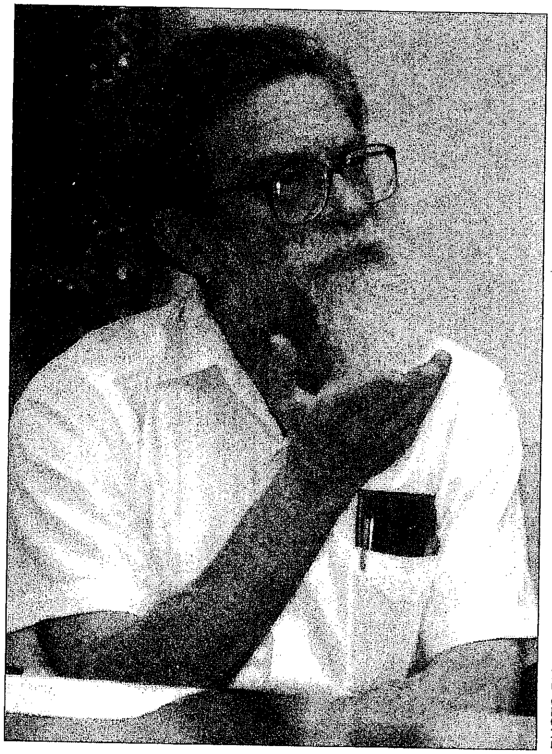

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HOWARD STEIN

Howard Stein was born on January 21, 1929 in New York City. He received a B.A. from Columbia College in 1947, a Ph.D. in Philosophy from the University of Chicago in 1958, and an M.S. in Mathematics from the University of Michigan in 1959. After teaching in the Natural Sciences Collegiate Division at the University of Chicago (1949–1958) and the Mathematics Department at Brandeis University (1959–1962), he spent five years in private industry, in the Systems Analysis and Computer Products divisions of Honeywell. A Professorship of Philosophy at Case Western Reserve University (1967–1973) brought him back to the academic world, after which he moved to Columbia University (1973–1980), and then returned to the University of Chicago, this time in the Department of Philosophy and the Committee on the Conceptual Foundations of Science. He retired in 2000. During his long and distinguished career, Howard Stein received fellowships from the National Science Foundation and the Guggenheim Foundation, and was elected to the American Academy of Arts and Sciences. A complete list of his many influential papers in the history and philosophy of science and mathematics is included in this volume.

Reading Natural Philosophy

Essays in the History and Philosophy of Science and Mathematics

EDITED BY
David B. Malament

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To Howard Stein
on the Occasion of His 70th Birthday

[6]

Maxwell and "the Method of Physical Analogy": Model-based Reasoning, Generic Abstraction, and Conceptual Change

NANCY J. NERSESSIAN

1. Introduction

As my teacher at Case Western Reserve University, Howard Stein gave me a piece of advice—one of many for which I will always be grateful—that I recall went something like this: if you want to understand the nature of science, read the scientists not just what philosophers have to say about science. To a young student who had just switched from physics to philosophy of science, this was a revelation. One could actually read the words of Newton, Maxwell, and Einstein and not just science textbooks! His advice was all the more significant because I found what Carnap, Nagel, Hempel, and the others we were assigned to read in classes had to say about science did not address the problems that had led me from physics to philosophy. Reading the scientists was the start of developing my own analysis of these problems, which is a task that still occupies me all these many years later. I started with Einstein, since we were working on the general theory of relativity, but interest in the origins of the field concept led me back to Faraday and Maxwell. In this paper I want to return to Maxwell for several reasons, not the least of which is Howard's often expressed admiration for his acuity and insight into scientific method and his encouragement to explore the nature of Maxwell's "method of physical analogy" and its role in his discovery of the electromagnetic field equations.

When I first read Maxwell I found it surprising how many commentators on his work failed to take seriously what seemed to me to be the generative role of the analogy developed in the 1861–62 paper (Maxwell 1861–62).[1] Maxwell's own comments on analogy as a method of discovery—in letters, publications, and lectures—were largely dismissed with his analogies characterized as at best "merely suggestive" (Heimann 1970), offering "slight" value as a heuristic guide (Chalmers 1973, 137),[2] and at worst as dishonest post-hoc fabrications (Duhem 1914, 98). In this last case, Duhem claimed that Maxwell had cooked up the analogy after the fact and even falsified an equation (see also Duhem 1902) while "the results he obtained were known to him by other means" (1914, 98).

The "known by other means" claim is one I frequently encounter from philosophers in response to presentations of my interpretation of Maxwell. When pressed as to what other means, the response is usually that the equations were derived "by induction from the experiments," with the nature of the inductive process left mysterious. Certainly experimental results played a key role in Maxwell's analysis, but the process of deriving the equations was not what philosophers usually call "induction." Another frequent response is the "symmetry argument," presented in figure 1. For example, Steiner states "Once the phenomenological laws of Faraday, Coulomb, and Ampère had been given differential form, Maxwell noted that they contradicted the conservation of electrical charge. . . . Yet, by tinkering with Ampère's law adding to it the 'displacement current,' Maxwell succeeded in getting the laws actually to imply charge conservation" (Steiner 1989, 458). The "tinkering" is usually interpreted as Maxwell's having noticed that equation for the magnetic field did not include a contribution from the electric field. This account is often given by physicists as well (see, e.g., Jackson 1962, 177).[3] But what is left out of the symmetry account is the central problem of how Maxwell gave the phenomenological laws differential form, which leads to a study of "the method of physical analogy" and ultimately to a quite different interpretation of the "tinkering" process. Another common move made by philosophers is to claim that since, in fact, the aether can be eliminated from Maxwell's laws,

Coulomb Law: $\text{div}\,\mathbf{D} = 4\pi\rho$

Ampère Law: $\mathbf{curl}\,\mathbf{H} = 4\pi\mathbf{J}$

Faraday Law: $\mathbf{curl}\,\mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$

Absence of free magnetic poles: $\text{div}\,\mathbf{B} = 0$

Conservation of charge requires

Equation of continuity: $\text{div}\,\mathbf{J} + \dfrac{\partial \rho}{\partial t} = 0$

Considerations of consistency and symmetry lead to alteration of Faraday Law

$$\mathbf{curl}\,\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

FIGURE 1
The Symmetry Account

its role in the development of the theory is insignificant, thus reducing the generative role of physical analogy. For example, Kitcher does not discuss Maxwell's analogies but indirectly eliminates them in claiming that the aether was not a "working posit" involved in problem solving, explanation, and prediction, but a "presuppositional posit," which was thought to be required to make the claims of the theory true (Kitcher 1993, 174). It can simply be removed from Maxwell's theory. Yet without the working posit of the existence of the aether as a continuum mechanical medium Maxwell could not have derived the equations at all.[4]

Failing to follow the well-known dictum of Einstein,[5] Maxwell's deeds are also dismissed, for in dismissing the role of analogy, we are left with no evidence in Maxwell's papers, drafts, and correspondence as to what "other means" he employed. I contend that in examining Maxwell's "deeds" we see they exhibit a remarkable harmony between word and deed. And, as with Einstein and Newton, Maxwell is one scientist from whom we can learn a great deal about the nature of scientific practice by listening to his words.

