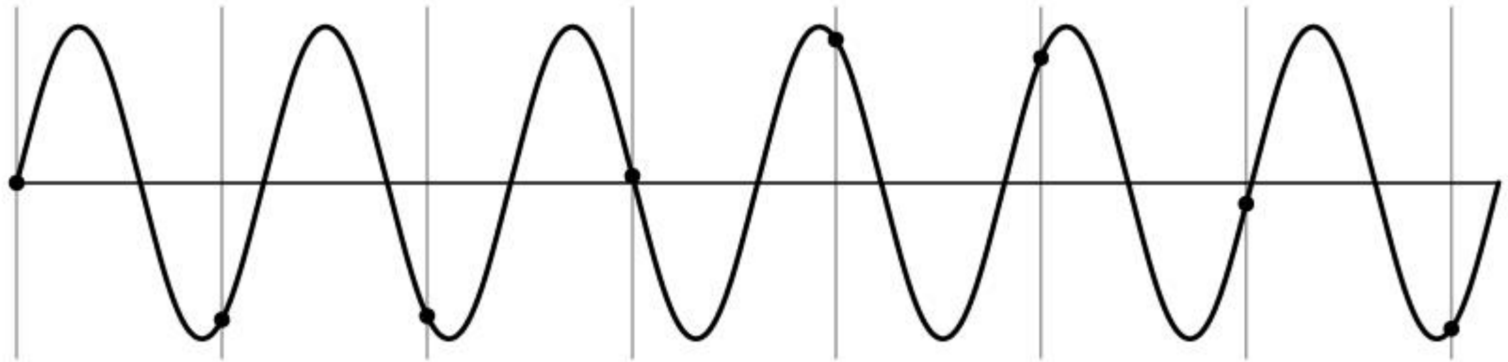
Subsampling by a factor of 2



Throw away every other row and column to create a 1/2 size image

Aliasing problem

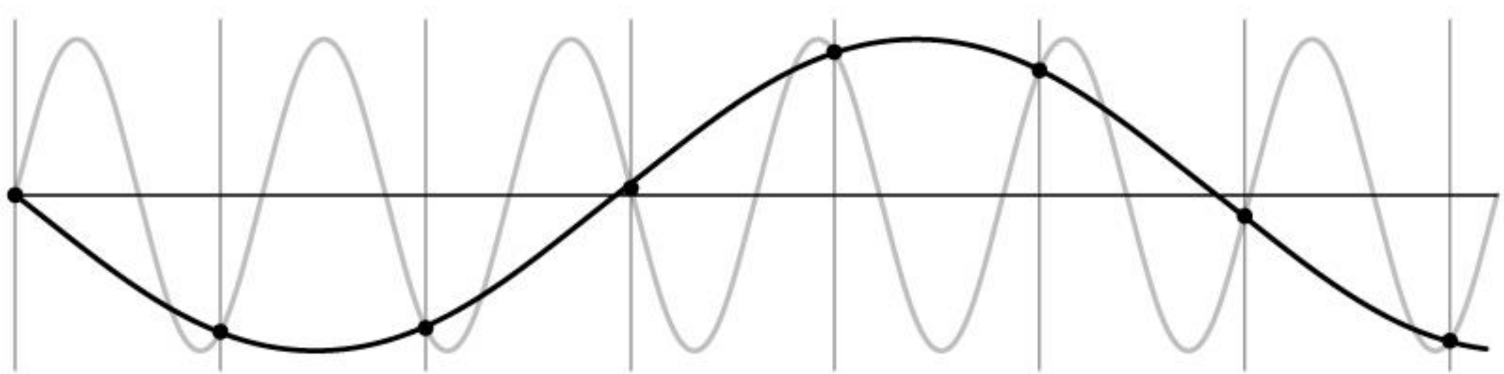
- 1D example (sinewave):



Source: S. Marschner

Aliasing problem

- 1D example (sinewave):



