



WODKA  
WODKA

MATHEMATICS  
MAN AND BEAST  
DRUGS

SNOW CRYSTALS  
FOTOGRAFIE UND DAS UNSICHERE  
STOWASSER

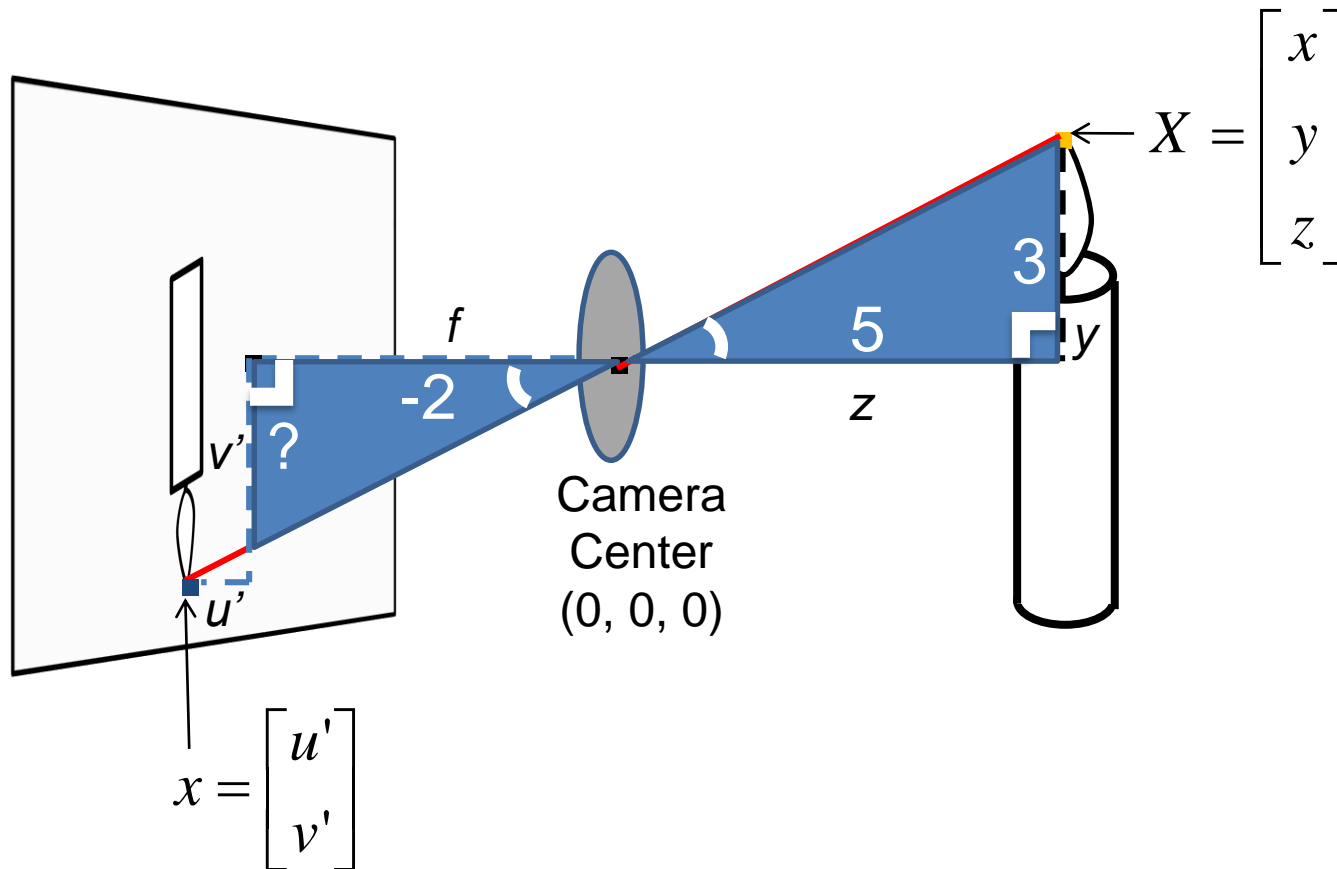
STACK-ON

# Light

Computer Vision

James Hays

# Projection: world coordinates $\rightarrow$ image coordinates



If  $X = 2$ ,  $Y = 3$ ,  
 $Z = 5$ , and  $f = 2$   
 What are  $U$  and  $V$ ?

$$\frac{v'}{-f} = \frac{y}{z}$$

$$u' = -x * \frac{f}{z}$$

$$v' = -y * \frac{f}{z}$$

$$u' = -2 * \frac{2}{5}$$

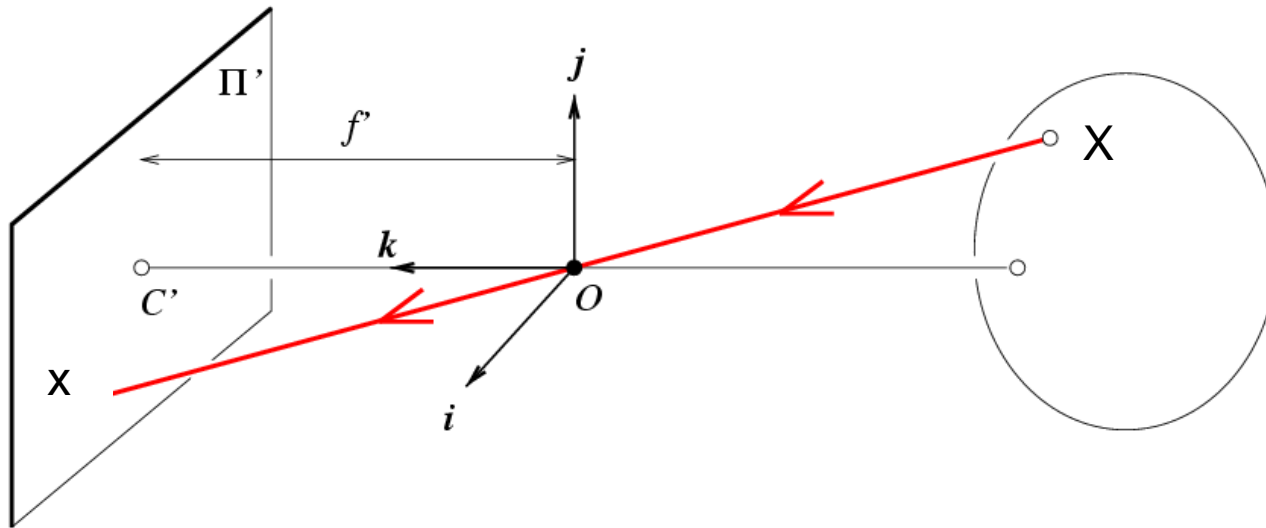
$$v' = -3 * \frac{2}{5}$$

Interlude: why does this matter?

# Relating multiple views



# Projection matrix



## Intrinsic Assumptions

- Unit aspect ratio
- Optical center at  $(0,0)$
- No skew

## Extrinsic Assumptions

- No rotation
- Camera at  $(0,0,0)$

$$\mathbf{x} = \mathbf{K} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \mathbf{X} \Rightarrow w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$\mathbf{K}$

# Remove assumption: optical center at origin

## Intrinsic Assumptions

- Unit aspect ratio
- No skew

## Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \mathbf{X} \Rightarrow w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



# Remove assumption: square pixels

Intrinsic Assumptions

- No skew

Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \mathbf{X} \Rightarrow w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & 0 & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

# Remove assumption: non-skewed pixels

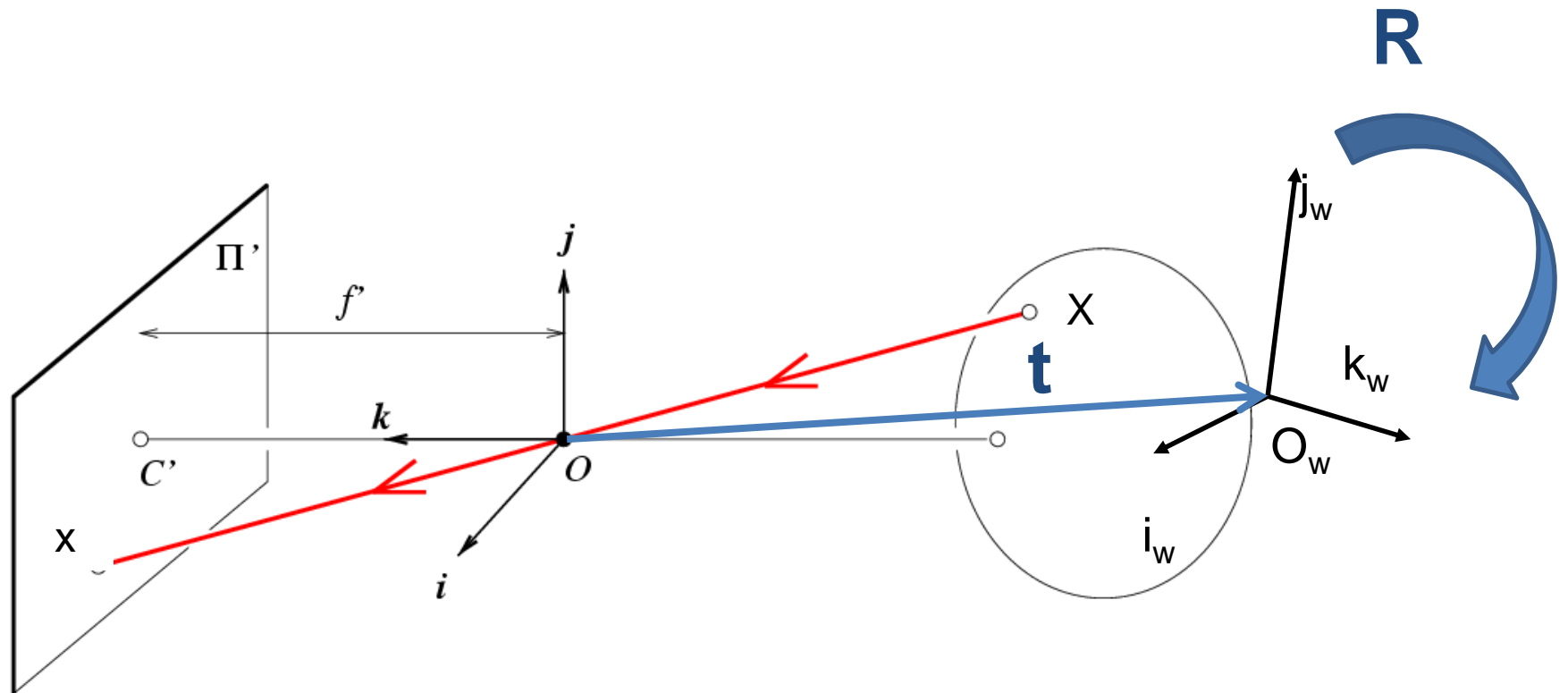
Intrinsic Assumptions    Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \mathbf{X} \Rightarrow w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Note: different books use different notation for parameters

# Oriented and Translated Camera



# Allow camera translation

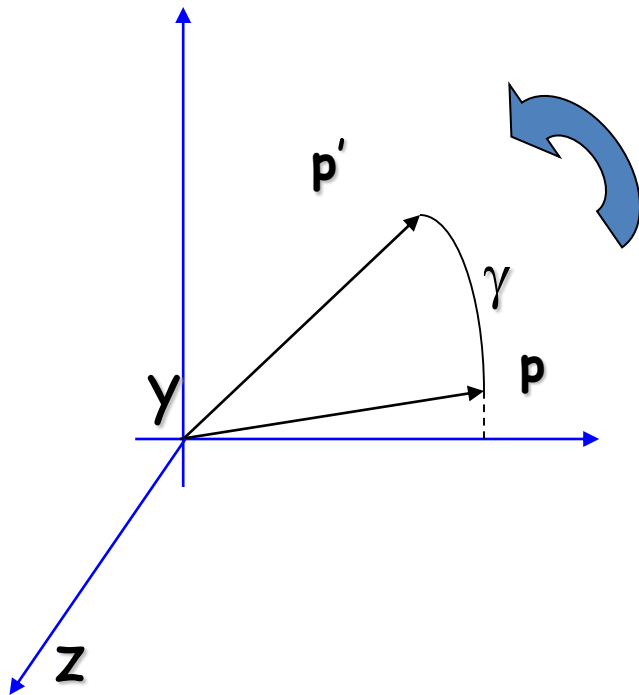
Intrinsic Assumptions    Extrinsic Assumptions

- No rotation

$$\mathbf{x} = \mathbf{K} \begin{bmatrix} \mathbf{I} & \mathbf{t} \end{bmatrix} \mathbf{X} \quad \Rightarrow \quad w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & 0 & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

# 3D Rotation of Points

Rotation around the coordinate axes, **counter-clockwise**:



$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix}$$

$$R_y(\beta) = \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix}$$

$$R_z(\gamma) = \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

# Allow camera rotation

$$\mathbf{x} = \mathbf{K}[\mathbf{R} \quad \mathbf{t}] \mathbf{X}$$



$$w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

# Degrees of freedom

$$\mathbf{x} = \mathbf{K} \begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix} \mathbf{X}$$



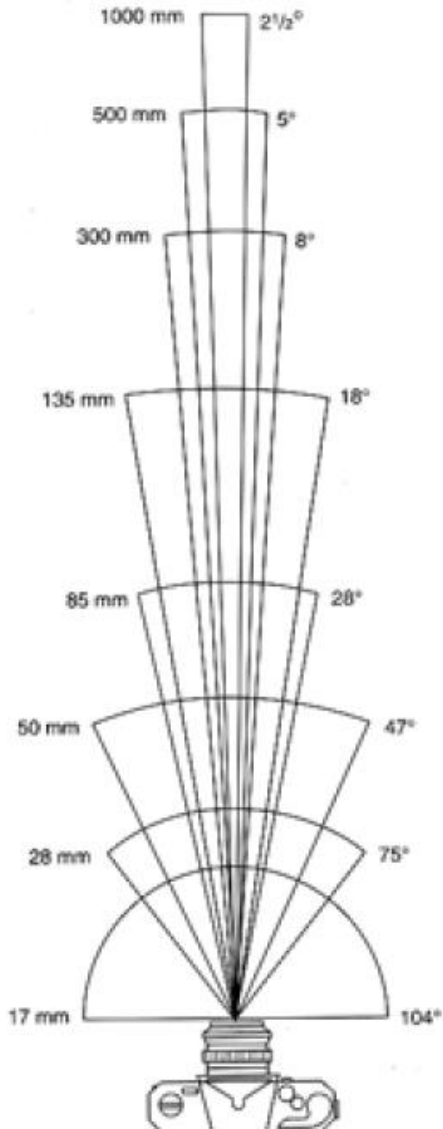
$$w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{matrix} 5 \\ \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \end{matrix} \begin{matrix} 6 \\ \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \end{matrix} \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

# Reminder: read your book

- Lectures have assigned readings
- Szeliski 2.1 and especially 2.1.5 cover the geometry of image formation