Finally, in coming to grips with how Maxwell employed models in constructing the electromagnetic field representation we gain significant insight into a central problem of creativity: Given that we must start from existing representations, how is it possible that we ever create anything genuinely novel? In this case, the problem is how, starting from Newtonian systems, did Maxwell derive the laws of a non-Newtonian dynamical system? What is needed in order to resolve both the general and the specific problem is an explanatory account of how analogy, and more generally what I call "model-based reasoning," functions to generate new representations—representations that ultimately come to transcend the specific models that were employed in their generation.

2. Cognitive-Historical Analysis

My research into conceptual change in several episodes in physics led to my characterizing specific concept formation practices as forms of "model-based reasoning" (Nersessian 1984a; Nersessian 1984b; Nersessian 1988; Nersessian 1992a; Nersessian 1992b; Nersessian 1995; Nersessian 1999). Here I will present just the main features of the case that model-based reasoning is generative of conceptual change in science, by focusing on three central forms: analogical modeling, visual modeling, and thought experimenting (simulative modeling). I will then provide an interpretation of the role of physical analogy in the development of Maxwell's theory. I present a unified analysis of them because they are often employed together in reasoning episodes. For example, as will be discussed in section 4, the idle

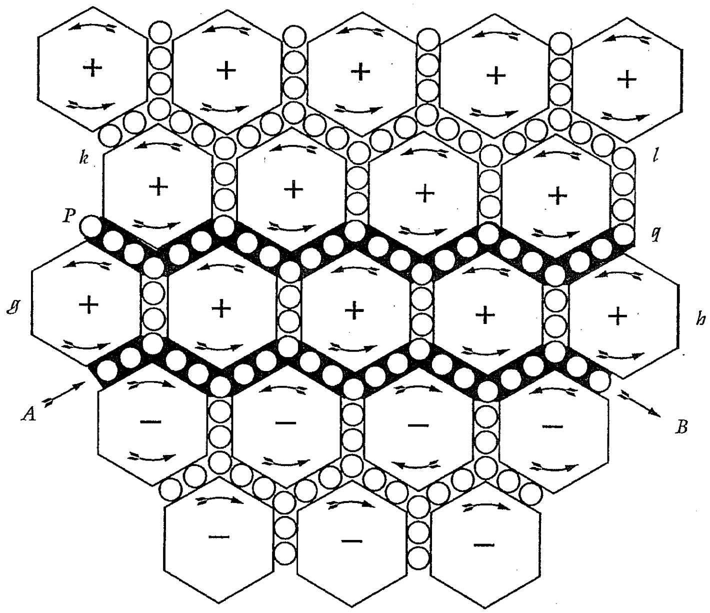

FIGURE 2
Maxwell's drawing of the vortex-idle wheel medium (Maxwell 1890, vol. 1, plate 7)

wheel–vortex model employed by Maxwell in his derivation of the electromagnetic field equations and illustrated by him in figure 2 exemplifies why a unified account is needed. On my interpretation this is a *visual* representation of an *analogical* model that is accompanied with instructions for *simulating* it correctly in thought: "Let the current from left to right commence in AB. The row of vortices *gh* above AB will be set in motion in the opposite direction to a watch. . . . We shall suppose the row of vortices *kl* still at rest, then the layer of particles between these rows will be acted on by the row *gh* on their lower sides and will be at rest above. If they are free to move, they will rotate in the negative direction, and will at the same time move from right to left, or in the opposite direction from the current, and so form an *induced* electric current" (Maxwell 1890b, vol. 1, 477, italics in original).

My analysis draws from practices employed in physics, but investigations of other sciences by philosophers, historians, and cognitive scientists establish that these practices are employed across the sciences (see, e.g., Darden 1980; Darden 1991; Gentner et al. 1997; Giere 1994; Giere

1988; Giere 1992; Gooding 1990; Griesemer 1991a; Griesemer 1991b; Griesemer and Wimsatt 1989; Holmes 1981; Holmes 1985; Latour 1986; Latour 1987; Lynch and Woolgar 1990; Rudwick 1976; Shelley 1996; Thagard 1991; Trumpler 1997; Tweney 1987; Tweney 1992). Further, although these practices are ubiquitous and significant they are, of course, not exhaustive of the practices that generate new conceptual structures.

The account of model-based reasoning developed here stems from an interdisciplinary method that I have called "cognitive-historical" analysis. The objective of that method is to create accounts of the nature and development of science that are informed by studies of historical and contemporary scientific practices and cognitive science investigations of aspects of human cognition pertinent to these practices. When used in analyzing conceptual change, the "historical" dimension of the method is required to uncover the practices scientists employ and to examine these over extended periods of time and as embedded within local communities and wider cultural contexts. The "cognitive" dimension assumes the need to factor into the analysis how human cognitive capacities and limitations could produce and constrain the practices of scientists. Neither the practices nor the cognitive factors can be known *a priori*, empirical research is needed in both cases. Thus cognitive-historical analyses make use of the customary range of historical records for gaining access to practices and draw on and conduct cognitive science investigations into how humans reason, represent, and learn.

In contemporary philosophical parlance, the cognitive-historical method is a "naturalistic" method of analysis. Underlying the method is a "continuum hypothesis": the cognitive practices of scientists are extensions of the kinds of practices humans employ in coping with their physical and social environments and in problem solving of the more ordinary kind. Scientists extend and refine basic cognitive strategies in explicit and critically reflective attempts to devise methods for understanding nature. From this perspective, scientific cognition is shaped by the evolutionary history of the human species, by the developmental processes of the human child, and by the cultural development of human societies. Biological and sociocultural factors co-determine human cognitive development and the various expressions of that development, such as science. The representational and reasoning practices of scientists are analyzed as bearing both the imprint of human cognitive development and the imprint of the sociocultural histories of the communities, internal and external to science, in which it has developed and has come to be practiced. Placing the scientific practices within the broader framework of human cognitive abilities and limitations provides a basis from which to develop an epistemological account that moves beyond the specific case study to more general conclusions about the nature and function of the scientific practices. Such placement aids in establishing that the fragments of scientific research and discovery investigated are more widely representative of scientific practices and thus acts to support drawing more general conclusions from specific aspects of case studies. That scientific cognition is contextual does not preclude developing generalizations from specific cases. As the cognitive anthropologist Edwin Hutchins has argued, "There are powerful regularities to be described at the level of analysis that transcends the details of the specific domain [case]. It is not possible to discover these regularities without understanding the details of the domain [case], but the regularities are not about the domain [case] specific details, they are about the nature of cognition in human activity" (Woods 1997, 15; see also Hutchins 1995).

That there is a continuum, however, does not rule out the possibility that there are salient differences between the scientific and ordinary cognition. Since most of the research in cognitive science has been conducted on ordinary cognition, the cognitive-historical method has to be reflexive in application. Cognitive theories and methods are drawn upon insofar as they help interpret the historical and contemporary practices, while at the same time cognitive theories are evaluated as to the extent to which they can be applied to scientific practices. The assumptions, methods, and results from both sides are subjected to critical evaluation, with corrective insights moving in both directions. One major impact the issues posed by this research into model-based reasoning can have on the field of cognitive science is to push the field toward integration and unification of phenomena largely treated in isolation. Accounting for scientific cognitive practices requires an analysis that integrates research in what are customarily separate research areas in cognitive science, such as analogy, imagery, conceptual change, categorization, problem solving, and decision making.