Source: S. Marschner

Aliasing problem

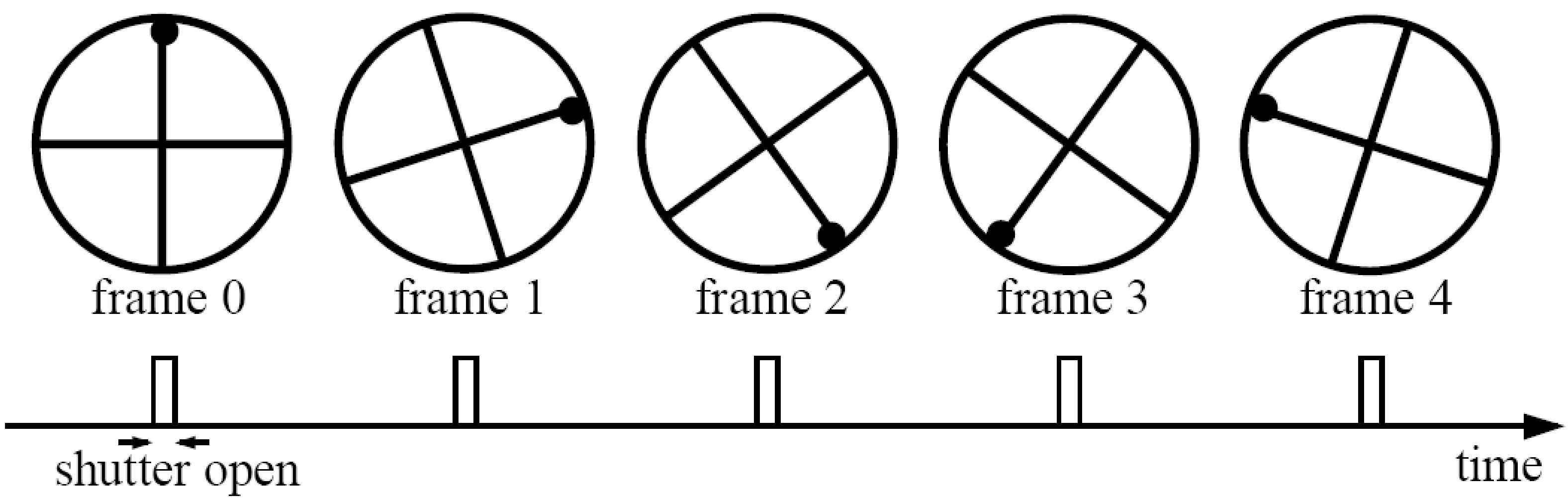
- Sub-sampling may be dangerous….
- Characteristic errors may appear:
  - “car wheels rolling the wrong way in movies”
  - “Checkerboards disintegrate in ray tracing”
  - “Striped shirts look funny on color television”

Source: D. Forsyth

Aliasing in video

Imagine a spoked wheel moving to the right (rotating clockwise). Mark wheel with dot so we can see what's happening.

If camera shutter is only open for a fraction of a frame time (frame time = 1/30 sec. for video, 1/24 sec. for film):



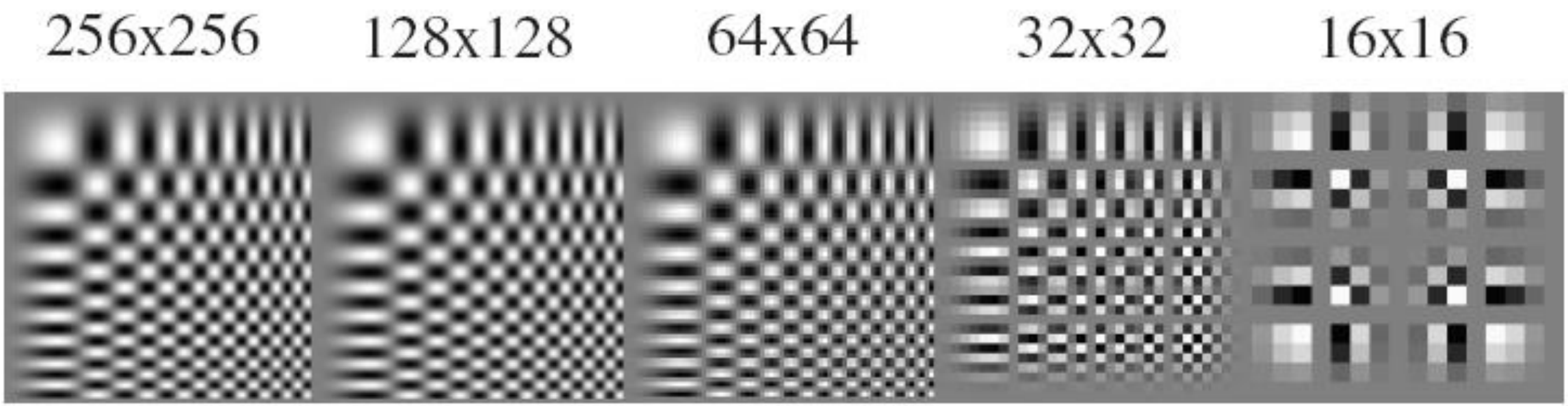
Without dot, wheel appears to be rotating slowly backwards! (counterclockwise)