# Field of View (Zoom, focal length)



17mm



28mm



50mm

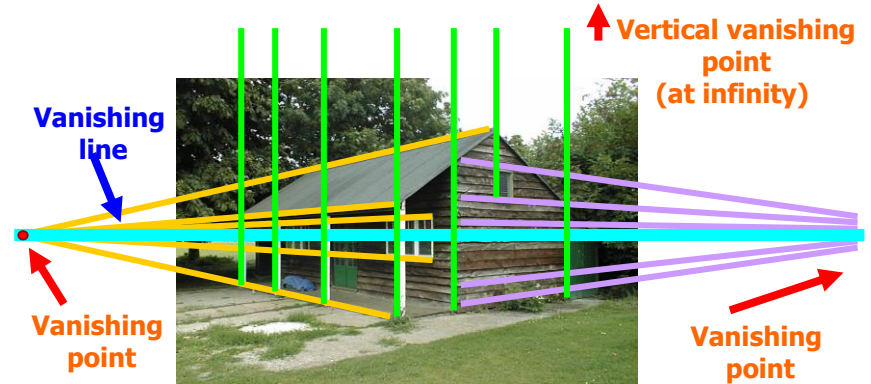


85mm

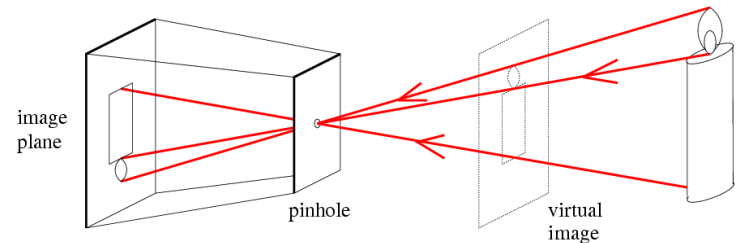
**From London and Upton**

# Things to remember

- Vanishing points and vanishing lines



- Pinhole camera model and camera projection matrix

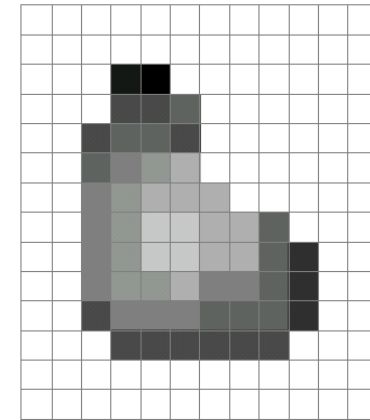
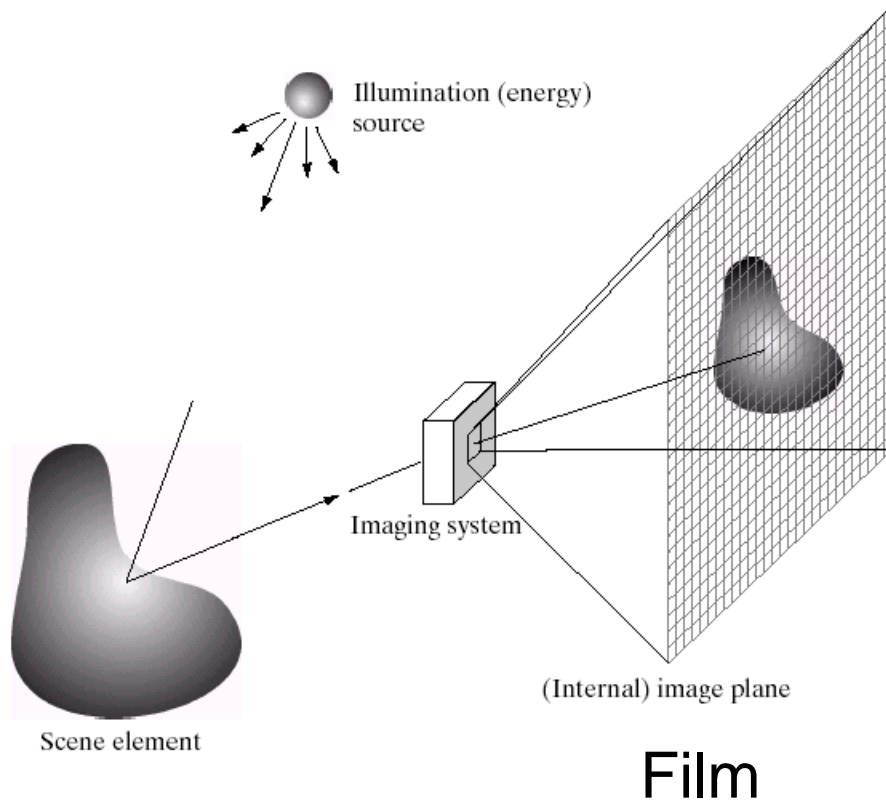


$$\mathbf{x} = \mathbf{K}[\mathbf{R} \quad \mathbf{t}] \mathbf{X}$$

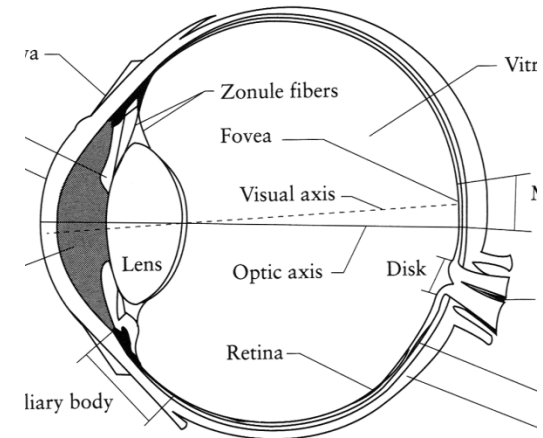
- Homogeneous coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

# Image Formation



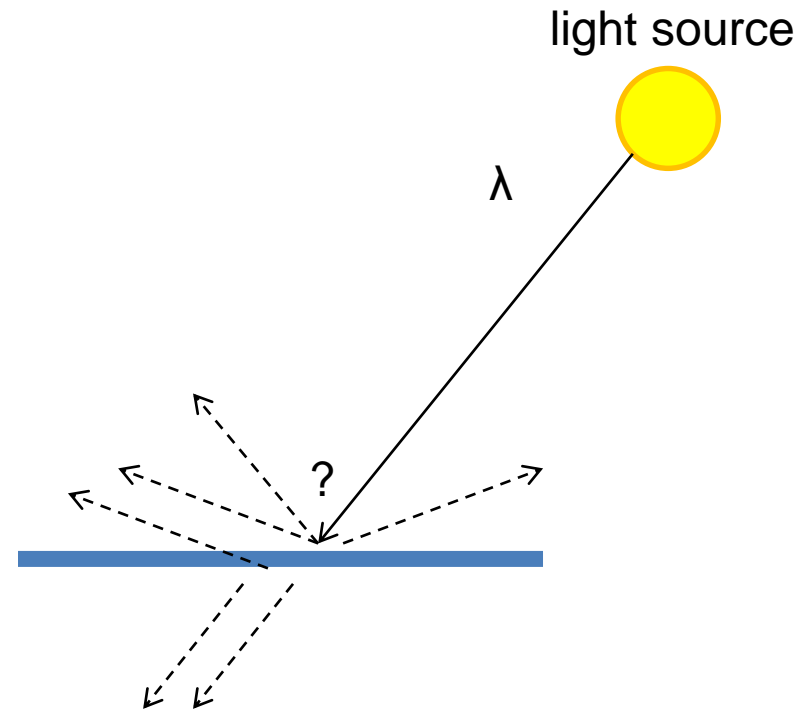
Digital Camera



The Eye

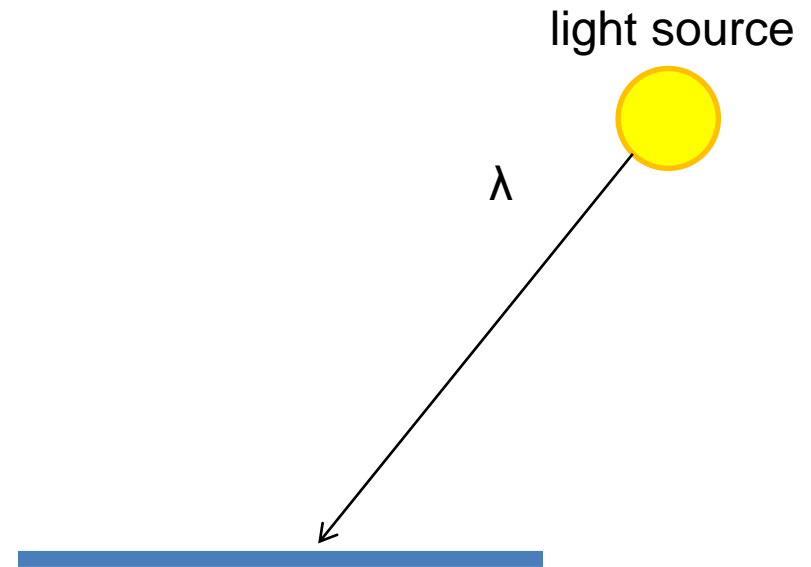
# A photon's life choices

- Absorption
- Diffusion
- Reflection
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- Interreflection



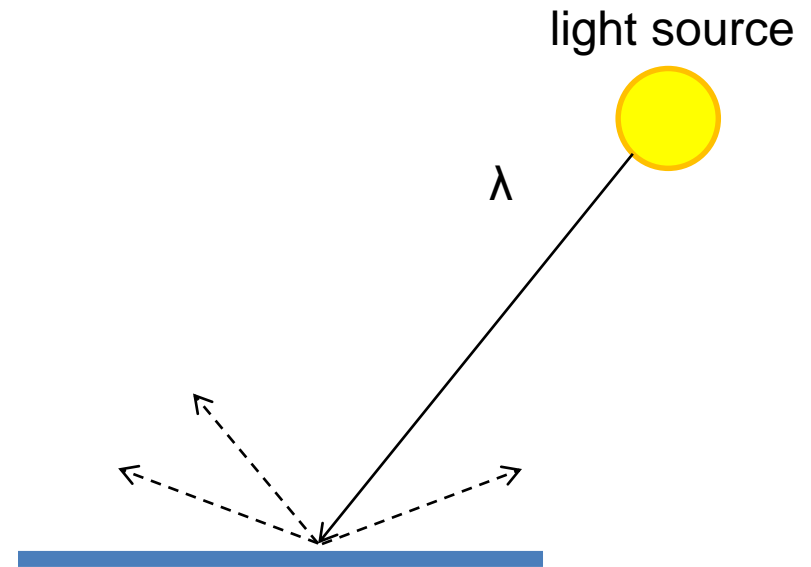
# A photon's life choices

- **Absorption**
- Diffusion
- Reflection
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- Interreflection



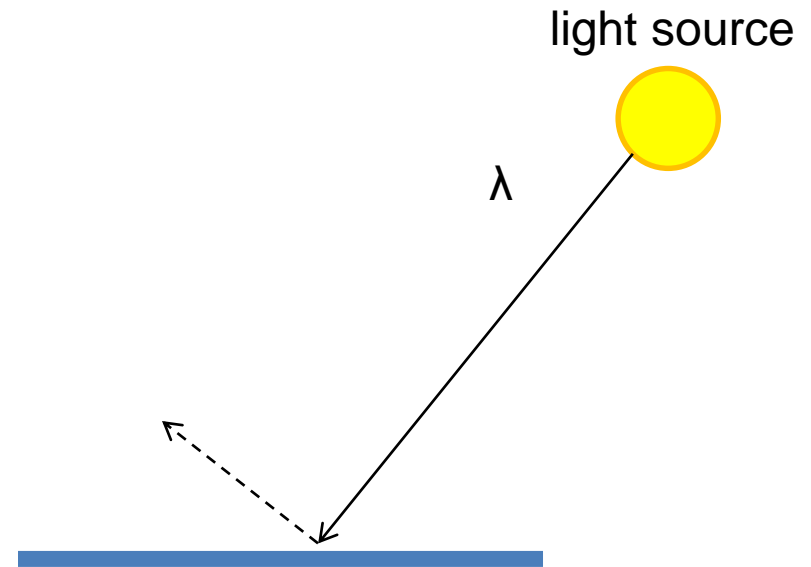
# A photon's life choices

- Absorption
- **Diffuse Reflection**
- Reflection
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- Interreflection



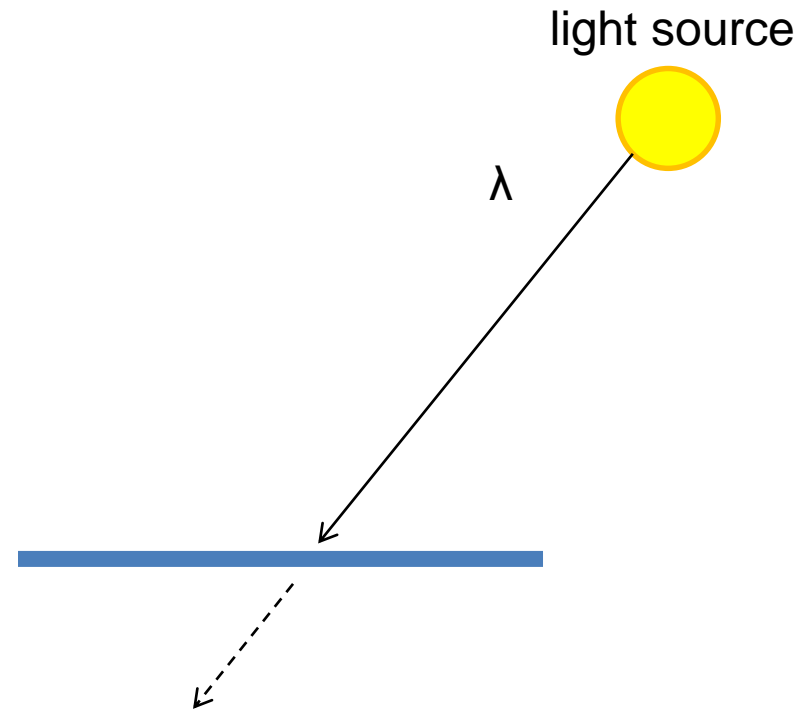
# A photon's life choices

- Absorption
- Diffusion
- **Specular Reflection**
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- Interreflection



# A photon's life choices

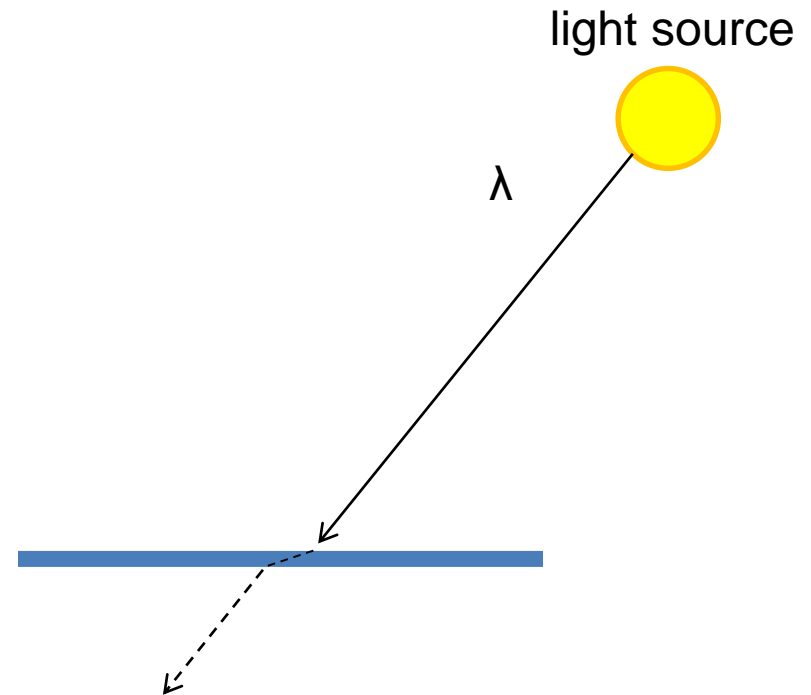
- Absorption
- Diffusion
- Reflection
- **Transparency**
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- Interreflection





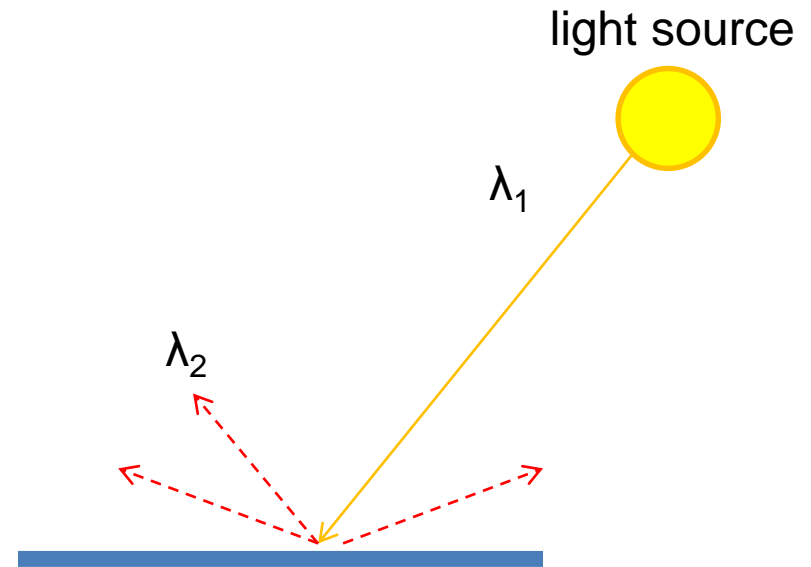
# A photon's life choices

- Absorption
- Diffusion
- Reflection
- Transparency
- **Refraction**
- Fluorescence
- Subsurface scattering
- Phosphorescence
- Interreflection