Although it is not possible to go into the details or give extensive references within the confines of this paper, my account of model-based reasoning derives from extensive historical and cognitive research. The historical research includes my own studies, mainly of but not limited to, nineteenth- and early-twentieth-century field physicists and pertinent research by historians and philosophers of science into other scientific domains. The cognitive research includes the literatures on analogy, mental modeling, mental simulation, mental imagery, imagistic and diagrammatic reasoning, expert/novice problem solving, and conceptual change. Further, my AI collaborators and I have developed and implemented a computational model, the ToRQUE system, that is a model-based reasoner derived from experimental problem-solving protocols that exhibit the kinds of abstraction and constraint satisfaction processes discussed in the next section (Griffith, Nersessian, and Goel 1996; Griffith 1999; Griffith, Nersessian, and Goel 2000).

3. Model-based Reasoning

Within philosophy the identification of reasoning with argument and logic is deeply ingrained. Traditional accounts of scientific reasoning have restricted the notion of reasoning primarily to deductive and inductive arguments. Embracing modeling practices as "methods" of conceptual change in science requires expanding philosophical notions of scientific reasoning to encompass forms of creative reasoning, many of which cannot be reduced to an algorithm in application, are not always productive of solutions, and where good usage can lead to incorrect solutions. Some accounts have proposed abduction as a form of creative reasoning, but the nature of the processes underlying abductive inference and hypothesis generation are left largely unspecified. Examining the modeling practices of scientists as forms of reasoning generative of conceptual change provides a means of specifying the nature of some forms of abductive inference.

The notion of model-based reasoning opens a set of issues about the role of representational format (internal and external) in the reasoning. Different kinds of representations such as linguistic, formulaic, imagistic, and analog/iconic enable different kinds of operations. Operations on linguistic and formulaic representations, for example, include the familiar operations of logic and mathematics. These representations are interpreted as referring to physical objects, structures, processes, or events descriptively. Customarily, the relationship between this kind of representation and what it refers to is "truth" and thus the representation is evaluated as being true or false. Operations on such expressions are rule based and truth preserving if the symbols are interpreted in a consistent manner and the properties they refer to are stable in that environment. Additional operations can be defined in limited domains provided they are consistent with the constraints that hold in that domain. On the other hand, analog models, diagrams, and imagistic representations are interpreted as representing demonstratively. The relationship between this kind of representation, which I will call "iconic," and what it represents is "similarity" or "goodness of fit." Iconic representations are similar in degrees and aspects to what they represent, and are thus evaluated as accurate or inaccurate. Operations on iconic representations involve transformations of the representations that change their properties and relations in ways consistent with the constraints of the domain. Significantly, transformational constraints represented in iconic representations can be implicit, for example, a person can do simple reasoning about what happens when a rod is bent without having an explicit rule such as "given the same force a longer rod will bend farther." The form of representation is such as to enable simulations in which the model behaves in accord with constraints that need not be stated explicitly during this process.

In cognitive psychology there is an ongoing controversy about the nature of human reasoning that parallels the issues raised about reasoning in philosophy. This is not surprising since many philosophers who adhere to the traditional view have played a significant role in shaping this debate. On the traditional psychological view, the mental operations underlying reasoning consist of applying a mental logic to proposition-like representations. For some time critics of this view have contended that a purely syntactical account of reasoning cannot account for significant effects of semantic information exhibited in experimental studies of reasoning (see, e.g., Johnson-Laird 1982; Johnson-Laird 1983; Mani and Johnson-Laird 1982; McNamara and Sternberg 1983; Oakhill and Garnham 1996; Perrig and Kintsch 1985; Wason 1960; Wason 1968). Instead, they propose adopting a hypothesis, first proposed by Craik (1943), that in many instances people reason by carrying out thought experiments on internal models. In its contemporary instantiation, the "mental modeling" hypothesis has been investigated for numerous domains, including: reasoning about causality in physical systems (see, e.g., DeKleer and Brown 1983); the role of representations of domain knowledge in reasoning (see, e.g., Gentner and Gentner 1983); logical reasoning (see, e.g., Johnson-Laird 1983); narrative comprehension (see, e.g., Perrig 1985); and induction (see, e.g., Holland et al. 1986). Additionally, there is considerable experimental protocol evidence collected by cognitive psychologists to support mental modeling as a fundamental form of problem solving employed by contemporary scientists (see, e.g., Chi, Feltovich, and Glaser 1981; Clement 1989; Dunbar 1995; Dunbar 1999; Griffith, Nersessian, and Goel 1996). These studies of reasoning processes provide further support for interpreting the modeling practices exhibited in the historical records of conceptual change as indicating that mental modeling has played a role in the past episodes.

Though "the" mental modeling hypothesis is far from unitary and in need of critical examination, my analysis of model-based reasoning has required adopting only a "minimalist" hypothesis: that in certain problem-solving tasks humans reason by constructing an internal iconic model of the situations, events, and processes that in dynamic cases can be manipulated through simulation. In constructing a model, information in various formats, including linguistic, formulaic, and imagistic, where the latter is taken here to include various perceptual modalities, can be used. In mundane cases the reasoning performed via mental modeling is usually successful because the models and manipulative processes embody largely correct constraints governing everyday real-world events. Think, for example, of how people often reason about how to get an awkward piece of furniture through a door. The reasoner usually figures out how to get a large chair through the door by mentally simulating turning over a geometrical

structure approximating the configuration of the chair through various rotations. The task is made easier when the physical chair is in front of the reasoner acting to support the structure in imagination. In the case of science where the situations are more removed from human sensory experience and the assumptions more imbued with theory, there is less assurance that a simulative reasoning process, even if carried out correctly, will yield success. Clearly scientists create erroneous models—revision and evaluation are crucial components of model-based reasoning. In the evaluation process, a major criterion is goodness of fit to the constraints of the target phenomena, but success can also include such factors as enabling the generation of a viable mathematical representation that can push the science along while other details of representing the phenomena are still to be worked out, as Newton did with the concept of gravitation and Maxwell did with the concept of electromagnetic field.

To explain how model-based reasoning could be generative of conceptual change in science requires a fundamental revision of the understandings of concepts, conceptual structures, conceptual change, and reasoning customarily employed explicitly in philosophy and at least tacitly in the other science studies fields. It is not possible to provide all the necessary details and arguments in the confines of this paper. Only an outline of my account will be developed here. Hopefully it will be sufficient to enable the reader to understand the interpretation provided of Maxwell's "method of physical analogy." A basic ingredient of the revision is to view the representation of a concept as providing sets of constraints for generating members of classes of models. Concept formation and change is a process of generating new, and modifying existing, constraints. This is accomplished through iteratively constructing models embodying specific constraints until a model of the *same type* with respect to the salient constraints of the phenomena under investigation, the "target" phenomena, is achieved. My hypothesis is that the prevalence of analogies, visual representations, and thought experiments in periods of radical conceptual change indicates that model-based reasoning is a highly effective means of examining, revising, and abstracting constraints of existing representational systems and, in light of constraints provided by the target problem, an effective means of generating new sets of constraints that the new representational structures come to embody. I will now provide brief encapsulations of how.