Slide by Steve Seitz

Aliasing in graphics

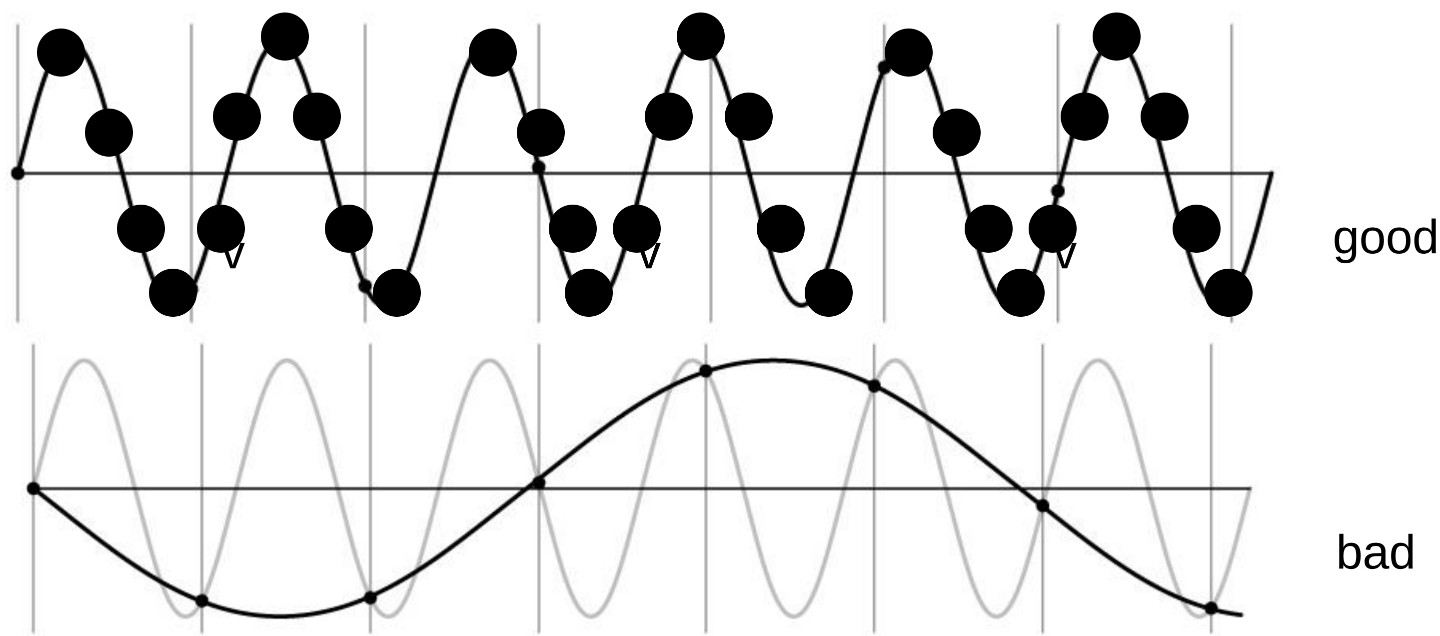
Source: A. Efros

Sampling and aliasing



Nyquist-Shannon Sampling Theorem

- When sampling a signal at discrete intervals, the sampling frequency must be $\geq 2 \times f_{max}$
- f_{max} = max frequency of the input signal
- This will allows to reconstruct the original perfectly from the sampled version



Anti-aliasing

Solutions:

- Sample more often

- Get rid of all frequencies that are greater than half the new sampling frequency
  - Will lose information
  - But it's better than aliasing
  - Apply a smoothing filter

Algorithm for downsampling by factor of 2

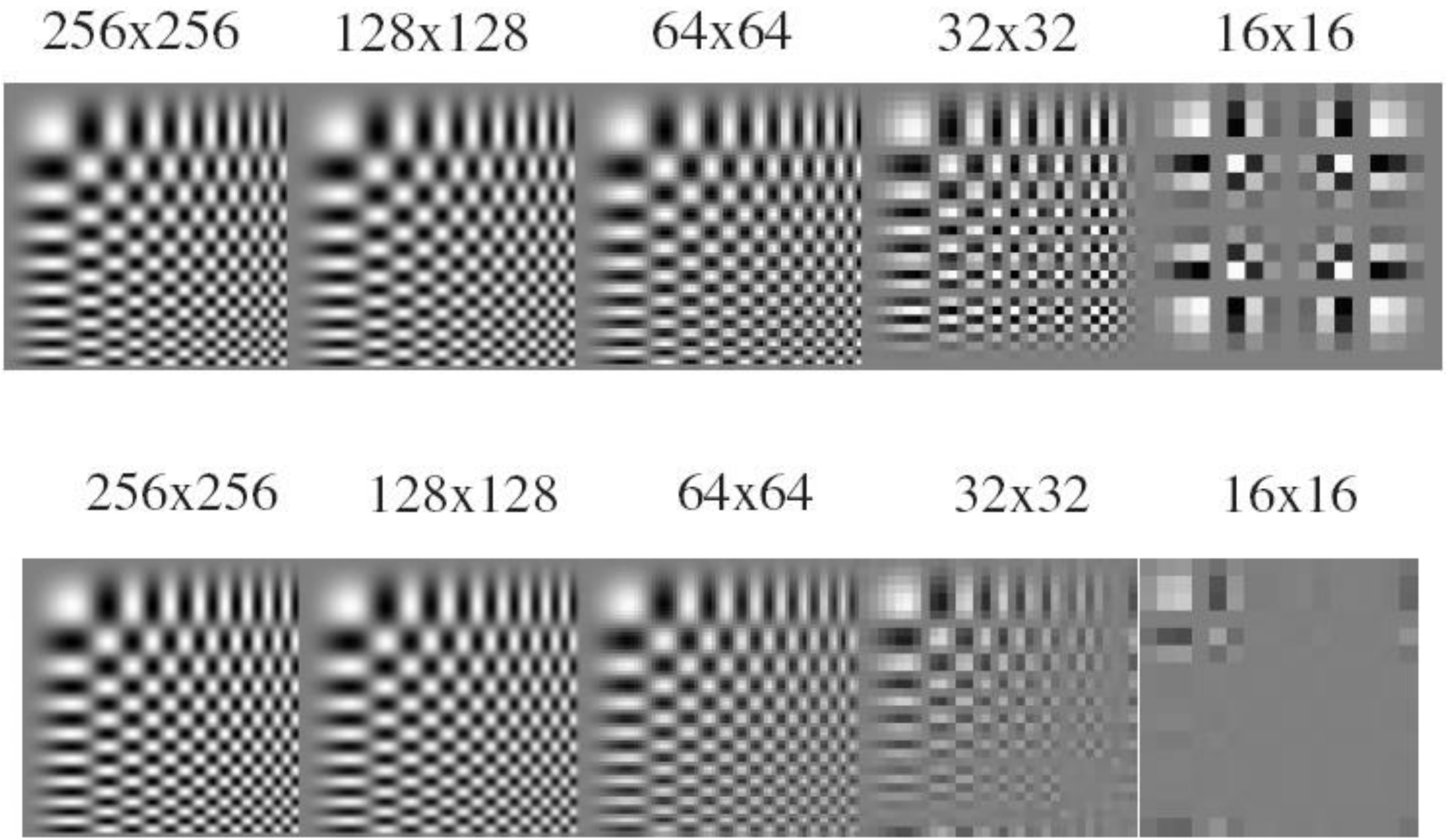
1. Start with image(h, w)
2. Apply low-pass filter