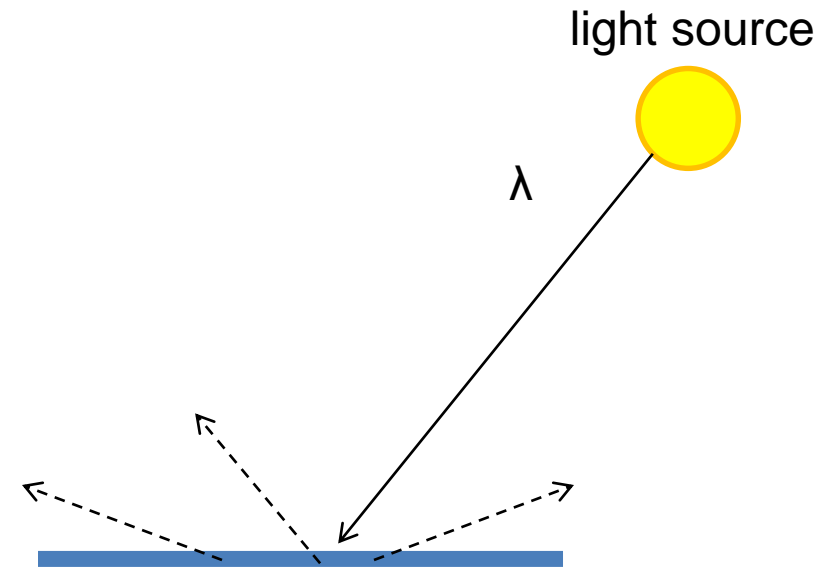
# A photon's life choices

- Absorption
- Diffusion
- Reflection
- Transparency
- Refraction
- **Fluorescence**
- Subsurface scattering
- Phosphorescence
- Interreflection



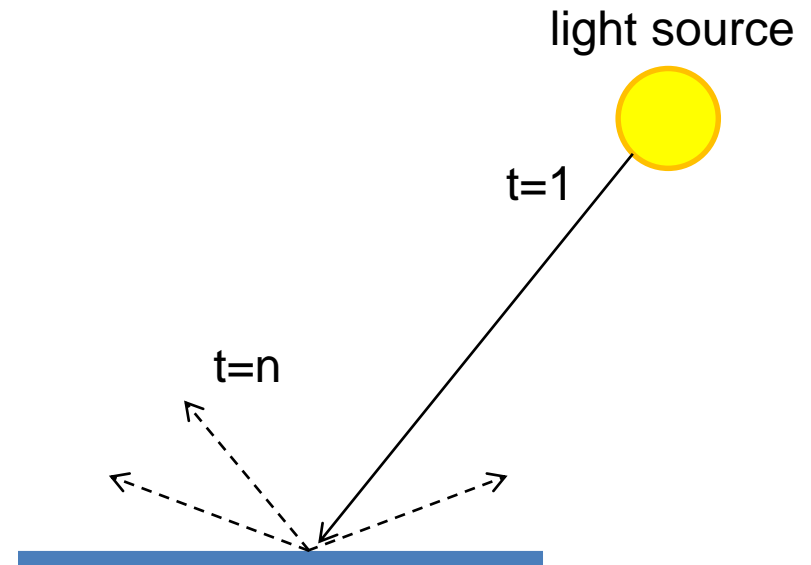
# A photon's life choices

- Absorption
- Diffusion
- Reflection
- Transparency
- Refraction
- Fluorescence
- **Subsurface scattering**
- Phosphorescence
- Interreflection



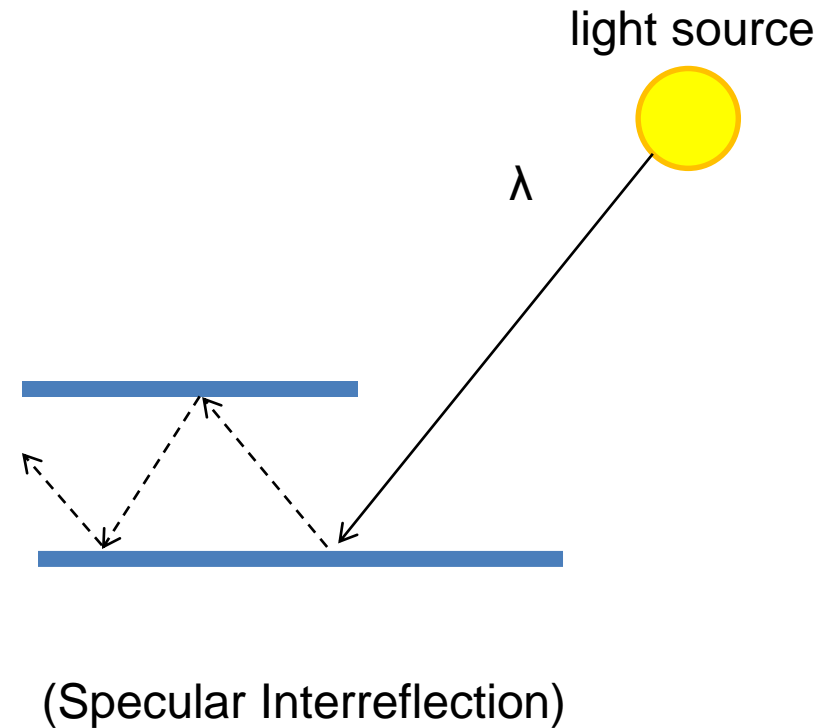
# A photon's life choices

- Absorption
- Diffusion
- Reflection
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- **Phosphorescence**
- Interreflection



# A photon's life choices

- Absorption
- Diffusion
- Reflection
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- **Interreflection**



# Lambertian Reflectance

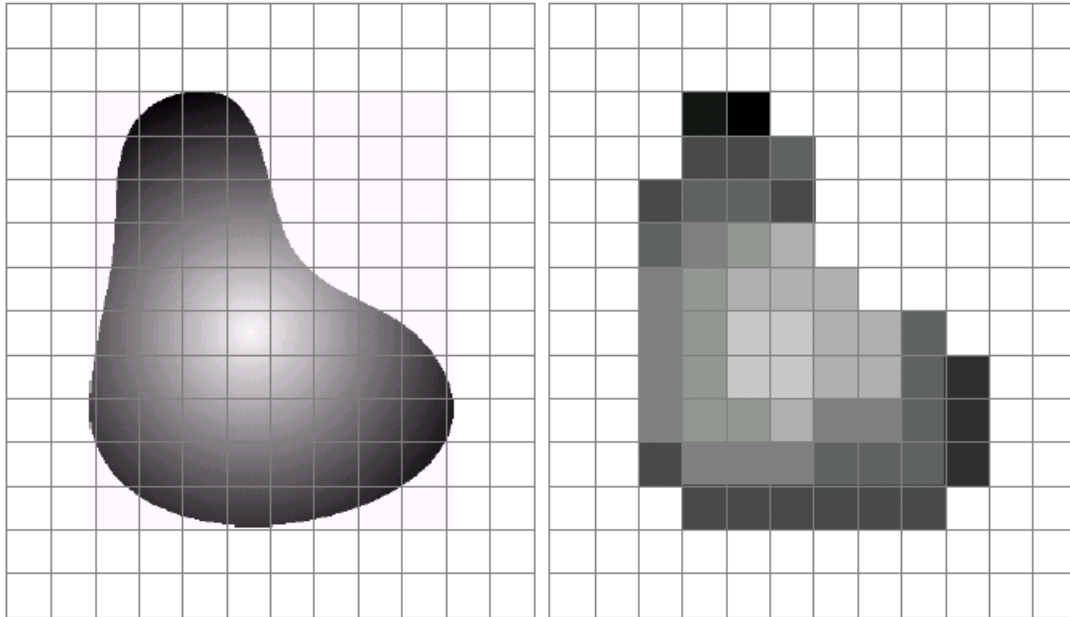
- In computer vision, surfaces are often assumed to be ideal diffuse reflectors with no dependence on viewing direction.

# Digital camera



- A digital camera replaces film with a sensor array
  - Each cell in the array is light-sensitive diode that converts photons to electrons
  - Two common types
    - Charge Coupled Device (CCD)
    - CMOS
  - <http://electronics.howstuffworks.com/digital-camera.htm>

# Sensor Array



a b

**FIGURE 2.17** (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.

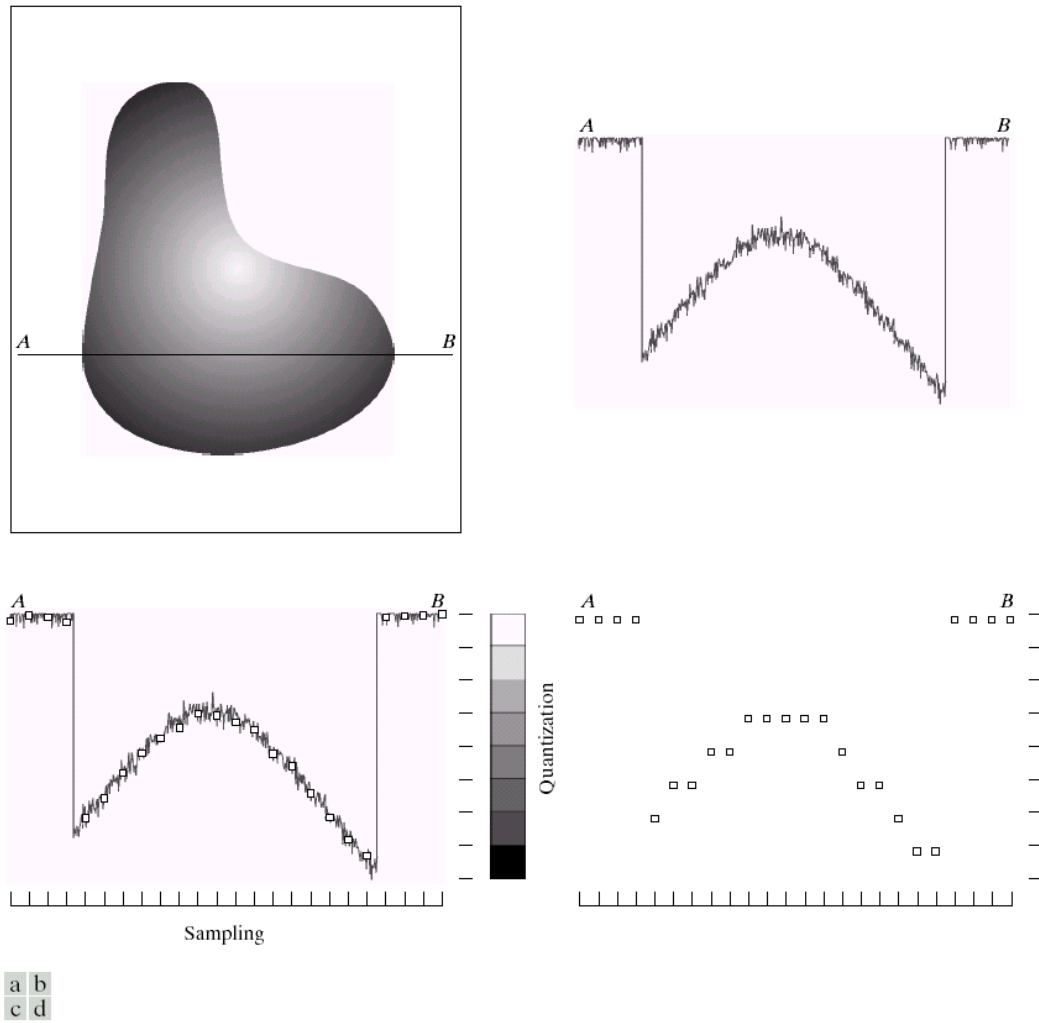
---



CMOS sensor

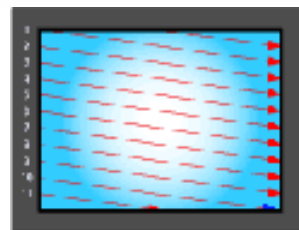
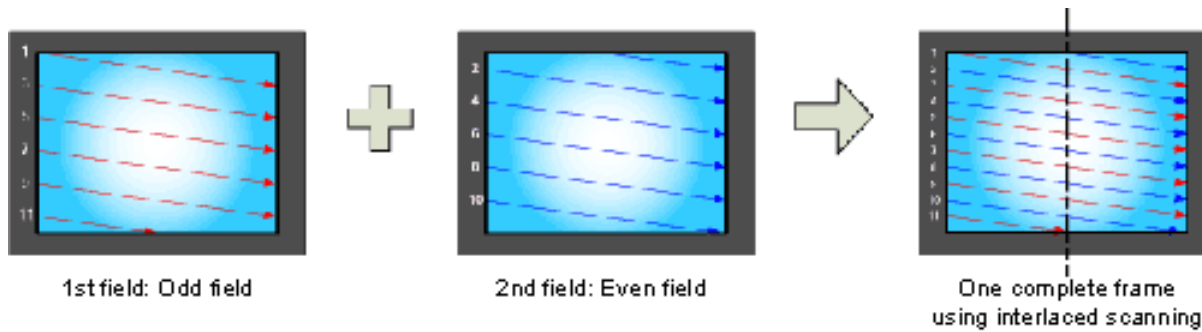


# Sampling and Quantization



**FIGURE 2.16** Generating a digital image. (a) Continuous image. (b) A scan line from *A* to *B* in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