To engage in analogical modeling one calls on knowledge of the generative principles and constraints for models in a known "source" domain. These constraints and principles can be represented mentally and externally in different informational formats and knowledge structures that act as explicit or tacit assumptions employed in constructing and transforming models during problem solving. Inter- or intra-domain models can be retrieved directly from the source domain and applied with suitable adaptation, but often, and especially in cases of conceptual change, no direct analogy exists and construction of an initial model itself is required. In these cases the source domain(s) provides constraints that are used together with those provided by the target problem to create the initial as well as subsequent models (Nersessian 1992a; Nersessian 1999; Nersessian 2000). Evaluation of the analogical modeling process is in terms of how well the salient constraints of a model fit the salient constraints of a target problem, with key differences playing a significant role in further model generation (Griffith 1999; Griffith, Nersessian, and Goel 1996). There is an extensive cognitive science literature on analogy with much empirical evidence that substantiates the claim that it is generative in instances of conceptual change. This literature provides theories of the processes of retrieval, mapping, transfer, elaboration, and learning employed in analogy and the syntactic, semantic, and pragmatic constraints operating on these processes (see, e.g., Gentner 1983; Gentner 1989; Gentner et al. 1997; Gick and Holyoak 1980; Gick and Holyoak 1983; Holyoak and Thagard 1989; Holyoak and Thagard 1996; Thagard et al. 1990). Although no current cognitive theory is able to handle the complexity of the Maxwell case, the literature does agree with my analysis in that analogies are not "merely" guides to reasoning but form the creative heart of the reasoning processes in which they are employed. There is also widespread agreement on criteria for good analogical reasoning, drawn from psychological studies of productive and nonproductive use of analogy and formulated by Gentner (Gentner 1983; Gentner 1989): 1. "structural focus": preserves relational systems, 2. "structural consistency": isomorphic mapping of objects and relations, and 3. "systematicity": maps systems of interconnected relationships, especially causal and mathematical relationships.

Constraints in both the target and source domains are domain-specific and need to be understood in the reasoning process at a sufficient level of abstraction for retrieval, transfer, and integration to occur. I call this level of abstraction "generic." That is, the various representations employed have to function with some of their features considered as unspecified. In model-based reasoning processes, a central objective is to create a model that is of the *same kind* with respect to salient dimensions of the target phenomena one is trying to represent. Thus, although an instance of a model is specific, inferences made with it in a reasoning process are generic. In viewing a model generically, one takes it as representing features, such as structure and behaviors, common to members of a class of phenomena. The relation between the generic model and the specific instantiation is similar to the type-token distinction used in logic. Generality in representation is achieved by interpreting the components of the representation as referring to object, property, relation, or behavior types rather than tokens of these. One cannot draw or imagine a "triangle in general" but only some specific instance of a triangle. However, in considering what it has in common with all triangles, humans have the ability to view the specific triangle as lacking specificity in its angles and sides. In considering the behavior of a physical system such as a spring, again one often draws or imagines a specific representation. However, to consider what it has in common with all springs, one needs to reason as though it lacked specificity in length and width and number of coils; to consider what it has in common with all simple harmonic oscillators, one needs to reason as though it lacked specificity in structure and aspects of behavior. That is, the reasoning context demands that the interpretation of the specific spring be generic.

The kind of creative reasoning employed in conceptual innovation involves not only applying generic abstractions but creating and transforming them during the reasoning process. The process of abstracting to the generic level is a significant reasoning process in analogical modeling in conceptual change which often requires recognition of potential similarities across disparate domains, and abstraction and integration of information from these. There are many significant examples of generic abstraction in conceptual change in science. In the domain of classical mechanics, for example, Newton can be interpreted as employing generic abstraction in reasoning about the commonalities among the motions of planets and of projectiles, which enabled his formulating a unified mathematical representation of their motions. The models he employed, understood generically, represent what is common among the members of specific classes of physical systems viewed with respect to a problem context. Newton's inverse-square law of gravitation abstracts what a projectile and a planet have in common in the context of determining motion, for example, that within the context of determining motion, planets and projectiles can both be represented as point masses. After Newton, the inverse-square-law model of gravitational force served as a generic model of action-at-at-distance forces for those who tried to bring all forces into the scope of Newtonian mechanics.

A variety of perceptual resources can be employed in modeling. Here I focus on the use of the visual modality since it figures prominently in cases of conceptual change across the sciences. A possible reason why is that employing the visual modality might enable the reasoner to bypass specific constraints inherent in current linguistic and formulaic representations of conceptual structures. There is a vast cognitive science literature on mental imagery that provides evidence that humans can perform simulative imaginative combinations and transformations that mimic perceptual spatial transformation (Kosslyn 1980; Shepard and Cooper 1982). These simulations are hypothesized to take place using internalized constraints assimilated during perception. Other research indicates that people use various kinds of knowledge of physical situations in imaginary simulations. For example, when objects are imagined as separated by a wall, the spatial transformations exhibit latency time consistent with having simulated moving around the wall rather than through it. There are significant differences between spatial transformations and transformations requiring causal and other knowledge contained in scientific theories. Although the research on imagery in problem solving is scant, recently cognitive scientists have undertaken several investigations examining the role of causal knowledge in mental simulation involving imagery, for example, experiments with problems employing gear rotation provide evidence of knowledge of causal constraints being utilized in imaginative reasoning (Hegarty 1992; Hegarty and Just 1994; Hegarty and Sims 1994; Schwartz and Black 1996).

In model-based reasoning, that the internal representations are iconic does not mean that they need to be picture like in format at all, but can be highly schematic. Thus this modality could be operative even in the reasoning of scientists, such as Bohr, who claim not to experience imagery, in other words pictures, in reasoning. The conflation of mental imagery with pictures-in-the-head stems from the fact that we presently lack an adequate means for expressing the notion of a representational format that is neither picturelike nor linguistic. External visual representations provide support for the processes of constructing and reasoning with a mental model. They aid significantly in organizing cognitive activity during reasoning, such as fixing attention on the salient aspects of a model enabling retrieval and storage of salient information, and exhibiting salient interconnections, such as structural and causal, in appropriate co-location. Further they facilitate construction of shared mental models within a community and transportation of scientific models out of the local milieu of their construction.

As used in model-based reasoning in physics, visual representations participate in modeling phenomena in several ways, including providing abstracted and idealized representations of aspects of phenomena and embodying aspects of theoretical models. For example, early in Faraday's construction of an electromagnetic field concept, the imagistic model he constructed of the lines of force provided an idealized representation of the patterns of iron filings surrounding a magnet (figure 3). But research substantiates that later in his development of the field concept, the imagistic model functioned as the embodiment of a dynamical theoretical model of the transmission and interconversion of forces, generally, through stresses and strains in, and various motions of, the lines (Gooding 1981; Gooding 1990; Nersessian 1984b; Nersessian 1985; Tweney 1985; and Tweney 1992). But, as I have argued, the visual representation Maxwell presented of the idle wheel–vortex model was intended as an embodiment of an imaginary system, displaying a generic dynamical rela-

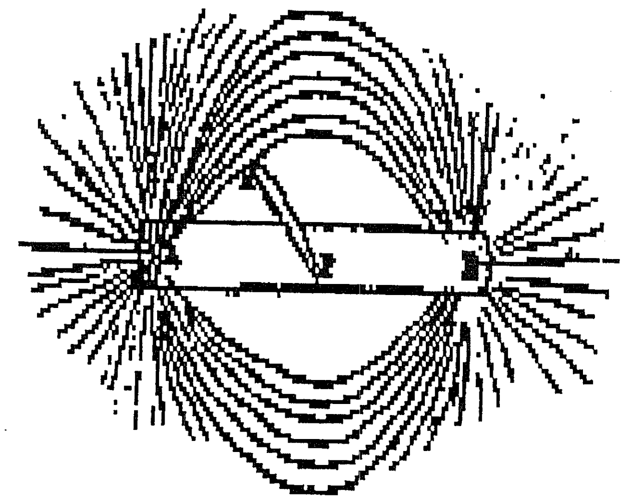

FIGURE 3
Faraday's drawing of the lines of force surrounding a bar magnet (Faraday 1839–55, vol. 1, Plate 1)

tional structure, and not as a representation of the theoretical model of electromagnetic field actions in the aether (figure 2).