   ```
   im_blur = imfilter(image, fspecial('gaussian', 7, 1))
   ```

3. Sample every other pixel

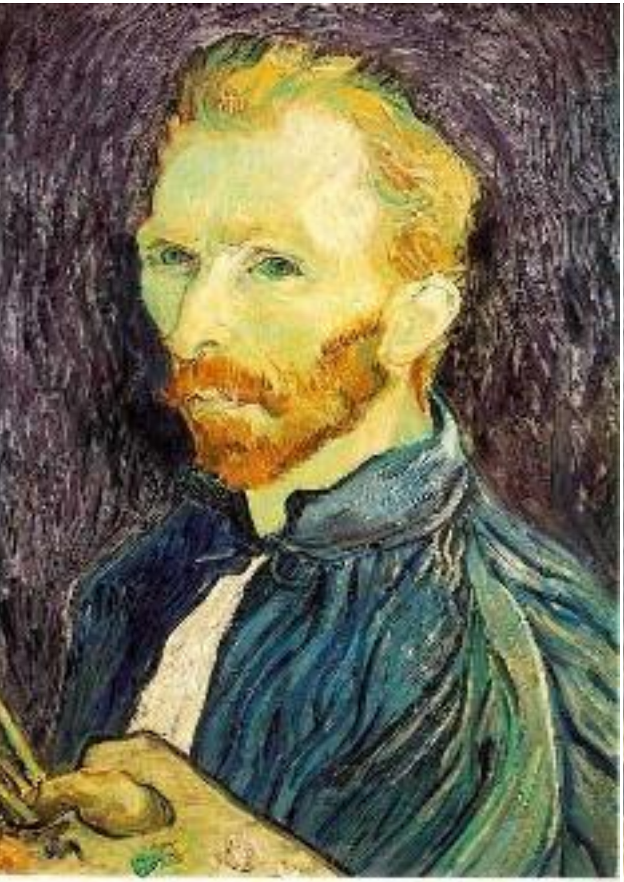
   ```
   im_small = im_blur(1:2:end, 1:2:end);
   ```

Anti-aliasing



Forsyth and Ponce 2002

Subsampling without pre-filtering



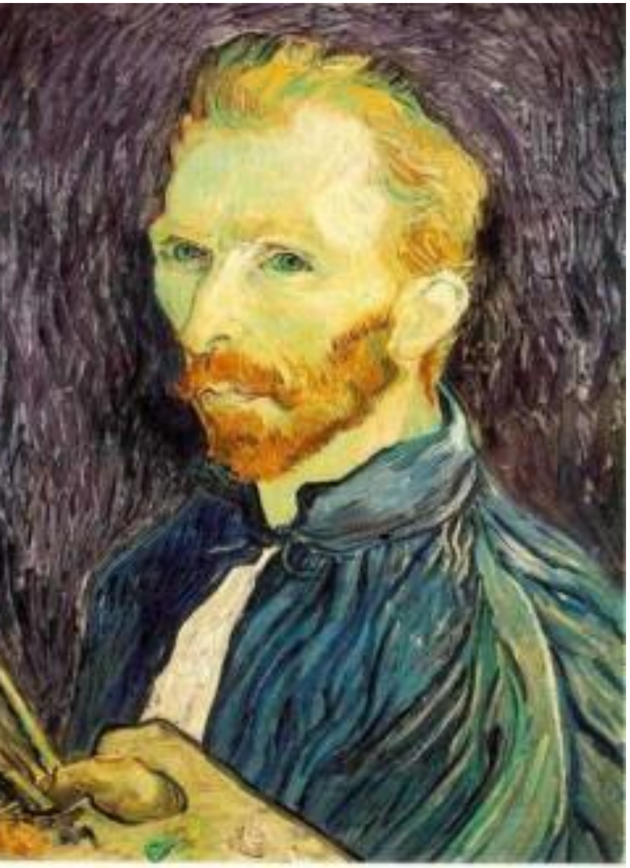
1/2

1/4 (2x zoom)

1/8 (4x zoom)

Slide by Steve Seitz

Subsampling with Gaussian pre-filtering



Gaussian 1/2

G 1/4

G 1/8

Slide by Steve Seitz