# Interlace vs. progressive scan



One complete frame using progressive scanning

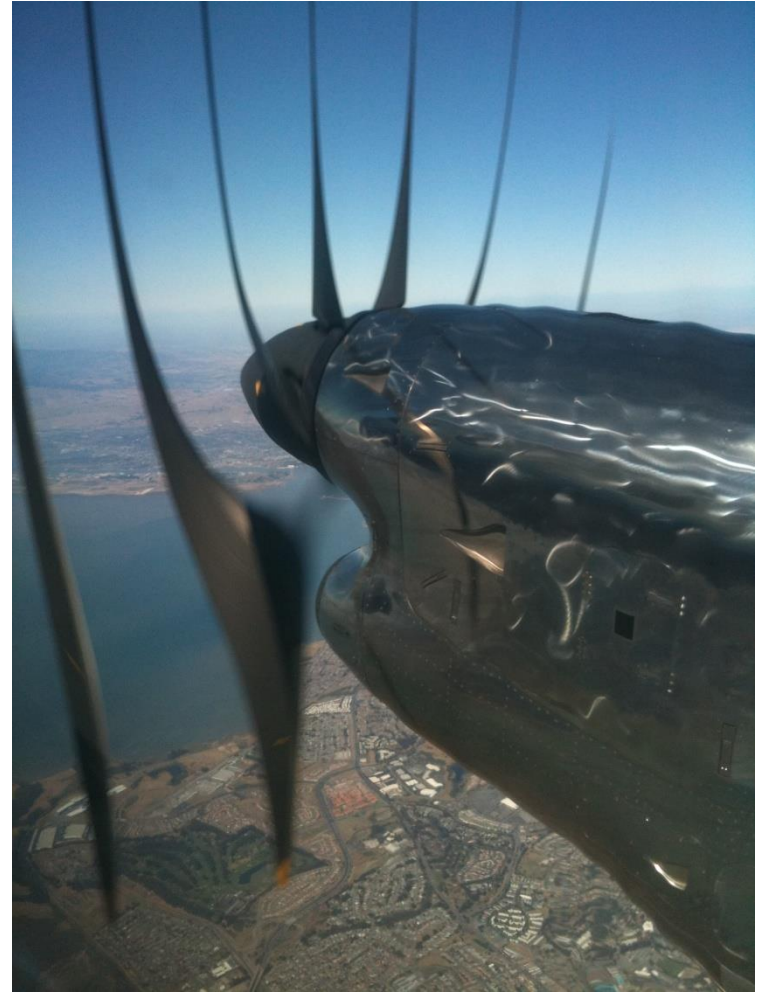
# Progressive scan



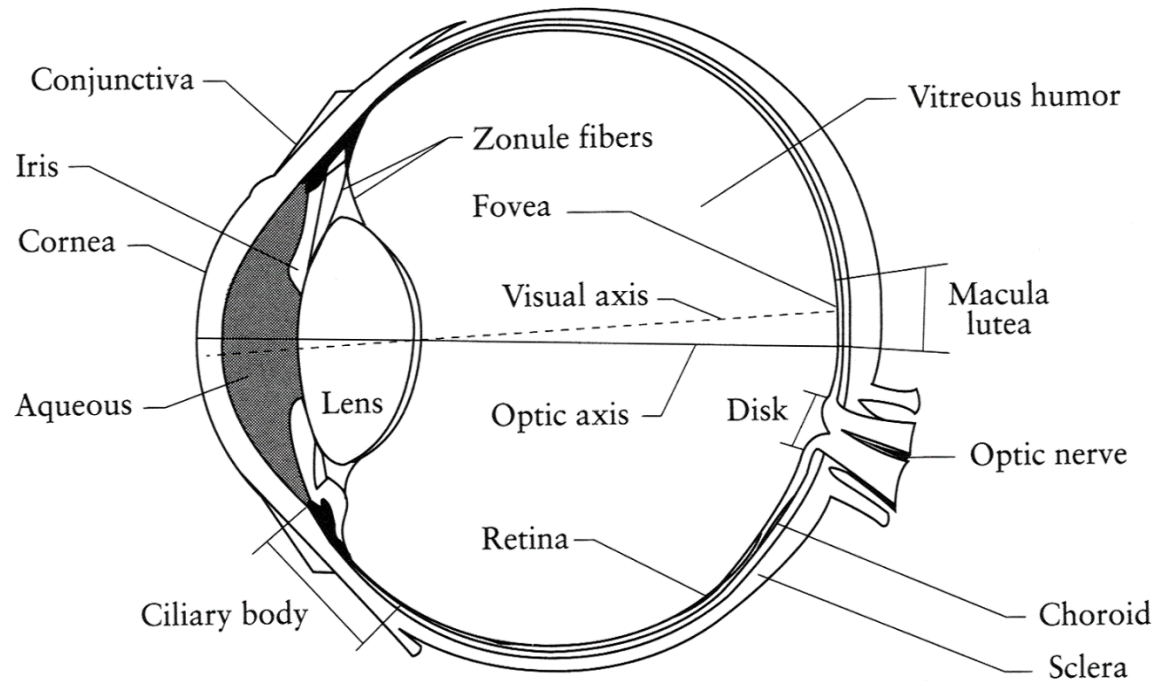
# Interlace



# Rolling Shutter



# The Eye

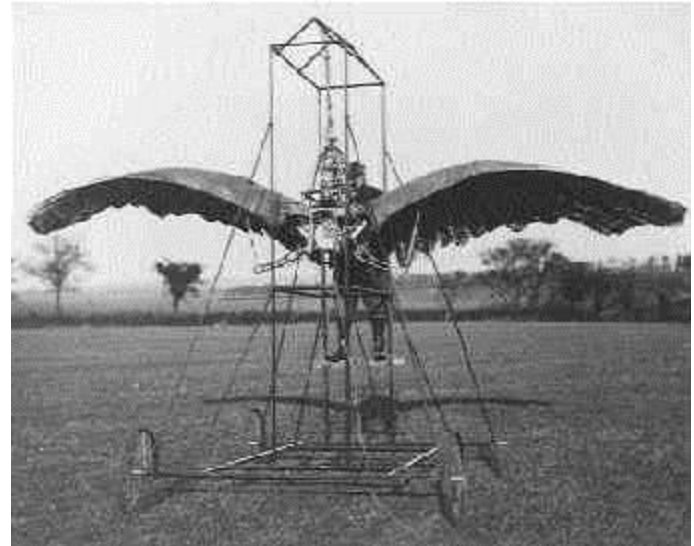
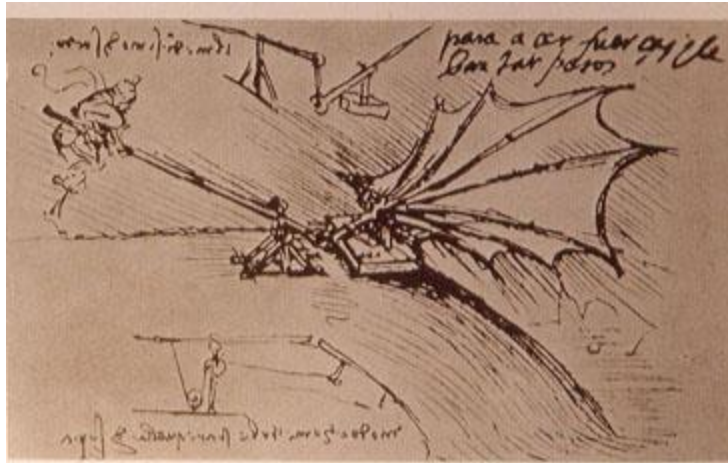


- The human eye is a camera!
  - **Iris** - colored annulus with radial muscles
  - **Pupil** - the hole (aperture) whose size is controlled by the iris
  - What's the "film"?
    - photoreceptor cells (rods and cones) in the **retina**

Aside: why do we care about human vision in this class?

- We don't, necessarily.

# Ornithopters





# Why do we care about human vision?

- We don't, necessarily.
- But cameras necessarily imitate the frequency response of the human eye, so we should know that much.
- Also, computer vision probably wouldn't get as much scrutiny if biological vision (especially human vision) hadn't proved that it was possible to make important judgements from 2d images.

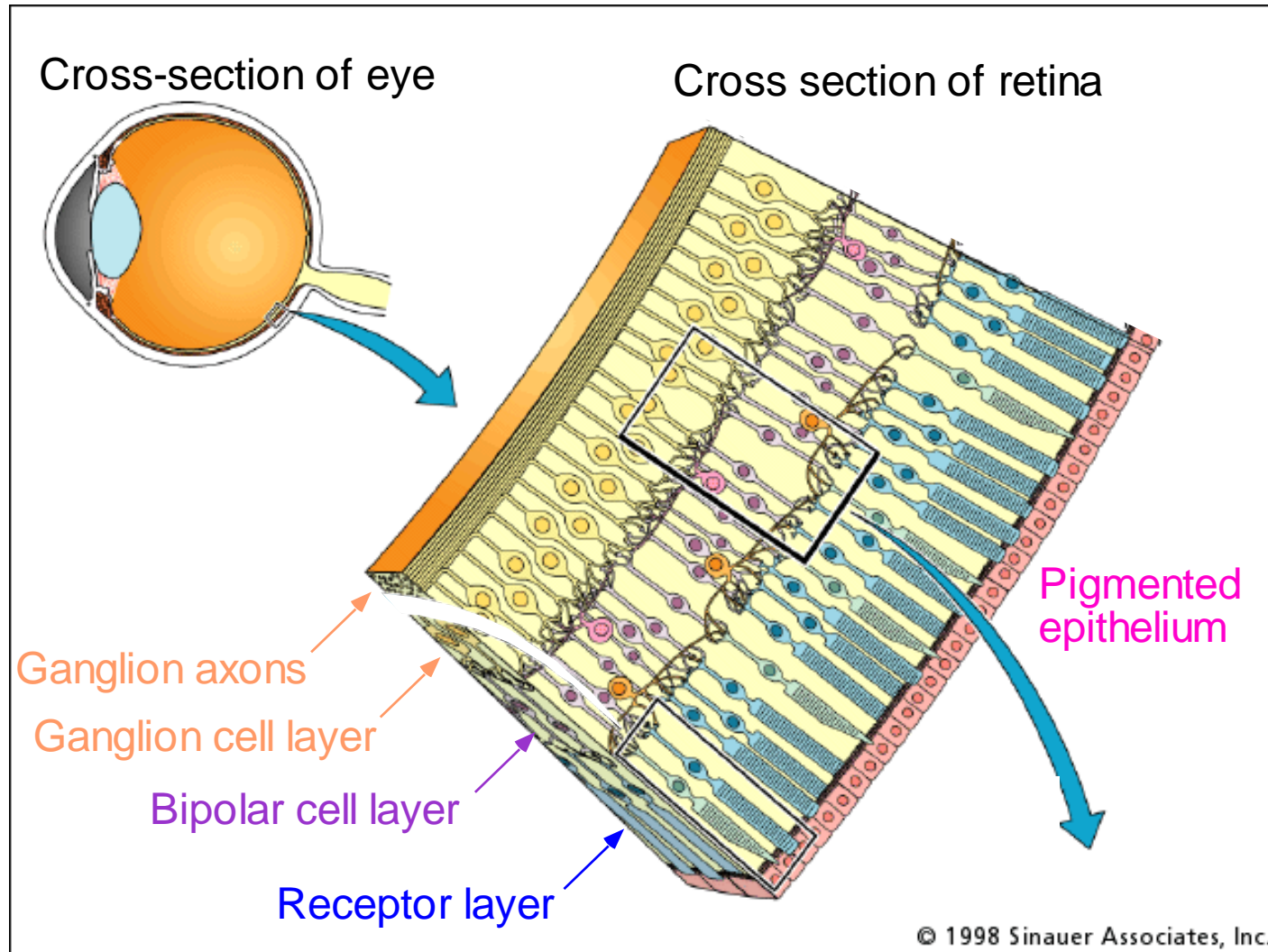
# Does computer vision “understand” images?

"Can machines fly?" The answer is yes, because airplanes fly.

"Can machines swim?" The answer is no, because submarines don't swim.

"Can machines think?" Is this question like the first, or like the second?

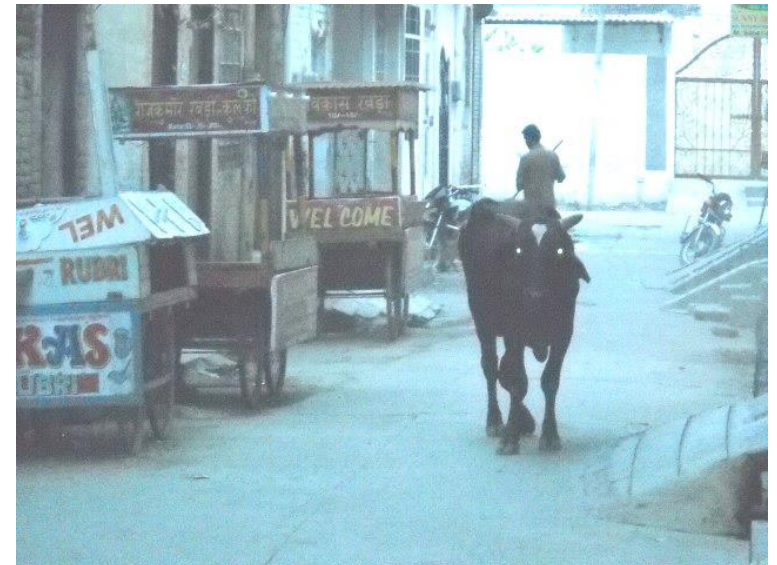
# The Retina



# What humans don't have: tapetum lucidum



Human eyes can reflect a tiny bit and blood in the retina makes this reflection red.



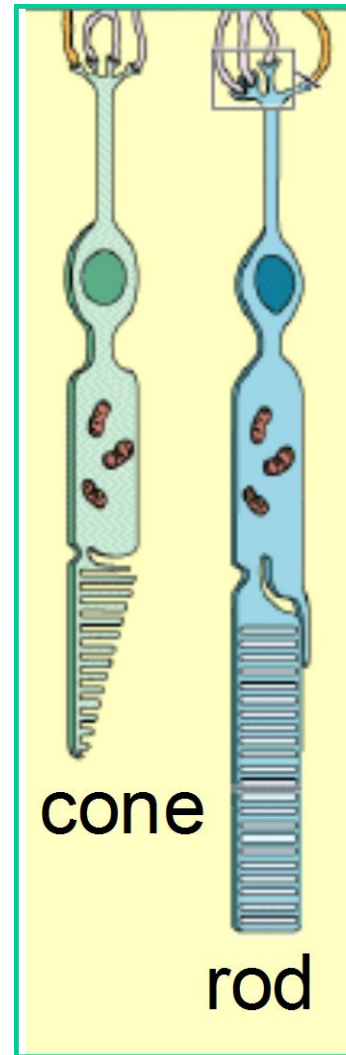
# Two types of light-sensitive receptors

## **C**ones

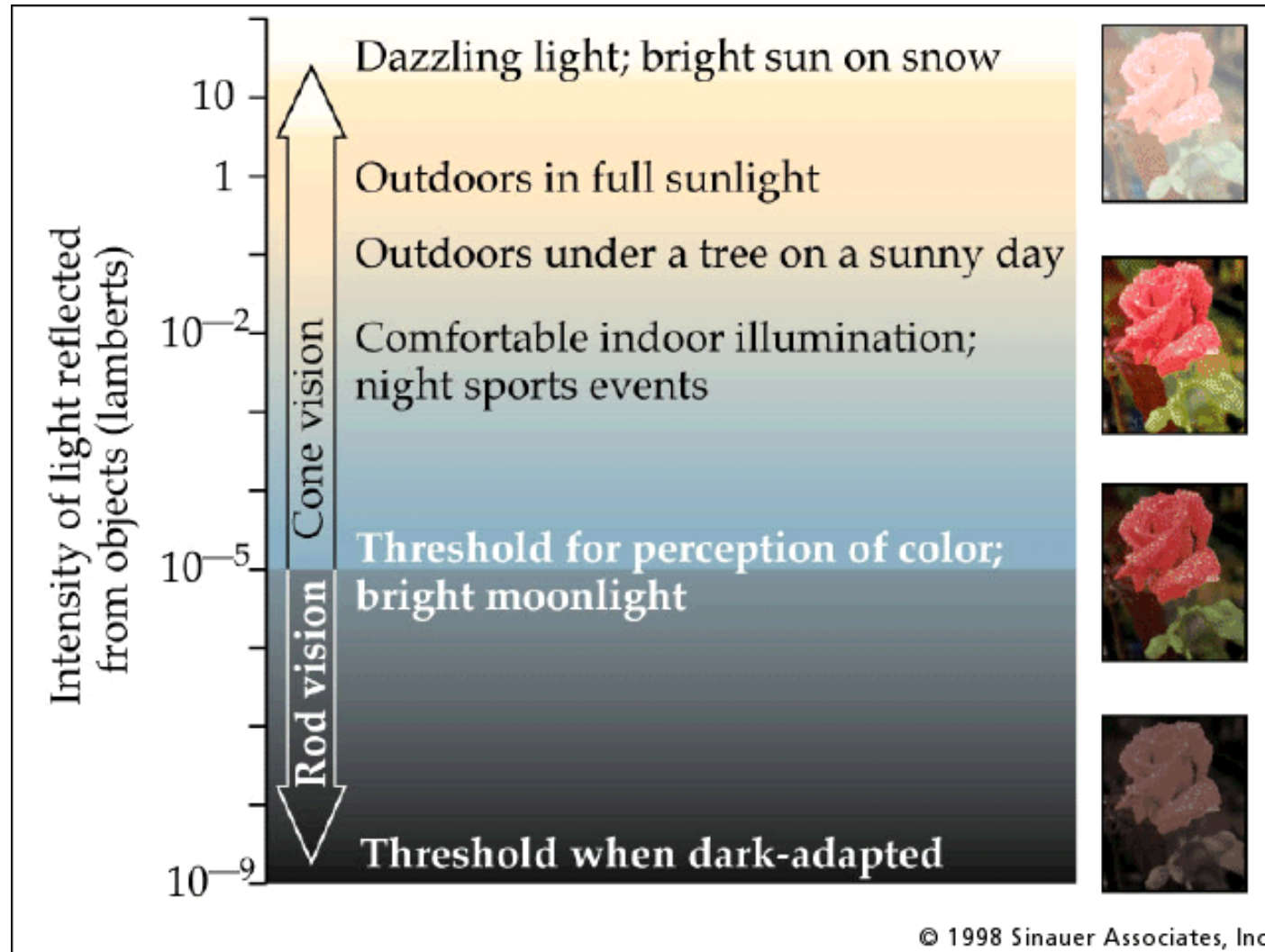
cone-shaped  
less sensitive  
operate in high light  
color vision

## **R**ods

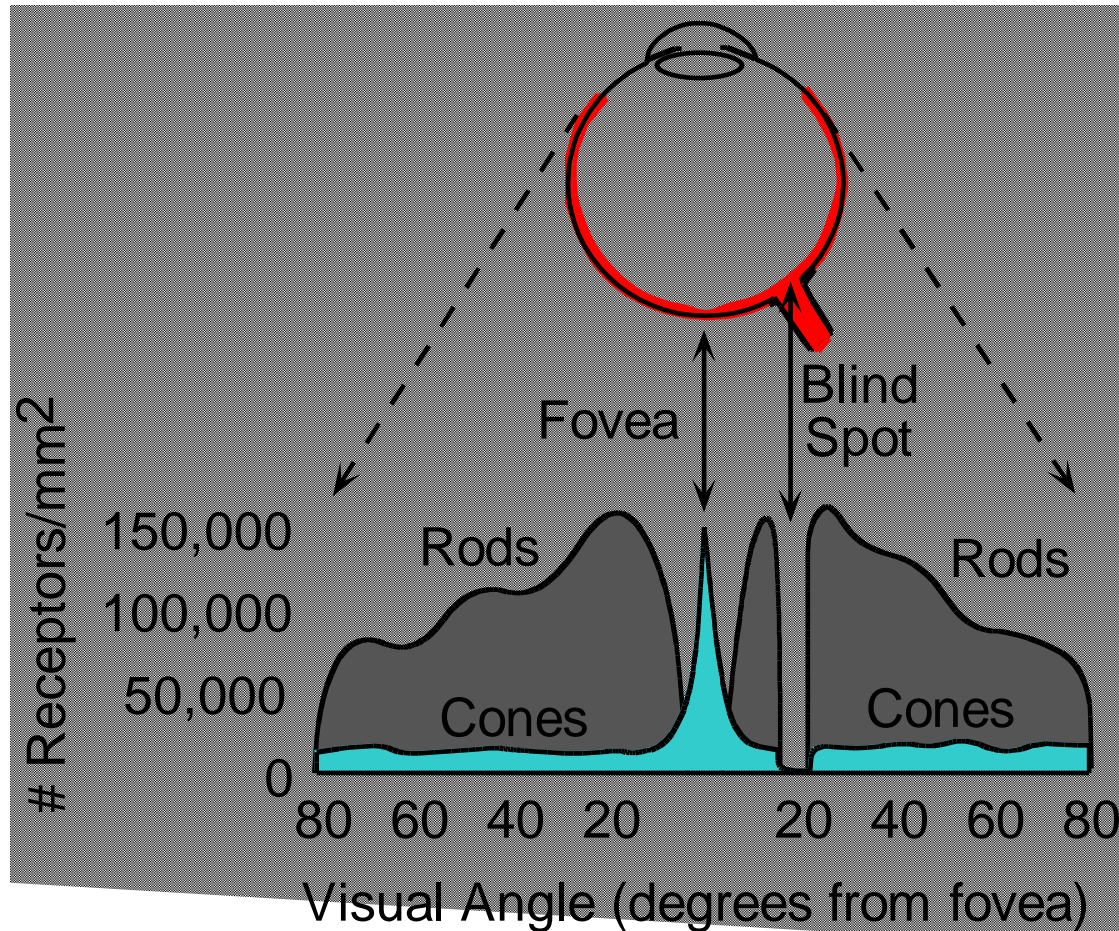
rod-shaped  
highly sensitive  
operate at night  
gray-scale vision



# Rod / Cone sensitivity



# Distribution of Rods and Cones



Night Sky: why are there more stars off-center?