As a form of model-based reasoning, thought experimenting can be construed as a specific form of the simulative reasoning that can occur in conjunction with the other kinds of model-based reasoning. Such simulative reasoning would involve constructing a model and using tacit and explicit knowledge to produce new states from it. Constructing a thought-experimental model requires understanding the salient constraints governing the kinds of entities or processes in the model and the possible causal, structural, and functional relations among them. Conducting a simulation requires tacit or explicit understanding of the constraints governing how those kinds of things behave and interact and how the relations can change. A simulation creates new states of a system being modeled, which in turn creates or makes evident new constraints. Changing the conditions of a model enables inferences about differences in the way that a system can behave. Because the simulation complies with the same constraints of the system it represents, performing a simulation with a model enables inferences about real-world phenomena.

In the case of scientific thought experiments implicated in conceptual change, the main historical traces are in the form of narrative reports constructed after the problem solving has taken place. These have often provided a significant means of effecting conceptual change within a scientific community. Accounting for the generative role of this form of model-based reasoning begins with examining how these thought-experimental narratives support modeling processes, and by means of cognitive-historical analysis infers that the original experiment involves a similar form of model-based reasoning. What needs to be determined is: (1) how a narrative facilitates the construction of a model of an experimental situation in thought and (2) how one can reach conceptual and empirical conclusions by mentally simulating the experimental processes.

From a mental modeling perspective, the function of the narrative form of presentation of a thought experiment would be to guide the reader in constructing a mental model of the situation described by it and to make inferences through simulating the events and processes depicted in it. A thought-experimental model can be construed as a form of "discourse" model studied by cognitive scientists, for which they argue that the operations and inferences are performed not on propositions but on the constructed model (see, e.g., Johnson-Laird 1982; Johnson-Laird 1989; Morrow, Bower, and Greenspan 1989; Perrig and Kintsch 1985). Unlike a fictional narrative, however, the context of the scientific thought experiment makes the intention clear to the reader that the inferences made pertain to potential real-world situations. The narrative has already made significant abstractions, which aid in focusing attention on the salient dimensions of the model and in recognizing the situation as prototypical (generic). Thus, the experimental consequences are seen to go beyond the specific situation of the thought experiment. The thought-experimental narrative is presented in a polished form that "works," which should make it an effective means of getting comparable mental models among the members of a community of scientists. Undoubtedly some experimental revision and tweaking goes on in the original reasoning and in the narrative construction, although accounts of this process are rarely presented by scientists.

Although some kinds of mental modeling may employ static representations, those derived from thought-experimental narratives are usually dynamic. The narrative delimits the specific transitions that govern what takes place. In constructing and conducting the experiment a scientist makes use of inferencing mechanisms, existing representations, and scientific and general world knowledge to make constrained transformations from one possible physical state to the next. Much of the information employed in these transformations is tacit. Thus, expertise and learning play a crucial role in the practice. The thought-experimental reasoning processes link the conceptual and the experiential dimensions of human cognitive processing (see also Gooding 1992). Thus, the constructed situation inherits empirical force by being abstracted both from our experiences and activities in the world and from our knowledge, conceptualizations, and assumptions of it. In this way, the data that derive from thought experimenting have empirical consequences and at the same time pinpoint the locus of the needed conceptual reform. The derived understanding forms the basis of further problem-solving efforts to construct an empirically adequate conceptualization.

In summation, there are several key ingredients common to the various forms of model-based reasoning considered in this section. The models are intended as interpretations of target physical systems, processes, phenomena, or situations. The models are retrieved or constructed on the basis of potentially satisfying salient constraints of the target domain. In the modeling process, various forms of abstraction, such as limiting case, idealization, generalization, generic modeling, are utilized, with generic modeling playing a highly significant role in the abstraction and integration of constraints. Evaluation and adaptation take place in light of structural, causal, and/or functional constraint satisfaction and enhanced understanding of the target problem that has been obtained through the modeling process. Simulation can be used to produce new states and enable evaluation of behaviors, constraint satisfaction, and other factors.

4. Maxwell's Use of "the Method of Physical Analogy": A Cognitive-Historical Interpretation

4.1 *The "Method of Physical Analogy"*

Maxwell's writings are peppered with talk of "mental operations" in physical and mathematical reasoning. One particularly nice expression of such concerns appears in an article he wrote for *Nature* about the new mathematical formalism, the method of quaternions, that he employed in the *Treatise:* "It does not . . . encourage the hope that mathematicians may give their minds a holiday, by transferring all their work to their pens. It calls upon us at every step to form a mental image of the geometrical features represented by the symbols, so that in studying geometry by this method, we have our minds engaged with geometrical ideas, and are not permitted to fancy ourselves geometers when we are only arithmeticians" (Maxwell 1873a, 137). Although I would not be so foolish as to place Maxwell in the ranks of early cognitive scientists, I nevertheless believe my cognitive-historical analysis of his reasoning to be in the spirit of a man who would make such assertions.

Although ignored by many philosophers and historians, Maxwell's own comments on his method of analysis are most insightful. In investigating a new area in science, Maxwell asserted that one begins with a process of "simplification and reduction of the results of previous investigation to a form in which the mind can grasp them" (Maxwell 1855–56, 155). That process requires a "method of investigation, which allows the mind at every step to lay hold of a clear physical conception, without being committed to any theory founded on they physical science from which that conception is borrowed so that it is neither drawn aside from the subject in pursuit of analytical subtleties, nor carries beyond truth by favourite hypotheses" (ibid., 156). A "physical analogy" is "that partial similarity between the laws of one science and those of another which makes each of them illustrate the other" (ibid.). As Howard has pointed out, "'analogy' in Maxwell's sense is an isomorphism, an equivalence of form" (Stein 1976, 35). This can also be said of the many analogies Thomson constructed, such as between heat and electrostatics, and Maxwell wrote to him about the method that he "intended to borrow it for a season . . . but applying it in a somewhat different way (Larmor 1937, 17–18).[6]

However, Thomson's method was to take an existing mathematical representation of a known physical system, in this case Fourier's analysis of heat, and substitute the parameters for the system under investigation into the equations, in this case electrostatic parameters. What makes Maxwell's modeling process different is that the analogical source to be mapped to the domain of electromagnetism was not ready to hand, but had to be constructed. That is, Maxwell did not know the mathematical structures that could be applied, but through the discovery of an equivalence of form between the dynamical structure of certain mechanical relations and certain electromagnetic relations, he was able to construct the requisite structures. This kind of model-based reasoning process has the potential to lead to genuinely new representational structures, in other words, conceptual change. It does not matter whether the mechanical systems employed in the models do or do not exist in nature; all that matters is that they are "mechanically conceivable." That is, that they supply mechanisms belonging to the classes of phenomena with dynamical relational structure common to mechanics and electromagnetism. Throughout his reasoning processes Maxwell abstracted from the specific mechanism to find the mathematical form of that class of mechanism, in other words, of the generic dynamical structure.