Averted vision: [http://en.wikipedia.org/wiki/Averted\\_vision](http://en.wikipedia.org/wiki/Averted_vision)

Wait, the blood vessels are in front of the photoreceptors??

[https://www.youtube.com/watch?v=L\\_W-IXqoxHA](https://www.youtube.com/watch?v=L_W-IXqoxHA)

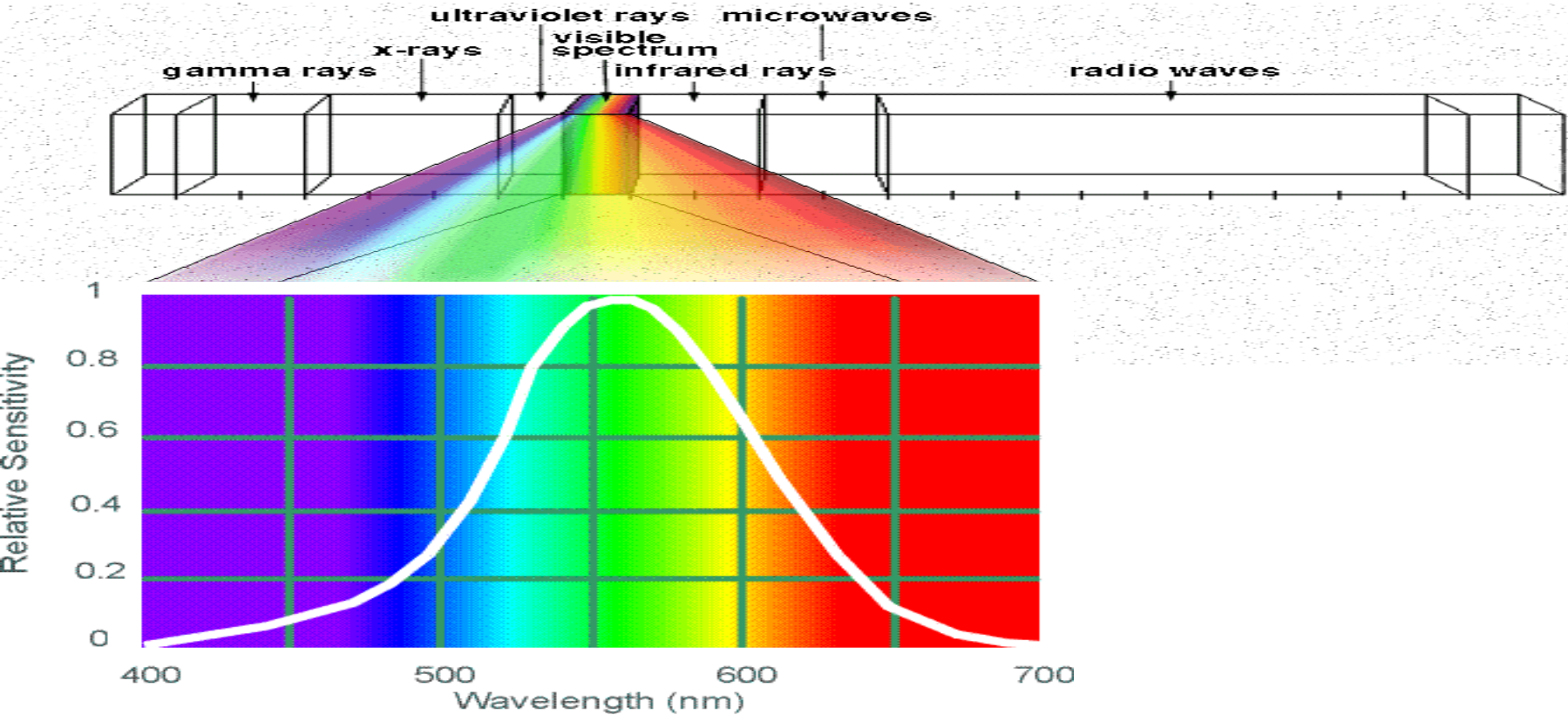




# Eye Movements

- Saccades
  - Can be consciously controlled. Related to perceptual attention.
  - 200ms to initiation, 20 to 200ms to carry out. Large amplitude.
- Microsaccades
  - Involuntary. Smaller amplitude. Especially evident during prolonged fixation. Function debated.
- Ocular microtremor (OMT)
  - involuntary. high frequency (up to 80Hz), small amplitude.
- Smooth pursuit – tracking an object

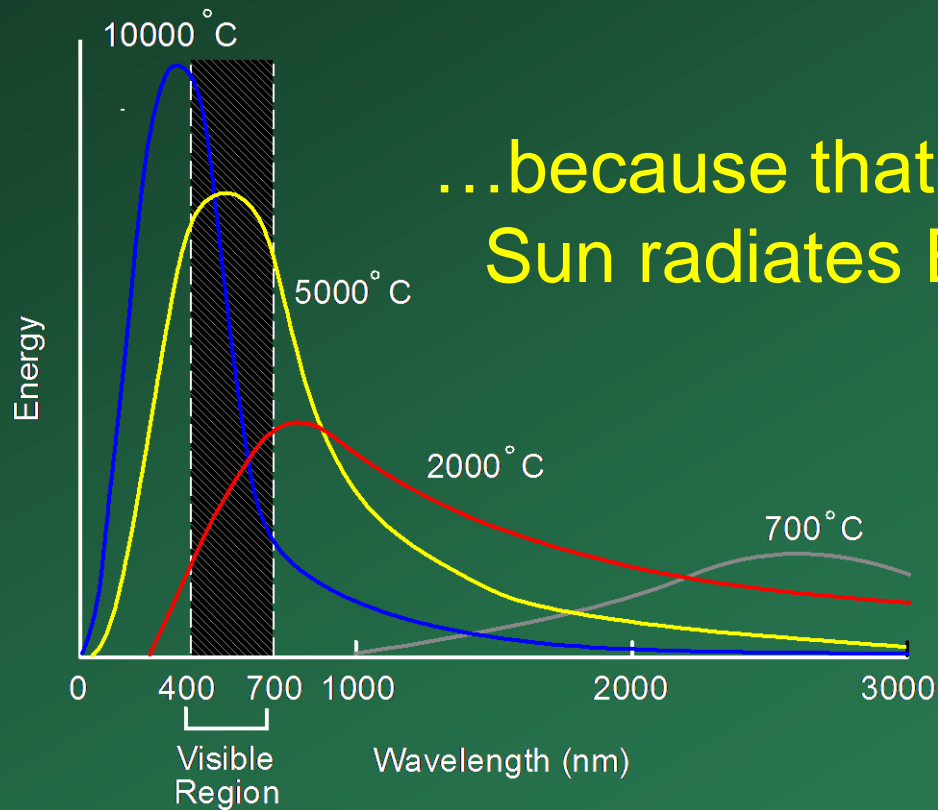
# Electromagnetic Spectrum



## Human Luminance Sensitivity Function

# Visible Light

Why do we see light of these wavelengths?

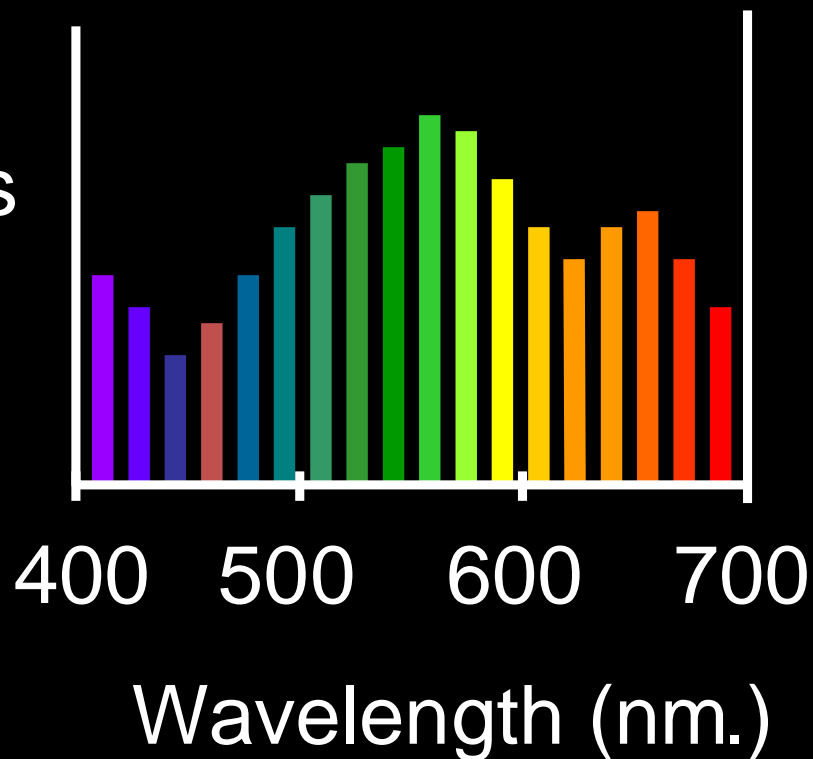


...because that's where the Sun radiates EM energy

# The Physics of Light

Any patch of light can be completely described physically by its spectrum: the number of photons (per time unit) at each wavelength 400 - 700 nm.

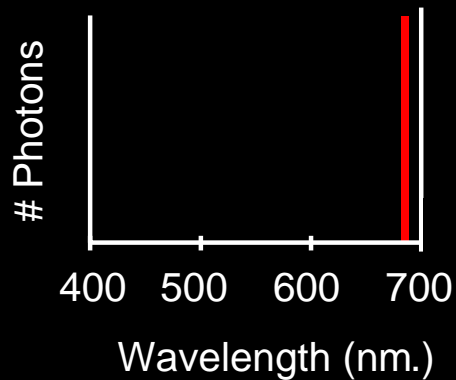
# Photons  
(per ms.)



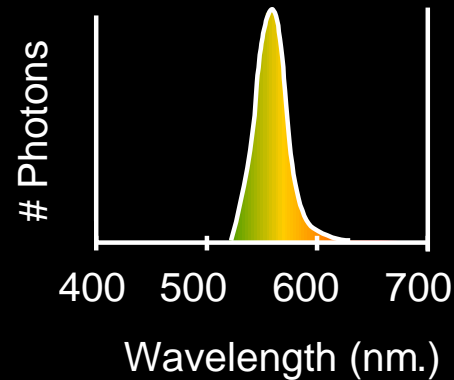
# The Physics of Light

## Some examples of the spectra of light sources

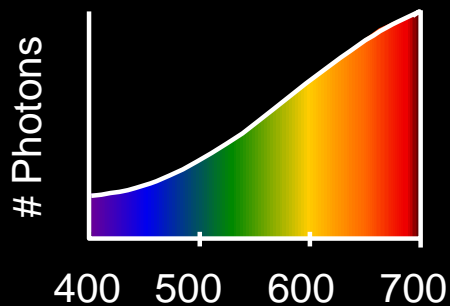
A. Ruby Laser



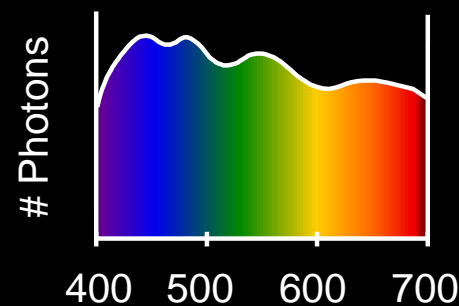
B. Gallium Phosphide Crystal



C. Tungsten Lightbulb



D. Normal Daylight

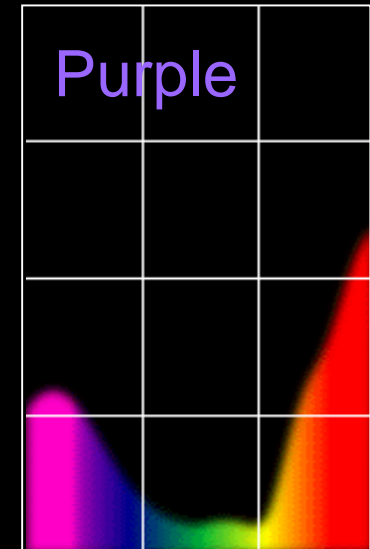
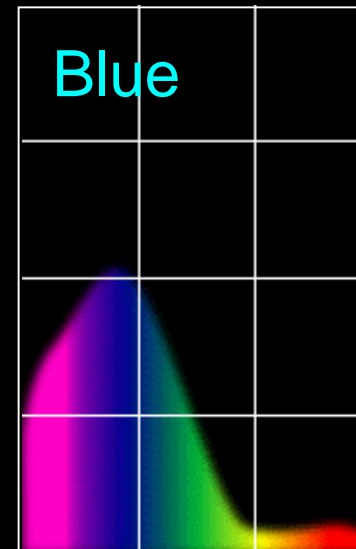
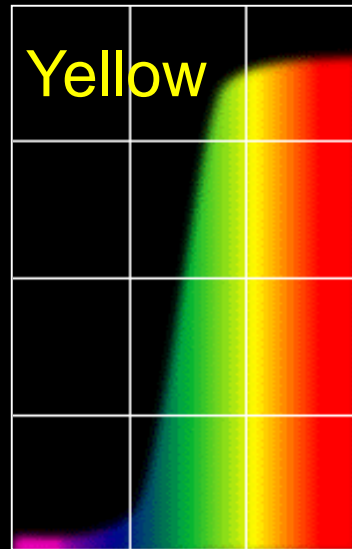
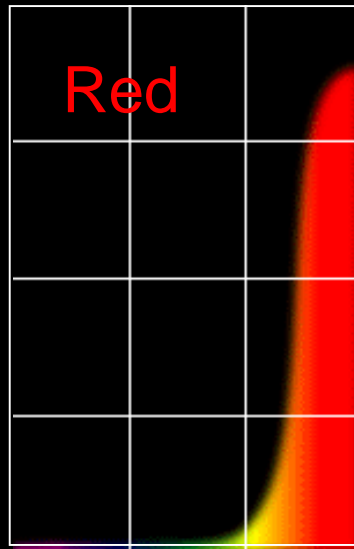


# The Physics of Light

Some examples of the reflectance spectra of surfaces



% Photons Reflected



400

700

400

700

400

700

400

700

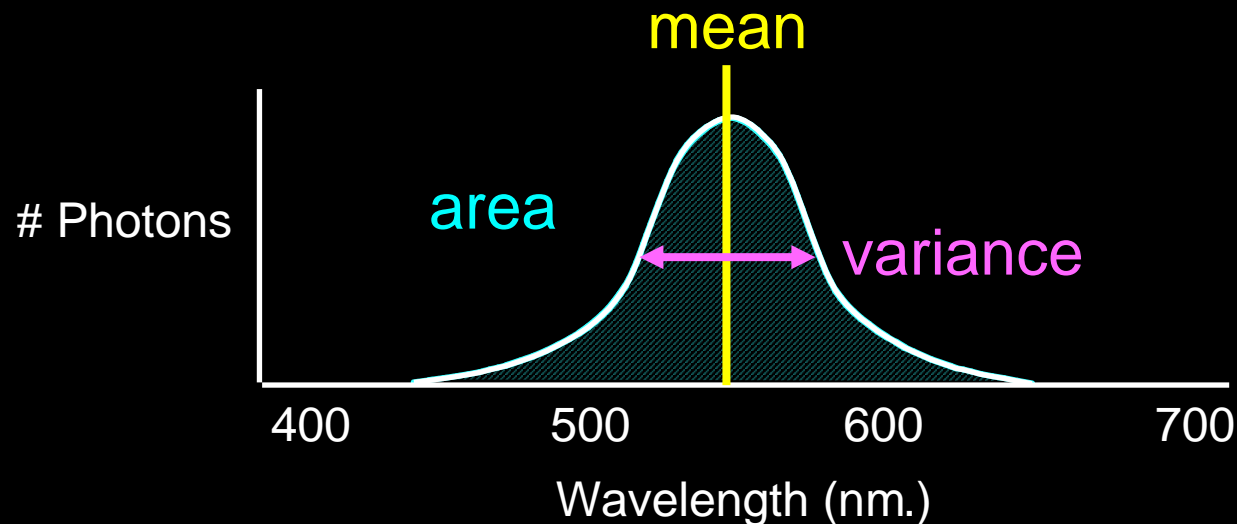
Wavelength (nm)

# The Psychophysical Correspondence

There is no simple functional description for the perceived color of all lights under all viewing conditions, but .....