In constructing the mathematical representation of the electromagnetic field concept, Maxwell created several models of an imaginary fluid medium drawing from the source domains of continuum mechanics and of machine mechanics. On my analysis, these analogical domains served as sources for constraints used together with those provided by the target problem to create the imaginary analog models that served as the basis of his reasoning. Maxwell also employed several imagistic representations, such as that in figure 2, which we discussed previously, and those in figure 4, which were

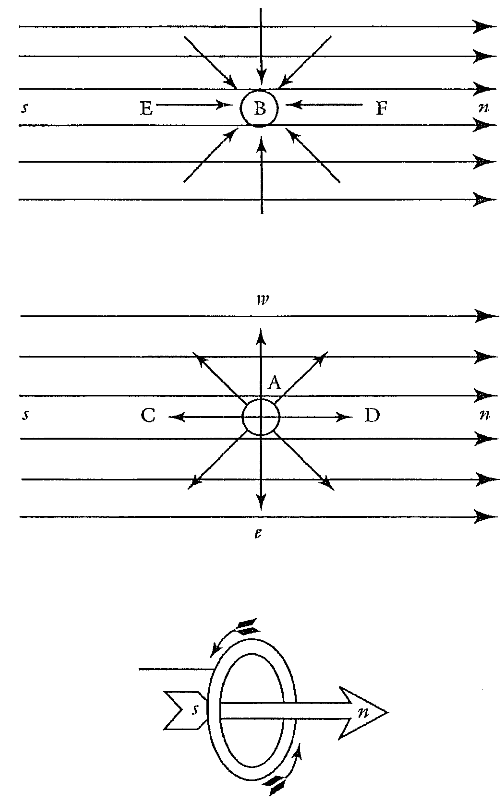

FIGURE 4
Maxwell 1890, vol. 1, p. 460, figs. 1–3

accompanied by text for how to imagine the motion of the vortices in the planes above and below the plane of the paper on which the figures were drawn. The analysis presented below is just of the published work, but my interpretation of Maxwell's reasoning also draws on his letters to Thomson during the period, what little draft material exists in the archives, and other published work. As I have argued elsewhere (Nersessian 1984; Nersessian 1992a) I believe there is sufficient evidence to support my contention that the reasoning in the published papers accurately presents Maxwell's own reasoning processes. We can look at him as attempting to lead his audience through his own reasoning processes as a rhetorical move to help his colleagues to understand the new field representation.

4.2 The Initial Model

Maxwell constructed a mathematical representation for the electromagnetic field concept over the course of several papers. I will focus on "On physical lines of force" (Maxwell 1861–62) and show the relevance of that analysis for "A dynamical theory of the electromagnetic field" (Maxwell 1864). In part 1 of the 1861–62 paper, the mathematical representation of various magnetic phenomena derives from a vortex-fluid model of the aether. Maxwell began with discussing general features of stress in a medium. Stress results from action and reaction of the contiguous parts of a medium and "consists in general of pressures or tensions different in different directions at the same point in the medium" (1861–62, 454). The force is a pressure along the axis of greatest pressure and a tension along the axis of least pressure. Maxwell hypothesized that the causes of electromagnetic phenomena are stresses in a mechanical continuum, the electromagnetic aether, which transmits electromagnetic actions continuously through the space surrounding bodies and charges. Given this hypothesis, one can assume a "resemblance in form" between dynamical relations that hold in the domains of continuum mechanics, such as fluids and elastic solids, and those that hold in the domain of electromagnetism. Thus, continuum mechanics can serve as a source domain for constructing models. The problem is to determine which relations, and determination of these was guided by constraints drawn from the domain of electromagnetism. As specific constraints on stresses that could account for magnetism, Maxwell considered (1) a tension in the direction of the lines of force and (2) a pressure greater in the equatorial than in the axial direction. These constraints are consistent with the geometrical configurations of the magnetic lines of force and Faraday's interpretation of them as resulting from lateral repulsion and longitudinal attraction. The magnetic constraints specify a configuration of forces in the medium and this configuration, in turn, is explained as resulting from the centrifugal forces of vortices in the medium with axes parallel to the lines of force. The centrifugal force of a vortex would cause it to expand equatorially and contract longitudinally. Each vortex is dipolar in that it rotates in opposite directions at its extremities. Further, the geometric constraints on the lines of force are satisfied by the shape of the vortex, which is wider the farther it is from its origin, so the lines become farther apart as they approach their midpoints. Thus, the vortex motion supplies a causal process that is capable of producing the configuration of the lines of force and the stresses in and among them. Figure 5 is my representation of such a vortex drawn from Maxwell's description. The vortex-fluid model is also consistent with constraints derived from known experimental results: (1) electric and magnetic actions are at right angles to each other; (2) magnetism is dipolar; and (3) the plane of polar-

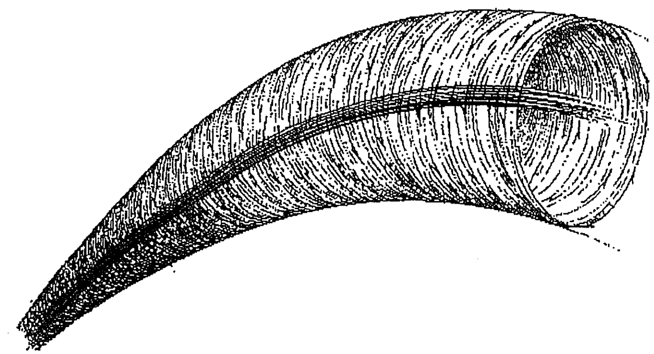

FIGURE 5
Vortex segment

ized light passed through a diamagnetic substance is rotated by magnetic action.

The system of infinitesimal vortices does not correspond to any known physical system. Maxwell constructed it to serve as the basis for deriving mappings between known dynamical relations in continuum mechanics and those thought to produce electromagnetic phenomena. The mathematical expressions for the magnetic phenomena are derived from the mathematical formula for the stresses in the vortex-fluid model by substitution. The vortex-fluid model is "generic" in that it is to be understood as satisfying constraints that apply to the *types* of entities and processes that can be considered as constituting either domain. The model represents the class of phenomena in each domain that are capable of producing the specific configurations of stresses. The modeling process Maxwell used throughout the analysis went as follows. First he constructed a model representing a specific mechanism. Then he treated the dynamical properties and relations generically by abstracting features common to the mechanical and the electromagnetic classes of phenomena. He proceeded to formulate the mathematical equations of the generic model and substituted in the electromagnetic variables.

In part 1 of the analysis Maxwell used the mathematical properties of the limiting case of a single vortex to derive formulas for quantitative relations consistent with the known constraints on magnetic systems. I will only give the highlights of that analysis. From the vortex-fluid model he derived an expression for the resultant force on an element of the medium due to variation in internal stress: $\mathbf{F} = [\mathbf{v}((1/4\pi)(\text{div } \mu\mathbf{v}))] + [(1/8\pi)(\mu \, \mathbf{grad} \, v^2)] + [\mu\mathbf{v} \times (1/4\pi)(\mathbf{curl} \, \mathbf{v})] - \mathbf{grad} \, p_1$ (equation 5, p. 458, where '**x**' denotes the cross product),[7] which we now call the general mechanical stress tensor). He then constructed the electromagnetic version by mapping quantitative properties as follows. He stipulated that the quantities related to the velocity of the vortex ($\alpha = vl$, $\beta = vm$, and $\gamma = vn$, with l, m, n the direction cosines of the axes of the vortices) be mapped to the components of the force acting on a unit magnetic bar pointing north. So, the magnetic intensity, which in contemporary notation is designated as 'H', is here related to the velocity gradient of the vortex at the surface. The quantity 'μ' is taken to represent the magnetic permeability, thus relating it to the mass of the medium. The quantity 'μH' represents the magnetic induction.

Substituting the magnetic quantities, Maxwell rewrote the first term of the mechanical stress tensor for the magnetic system as $\mathbf{H}(1/4\pi\,(\text{div}\,\mu\mathbf{H}))$ (equation 7, p. 459). He followed the same procedure of constructing a mapping between the model and the magnetic quantities and relations to rewrite all of the components of the stress tensor for magnetism. The resulting *electromagnetic* stress tensor represents the resultant force on an element of the magnetic medium due to its internal stress. The four components of the mechanical stress tensor, as interpreted for the electromagnetic medium, are $\mathbf{F} = [\mathbf{H}(1/4\pi\,(\text{div}\,\mu\mathbf{H}))] + [(1/8\pi)(\mu\,\mathbf{grad}\,H^2)] + [\mu\mathbf{H} \times (1/4\pi)(\mathbf{curl}\ \mathbf{H})] - \mathbf{grad}\ p_1$ (equations 12–14, p. 463). By component they are (1) the force acting on magnetic poles, (2) the action of magnetic induction, (3) the force of magnetic action on currents, and (4) the effect of simple pressure. The last component is required by the model—it is the pressure along the axis of a vortex—but had not yet been given an electromagnetic interpretation.

I will not go through the details of additional constructions and mappings, except to point out that he also derived an expression relating current density to the circulation of the magnetic field around the current-carrying wire $j = 1/4\pi\,(\mathbf{curl}\ \mathbf{H})$ (equation 9, p. 462). This equation agreed with the differential form of Ampère's law he had derived from kinematic considerations in the first paper of this series of three (1855–56, p. 194). The derivation given here still did not provide a mechanism connecting current and magnetism.

4.3 Introducing Idle Wheels

Thus far, then, Maxwell had been able to provide a mathematical formulation for magnetic induction, paramagnetism, and diamagnetism through modeling these phenomena by means of a nonexistent, but mechanically conceivable dynamical system. A mechanical inconsistency in the vortex-fluid model led Maxwell to a means of representing the causal relationships between magnetism and electricity. He began part 2 by stating that his purpose was to inquire into the connection between the magnetic vortices

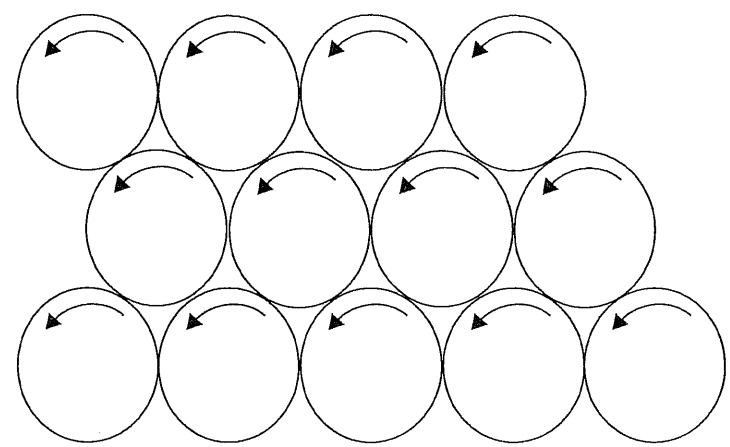

FIGURE 6
Cross section of model of vortex-fluid medium

and current. Thus he could no longer simply consider a single generic vortex in his analysis. He admitted a serious problem with the model in that he "found great difficulty in conceiving of the existence of vortices in a medium, side by side, revolving in the same direction" (468). Figure 6 is my drawing of a cross section of the vortex-fluid model as described by Maxwell. By imagining the motion of the vortices in this figure, it becomes evident that direct contact between consecutive vortices poses a problem in that there will be friction and, thus, jamming. Further, since they are all going in the same direction and, thus, at points of contact they would be going in opposite directions, in the case where they are revolving at the same rate, the whole mechanism should stop. Maxwell noted that in machine mechanics this kind of problem is solved by the introduction of "idle wheels." On that basis he proposed to enhance his imaginary model by supposing that "a layer of particles, acting as idle wheels is interposed between each vortex and the next" (468). In introducing the idle wheels, Maxwell stipulated that the particles would revolve in place in direction opposite to the vortices without slipping or touching. This is consistent with the constraint that the lines of force around a magnetic source can exist for an indefinite period of time, so there can be no loss of energy in the model. He also stipulated that there should be no slipping between the interior and exterior layers of the vortices, making the angular velocity constant. This constraint simplified calculations, but is inconsistent with the mechanical constraint that the vortices have elasticity, and would be eliminated in part 3. Figure 2 is Maxwell's rendering of the idle wheel–vortex model. The diagram shows a cross section of the medium. The vortex cross sections are represented by hexagons rather than circles, presumably to provide a better representation of how the particles are packed around the vortices, with the three-dimensional dodecahedra approximating to spheres in the limit.

The idle wheel–vortex model is a hybrid constructed from two source domains: fluid dynamics and machine mechanics. As discussed earlier, to combine salient entities and processes from two disparate domains requires abstraction of these to a sufficient level. My explanation of how generic abstraction could have led to the introduction of the idle-wheel particles is illustrated in figure 7. First Maxwell abstracted a generic model of spinning