A helpful constraint:

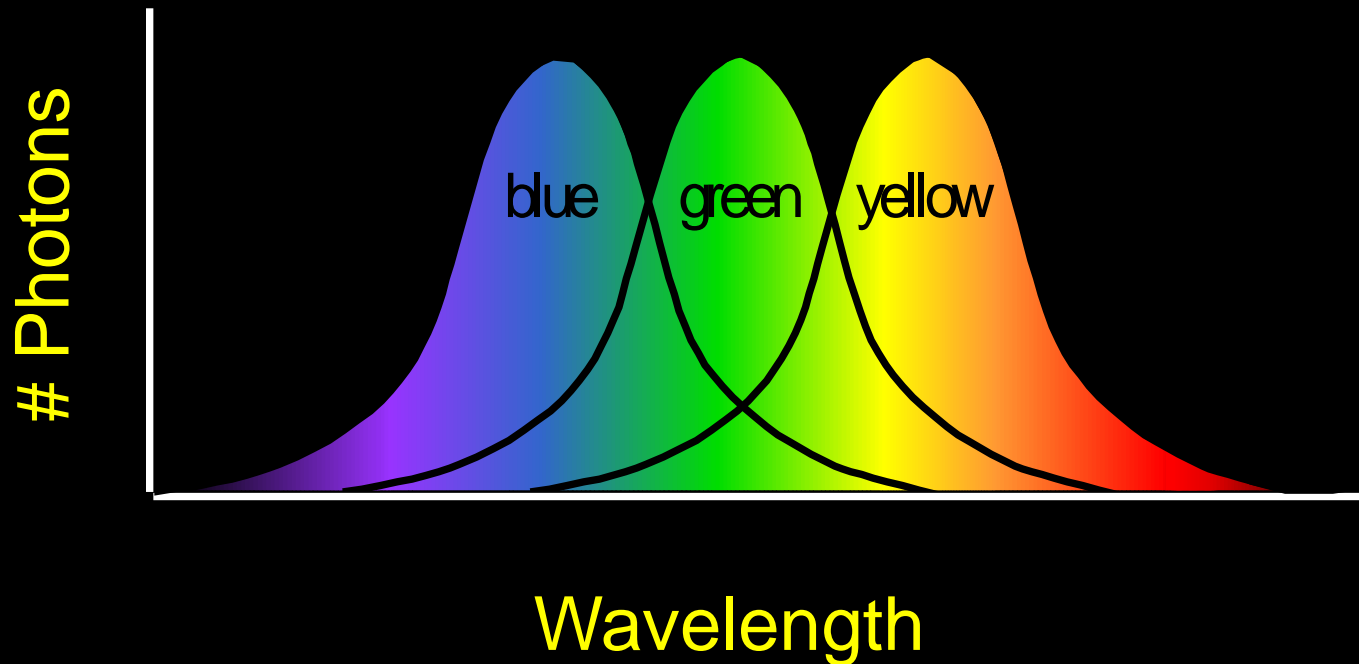
Consider only physical spectra with normal distributions





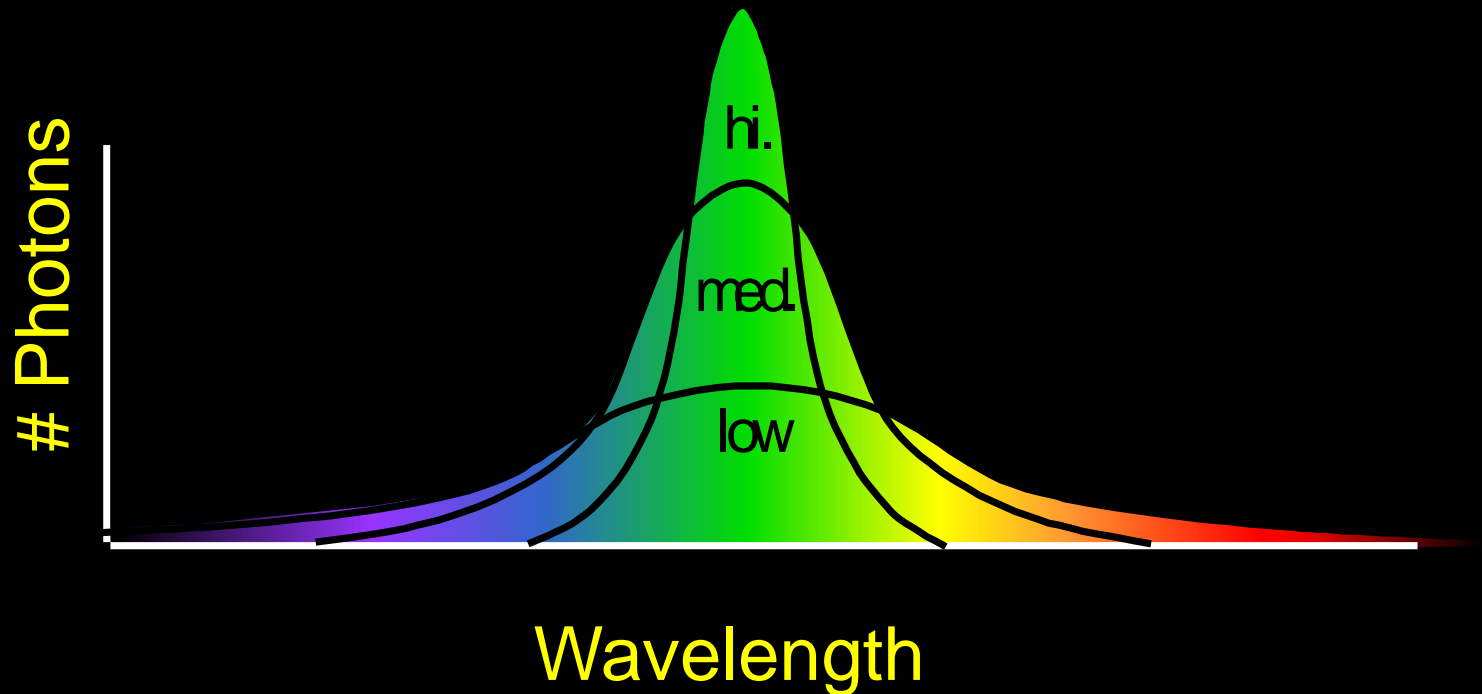
# The Psychophysical Correspondence

Mean ↔ Hue



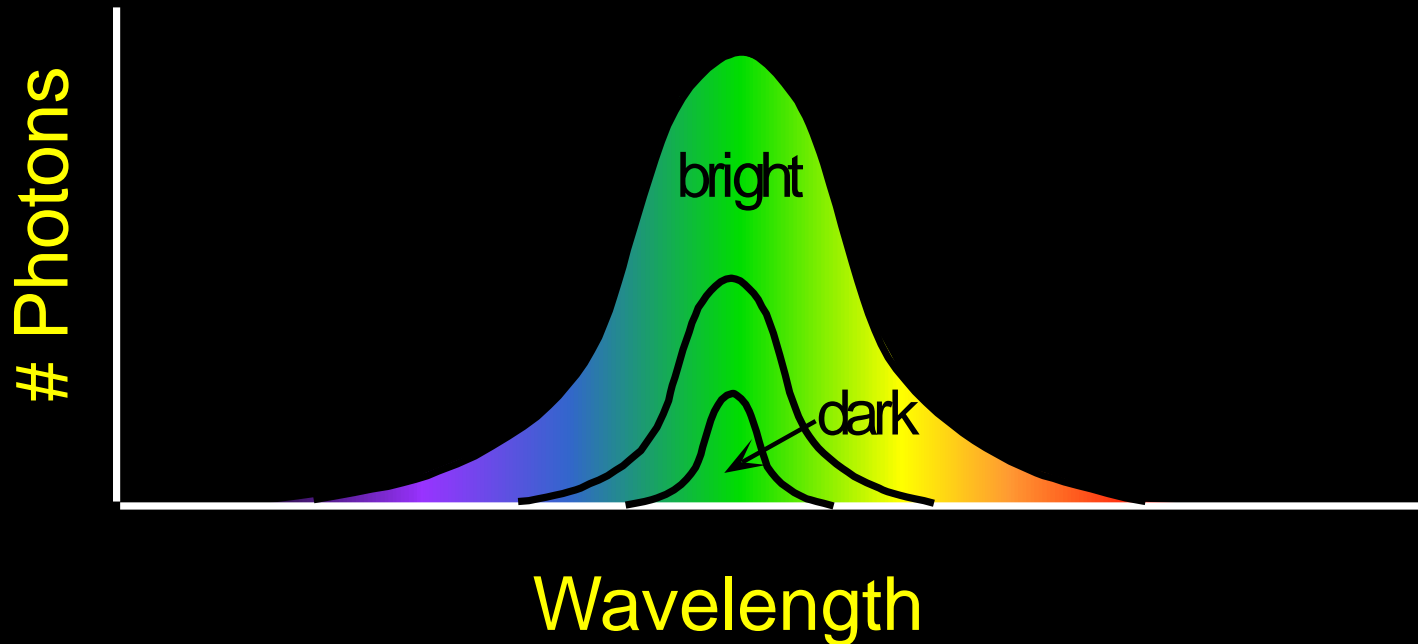
# The Psychophysical Correspondence

Variance  $\longleftrightarrow$  Saturation



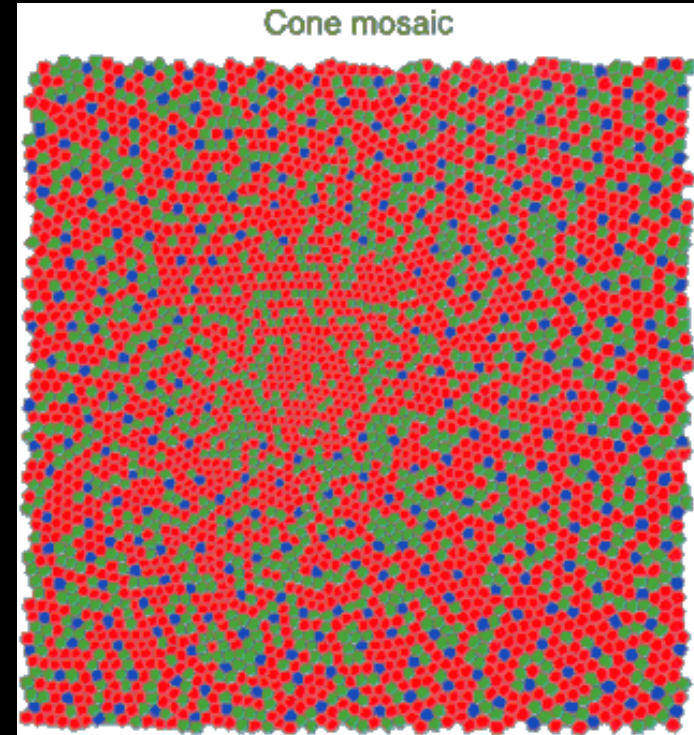
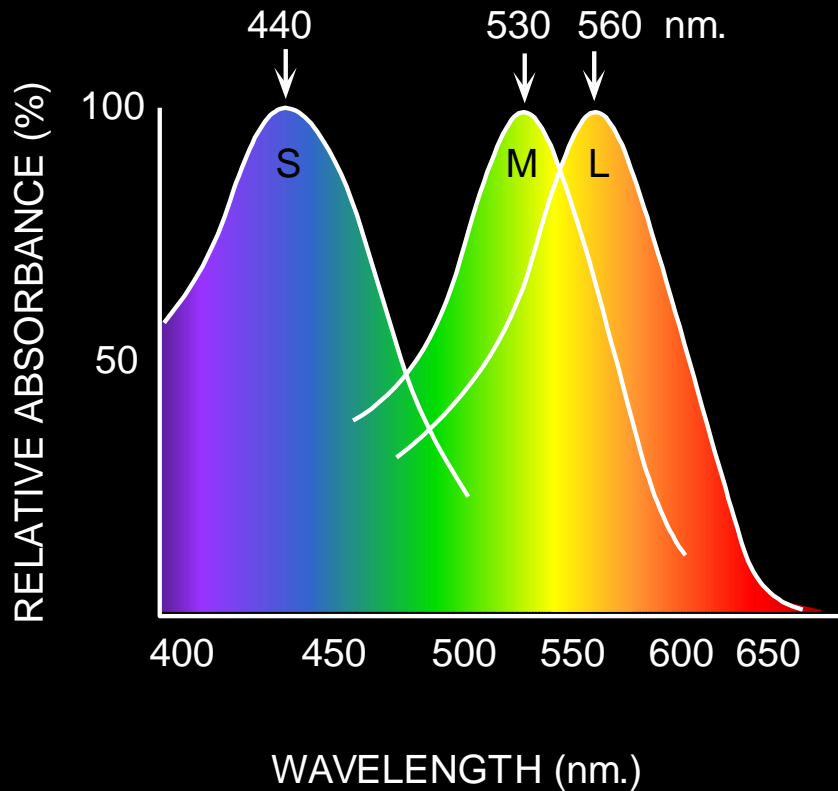
# The Psychophysical Correspondence

Area  $\longleftrightarrow$  Brightness



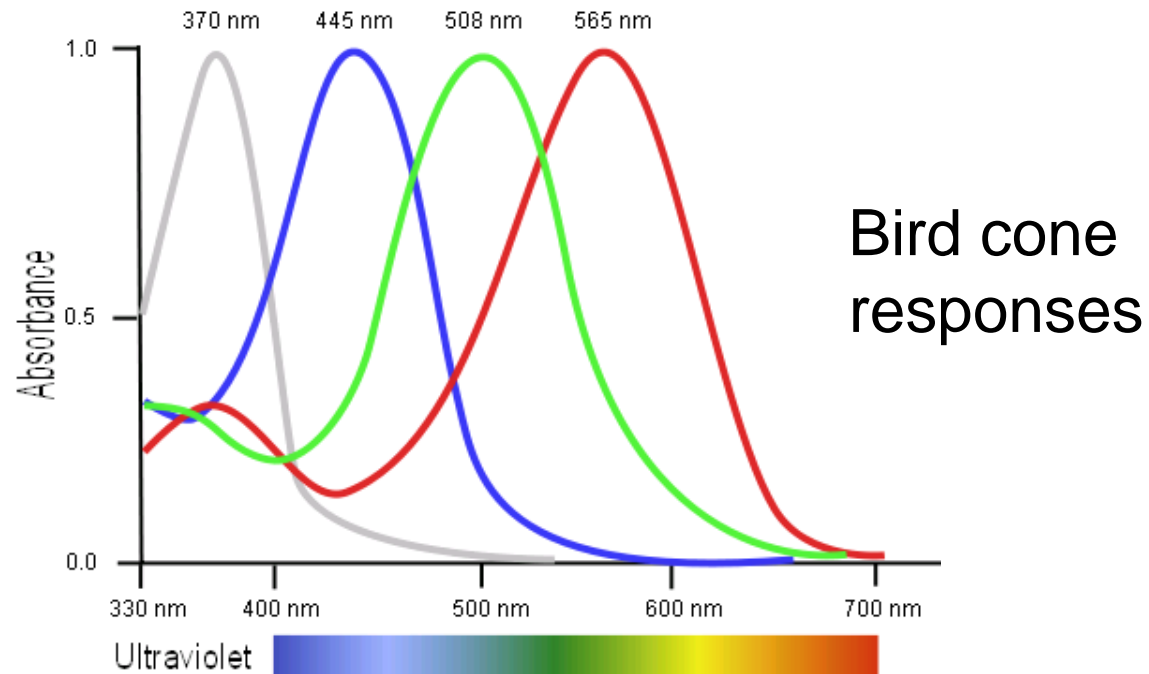
# Physiology of Color Vision

Three kinds of cones:



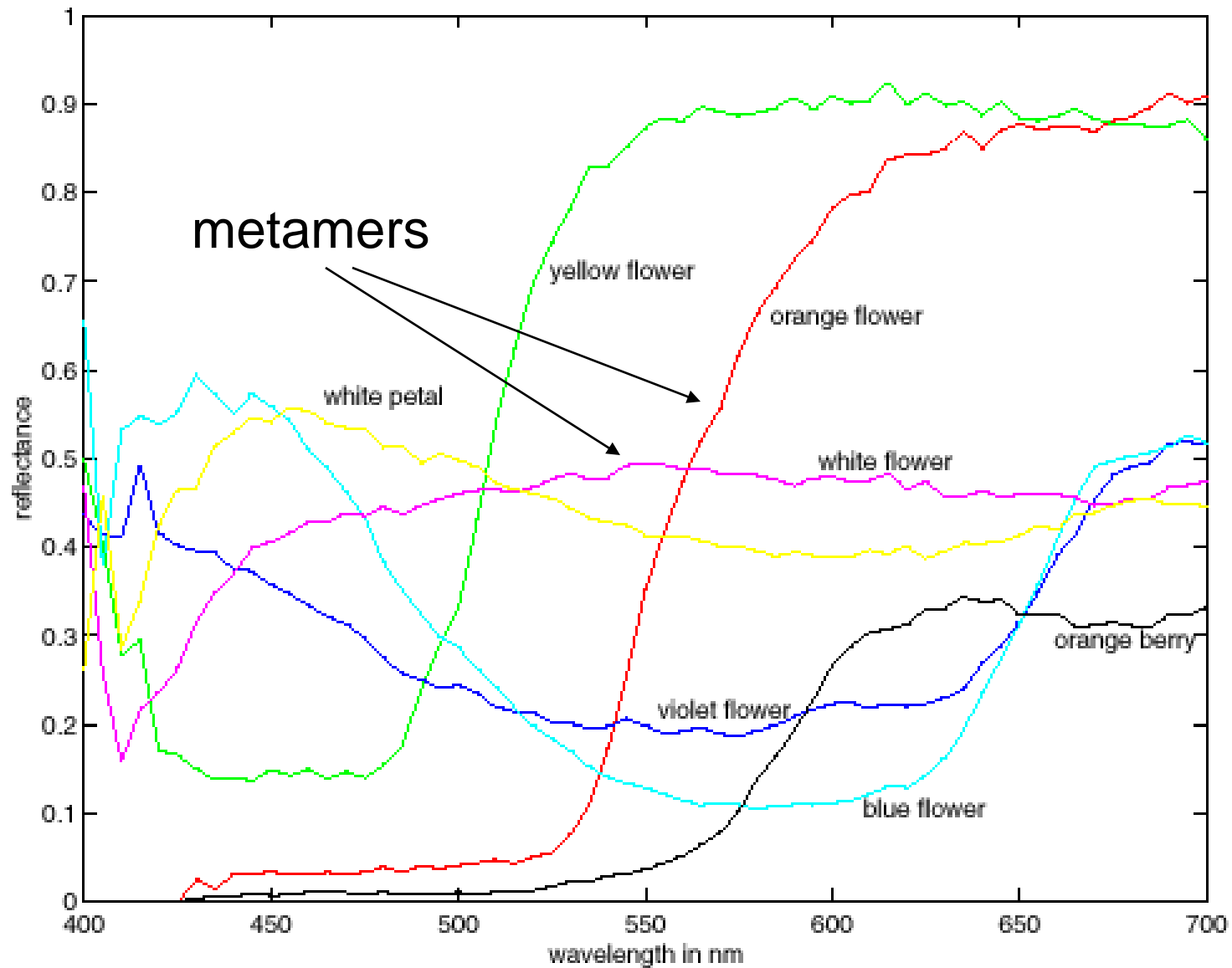
- Why are M and L cones so close?
- Why are there 3?

# Tetrachromatism

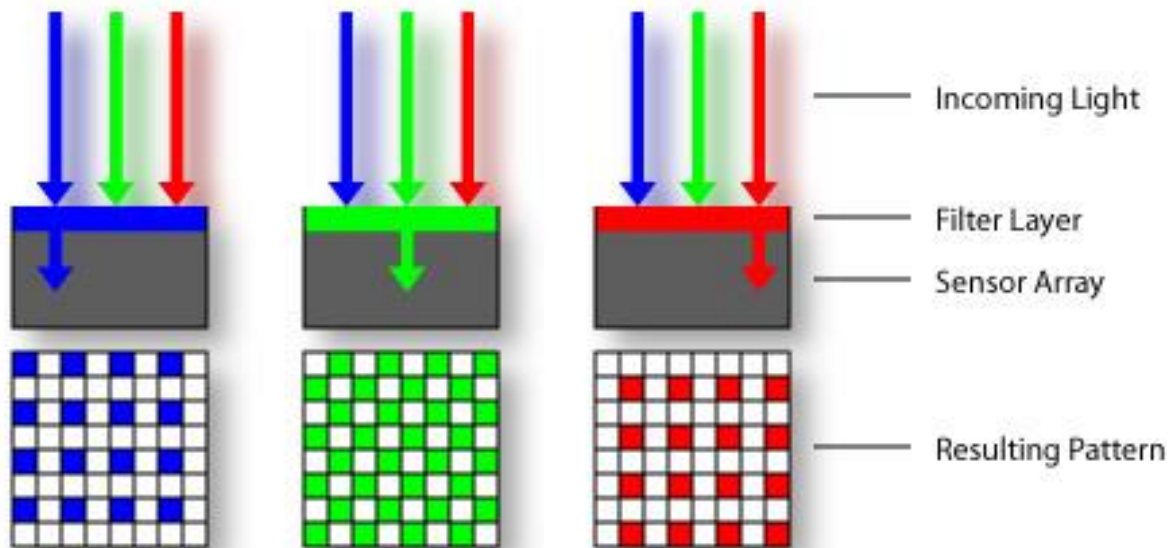
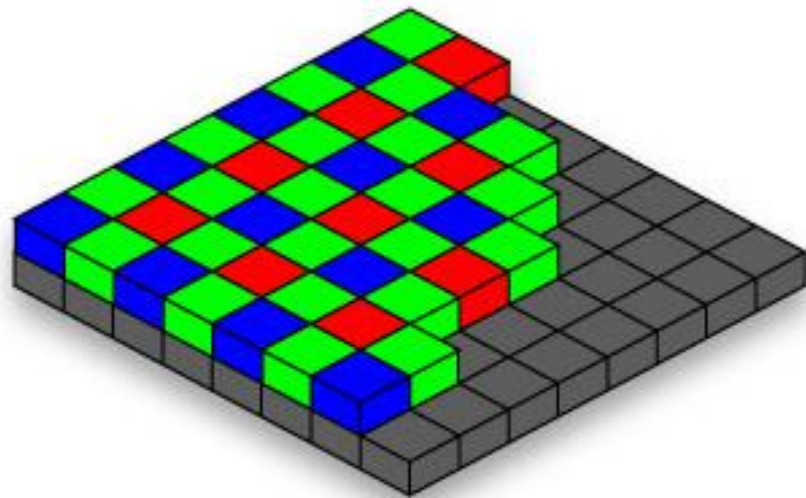


- Most birds, and many other animals, have cones for ultraviolet light.
- Some humans, mostly female, seem to have slight tetrachromatism.

# More Spectra

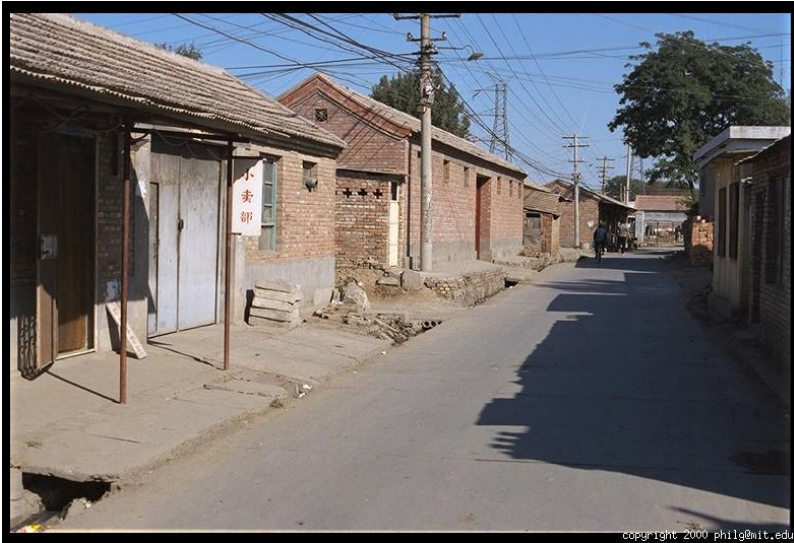


# Practical Color Sensing: Bayer Grid



- Estimate RGB at 'G' cells from neighboring values

# Color Image

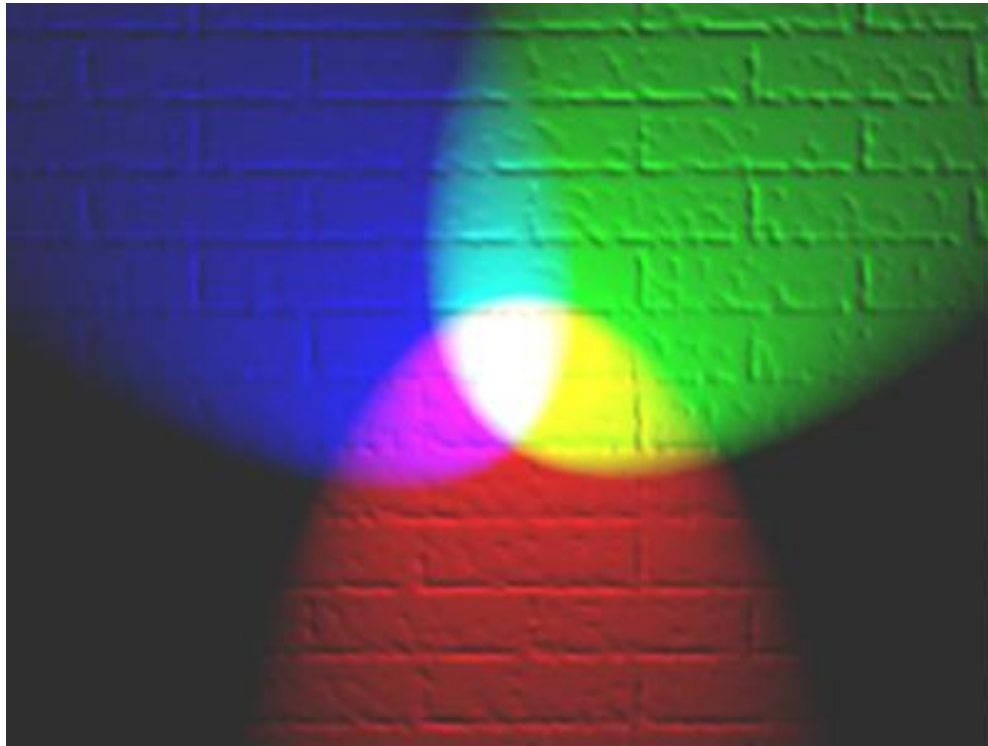






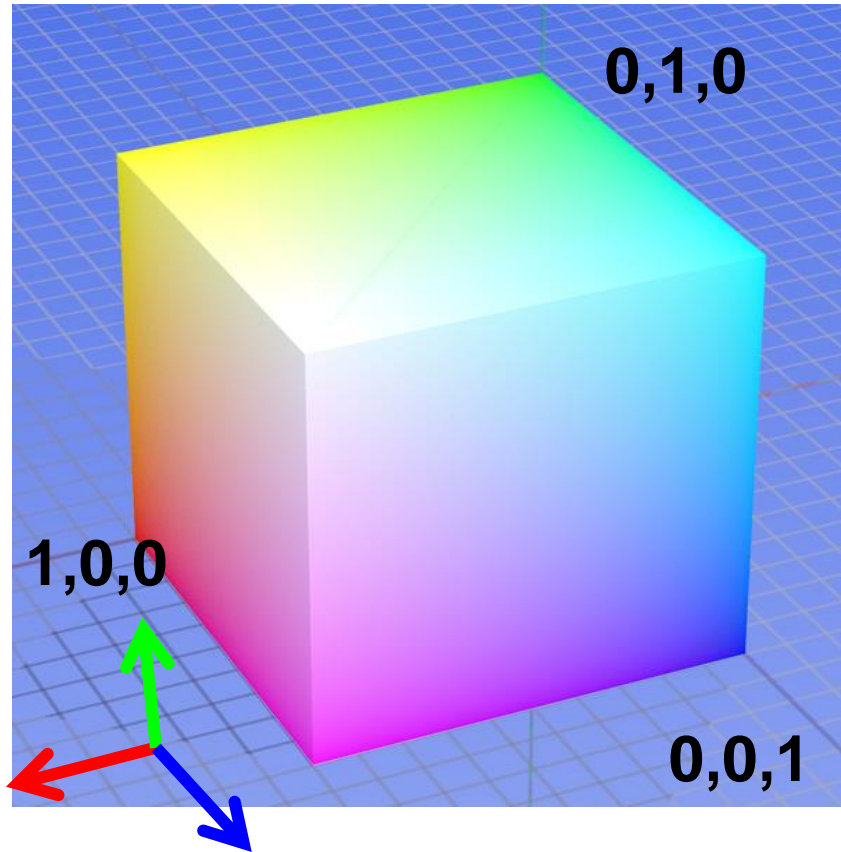
# Color spaces

- How can we represent color?



# Color spaces: RGB

Default color space



**R**  
(G=0,B=0)



**G**  
(R=0,B=0)



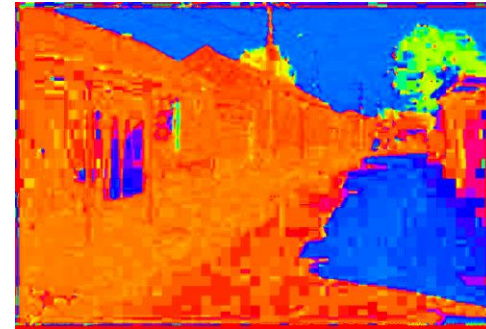
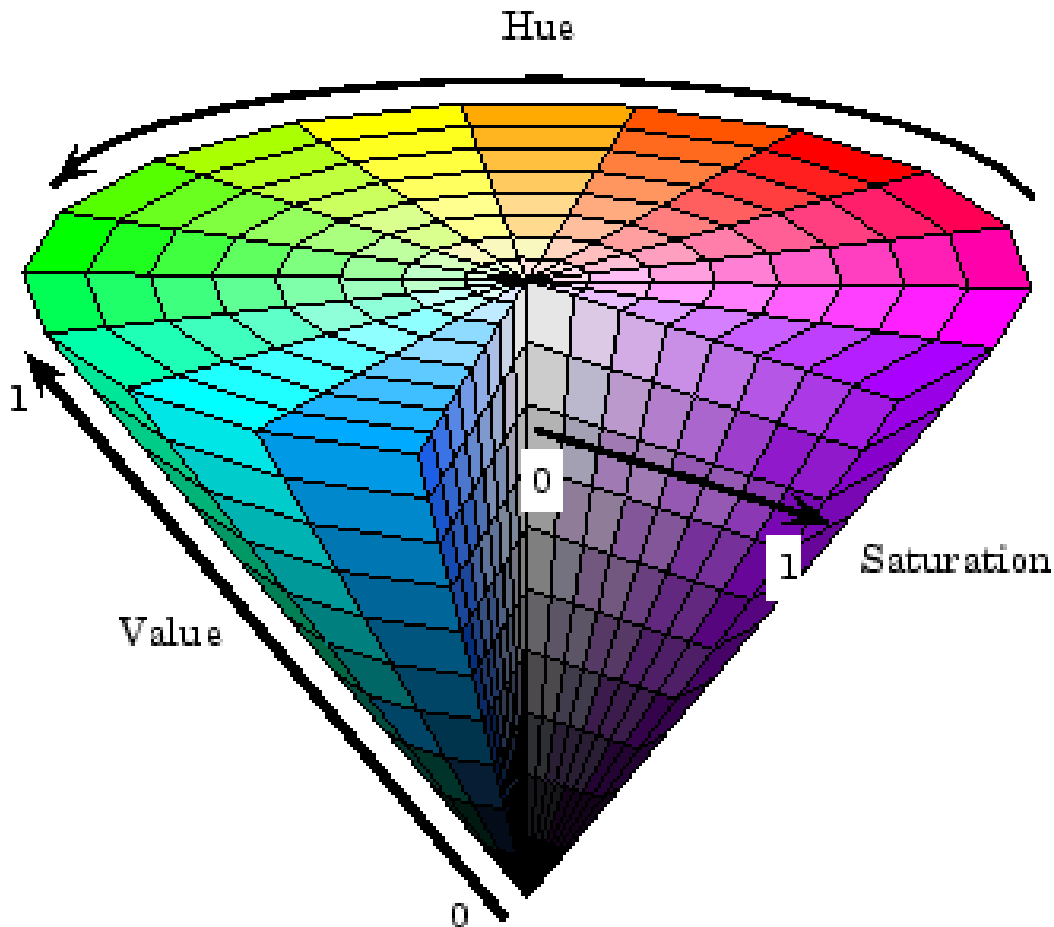
**B**  
(R=0,G=0)

## Some drawbacks

- Strongly correlated channels
- Non-perceptual

# Color spaces: HSV

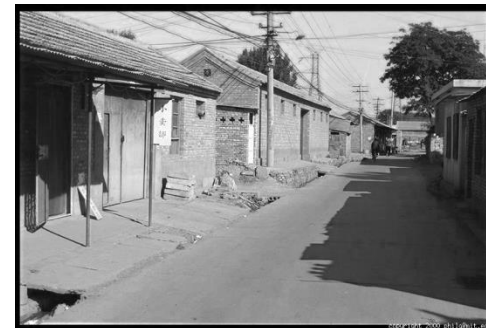
Intuitive color space



**H**  
(S=1,V=1)



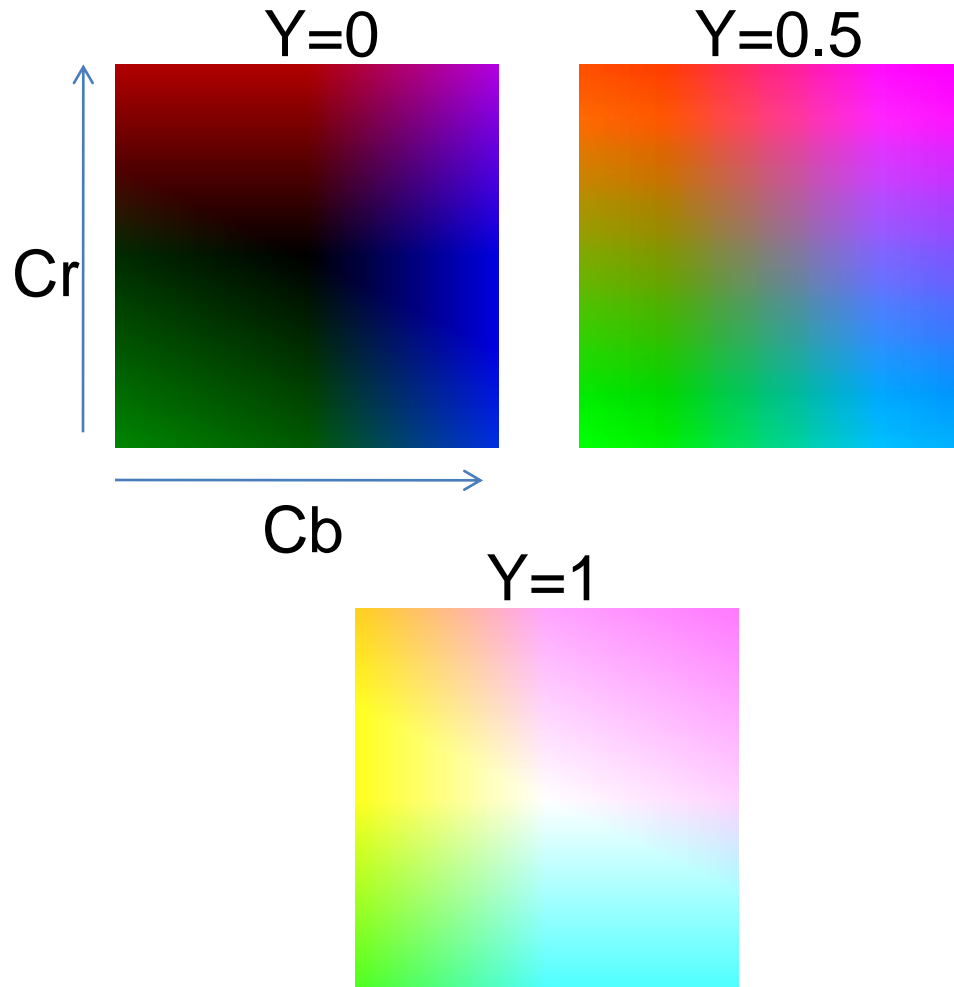
**S**  
(H=1,V=1)



**V**  
(H=1,S=0)

# Color spaces: YCbCr

Fast to compute, good for compression, used by TV



**Y**  
(Cb=0.5,Cr=0.5)



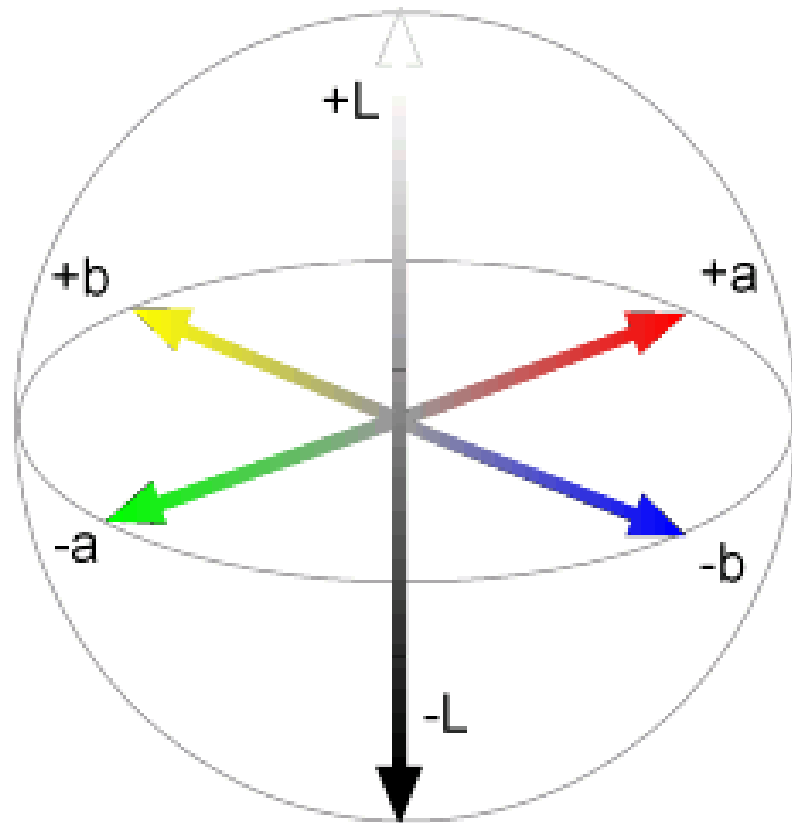
**Cb**  
(Y=0.5,Cr=0.5)



**Cr**  
(Y=0.5,Cb=0.5)

# Color spaces: L\*a\*b\*

“Perceptually uniform”\* color space



**L**  
(a=0,b=0)



**a**  
(L=65,b=0)



**b**  
(L=65,a=0)

If you had to choose, would you rather go without luminance or chrominance?

If you had to choose, would you rather go  
without **luminance** or chrominance?



# Most information in intensity



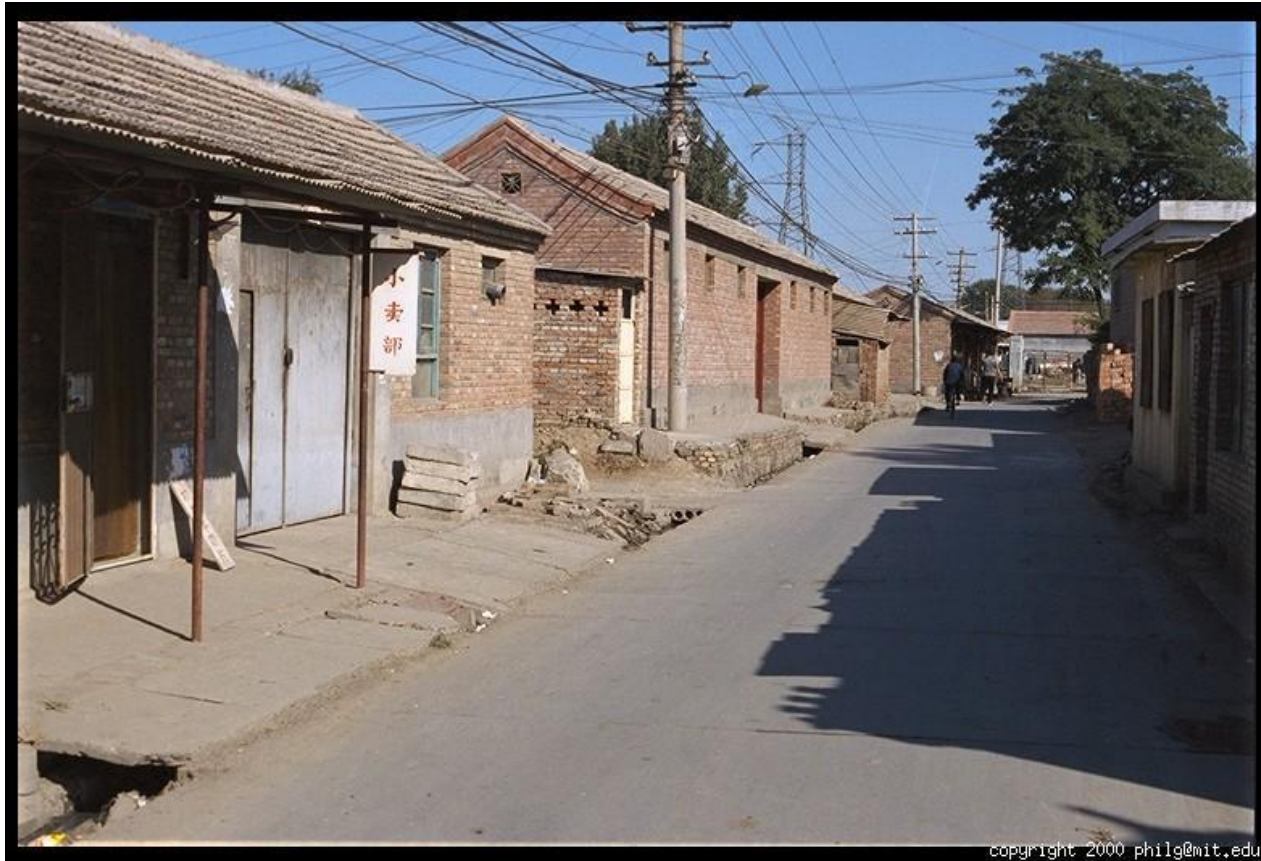
Only color shown – constant intensity

# Most information in intensity



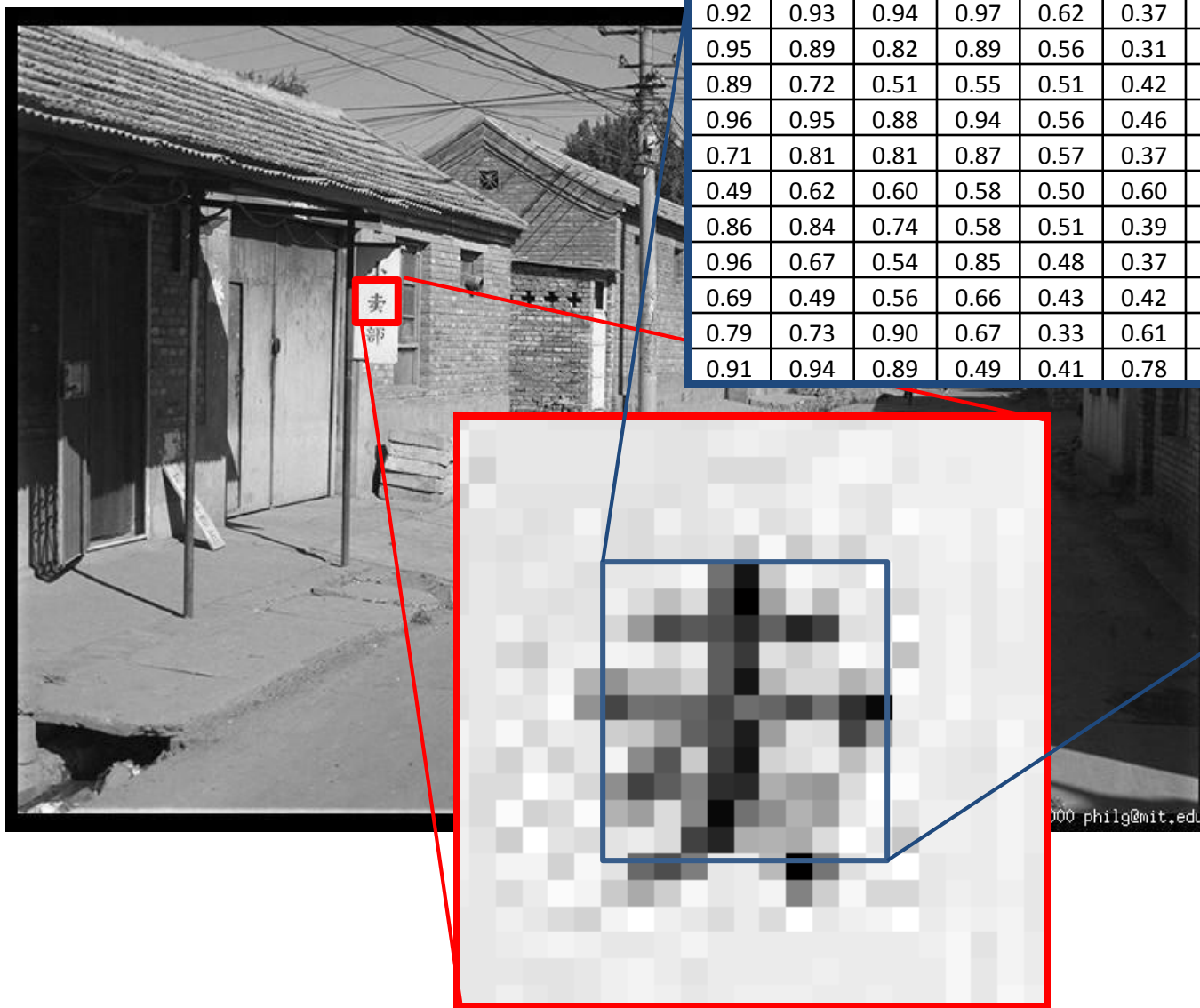
Only intensity shown – constant color

# Most information in intensity



Original image

# Back to grayscale intensity



0.92	0.93	0.94	0.97	0.62	0.37	0.85	0.97	0.93	0.92	0.99
0.95	0.89	0.82	0.89	0.56	0.31	0.75	0.92	0.81	0.95	0.91
0.89	0.72	0.51	0.55	0.51	0.42	0.57	0.41	0.49	0.91	0.92
0.96	0.95	0.88	0.94	0.56	0.46	0.91	0.87	0.90	0.97	0.95
0.71	0.81	0.81	0.87	0.57	0.37	0.80	0.88	0.89	0.79	0.85
0.49	0.62	0.60	0.58	0.50	0.60	0.58	0.50	0.61	0.45	0.33
0.86	0.84	0.74	0.58	0.51	0.39	0.73	0.92	0.91	0.49	0.74
0.96	0.67	0.54	0.85	0.48	0.37	0.88	0.90	0.94	0.82	0.93
0.69	0.49	0.56	0.66	0.43	0.42	0.77	0.73	0.71	0.90	0.99
0.79	0.73	0.90	0.67	0.33	0.61	0.69	0.79	0.73	0.93	0.97
0.91	0.94	0.89	0.49	0.41	0.78	0.78	0.77	0.89	0.99	0.93

000 philg@mit.edu

# Next week

- Convolution, Filtering, Image Pyramids, Frequencies, Project 1