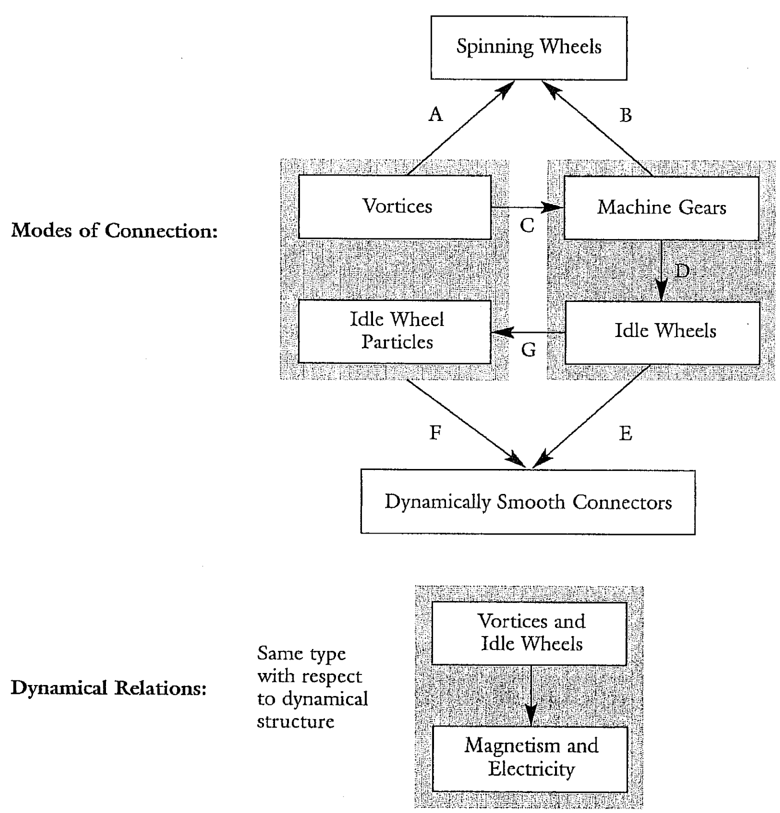

FIGURE 7
Introducing idle wheels via generic modeling

wheels from the vortex-fluid model (A). The generic model of spinning wheels reminded him of specific mechanical systems containing machine gears (B). He noticed an analogy between the vortices and the gears (C), but how this analogy would provide a new mode of connection for the vortices was not immediately evident. Next, from the model of the machine gears he abstracted the generic model of idle wheels (D), and then further abstracted that model into the generic model of dynamically smooth connectors (E). Finally, he instantiated the generic model of dynamically smooth connectors in the vortex-fluid model in the form of idle-wheel particles (F), where the instantiation is guided by both the analogous case of idle wheels (G) and constraints of the continuum mechanical system.

Maxwell, himself, stressed that the idle-wheel mechanism is not to be considered "a mode of connexion existing in nature" (486). Rather it is "a mode of connexion which is mechanically conceivable and easily investigated, and it serves to bring out the actual mechanical connexions between the known electro-magnetic phenomena" (ibid). It does so because the dynamical relations between the idle wheels and vortices are of the same kind as those between electricity and magnetism in the process of induction. That is, although a concrete mechanism is provided, in the reasoning process, the idle wheel–vortex system is taken to represent the class of dynamical systems having certain abstract relations in common. This class includes electric and magnetic interactions on the assumptions of Maxwell's treatment. Thus, in the analysis of electromagnetic induction discussed below, the idle wheel–vortex mechanism is not the *cause* of electromagnetic induction; it represents the *causal structure* of that kind of process. Throughout part 2 Maxwell provided analogies with machinery as specific mechanical interpretations of the relations he had derived between the idle-wheel particles and the fluid vortices to establish for the reader that there are real physical systems that instantiate the generic relations.

In ordinary mechanisms, idle wheels rotate in place. In the model this allows representation of action in a dielectric, or insulating, medium. To represent current, though, the idle wheels need to be capable of translational motion in a conducting medium. Maxwell noted that there are mechanical systems such as the "Siemens governor for steam-engines" which have idle wheels that can translate out of place. The major constraints that need to be satisfied are that (1) a steady current produces magnetic lines of force around it, (2) commencement or cessation of a current produces a current, of opposite orientation, in a nearby conducting wire, and (3) motion of a conductor across the magnetic lines of force induces a current in it. The dynamical relations between the vortices and the idle wheels serve to model the constraints governing the dynamical relations between electric currents and magnetism.

Going through the piece of the analysis in which Maxwell derived the equations for the translational motion of the particles in the imaginary system will help us to understand more fully the relationship between the model and the electromagnetic system. There is a tangential pressure between the surfaces of the spherical particles and the surfaces of the vortices, treated here as approximating rigid pseudospheres. So conceived, these vortices appear to be inconsistent with the geometrical constraints of the vortices in part 1 which should require the vortices to be elastic, but this would not be addressed until the analysis of static electricity in part 3. Maxwell derived the equation for the average flux density of the particles as a function of the circumferential velocity of the vortices $\mathbf{p} = \frac{1}{2}(\rho\ \mathbf{curl\ v})$ (equation 33, p. 471) and noted that it is of the same form as the equation relating current density and magnetic field intensity $\mathbf{j} = 1/4\pi\ (\mathbf{curl\ H})$ (equation 9), the form of Ampère's law for closed circuits he had derived in part 1. All that is required to make the equations identical is to set 'ρ', the quantity of particles on a unit of surface, equal to $1/2\pi$. He concluded that "it appears therefore, according to our hypothesis, an electric current is represented by the transference of the moveable particles interposed between the neighboring vortices" (471). That is, the flux density of the particles is taken to represent the electric current density.

We can see how the model provides a mathematical interpretation for constraint (1). Current is represented by translational motion of the particles. In a conductor the particles are free to move but in a dielectric (which the aetherial medium is assumed to be) the particles can only rotate in place. In a nonhomogeneous conducting medium, different vortices would have different velocities and the particles would experience translational motion. They would experience resistance and waste energy by generating heat, as is consistent with current. A continuous flow of particles would thus be needed to maintain the configuration of the magnetic lines of force about a current. The causal relationship between a steady current and magnetic lines of force is captured in the following way. When an electromotive force, such as from a battery, acts on the particles in a conductor it pushes them and starts them rolling. The tangential pressure between them and the vortices sets the neighboring vortices in motion in opposite directions on opposite sides—thus capturing the polarity of magnetism—and this motion is transmitted throughout the medium. The mathematical expression (equation 33) connects current with the rotating torque the vortices exert on the particles. Maxwell went on to show that this equation is consistent with the equations he had derived in part 1 for the distribution and configuration of the magnetic lines of force around a steady current (equations 15–16, p. 464).

Although I won't go through the derivations, Maxwell derived the laws of electromagnetic induction in two parts. Again we see the role of the model in the derivation, since the two cases are different mechanically in the idle wheel–vortex system. In the first case, the mechanism for communicating rotational velocity in the medium accounts for induction of currents by starting or stopping a primary current (constraint [2]), such as switching current off and on in a conducting loop and thereby inducing a current in a nearby conducting loop. A decrease or increase in current will cause a corresponding change in the velocity of the adjacent vortices. The difference in velocity between this row and the next adjacent row will cause the particles surrounding those vortices to speed up or slow down, and this motion will in turn be communicated to the next row and so on until the conducting wire is reached. The particles in the wire will be set in translational motion by the differential electromotive force between the vortices, thus inducing a current oriented in direction opposite to the initial current, which agrees with experimental results. In the second case, in which a current is induced by motion of a conductor across the lines of force (constraint [3]), Maxwell used considerations pertaining to the changing form and position of the medium. Briefly, the portion of the medium in the direction of the conducting wire would become compressed, causing the vortices to elongate and speed up, while vortices behind the wire contract back into place and decrease in velocity. The net force pushes the particles inside of the conductor, producing a current provided there is a circuit connecting the ends of the wire. The case of open circuits is considered in part 3.

4.4 *The "Displacement Current" and Inconsistent Signs*

By the end of part 2, Maxwell had given mathematical expression to some electromagnetic phenomena in terms of actions in a mechanical medium and had shown the representation coherent and consistent with known phenomena. The full mathematical representation of the electromagnetic field was constructed in part 3 with Maxwell's treatment of static electricity. I will focus on one piece of Maxwell's analysis—the introduction of what he called "the displacement current"—since this feature of the model leads to a formal inconsistency in the equations Maxwell presented in his next paper on the subject (1864). Understanding why he tolerated it and how he eliminated it in 1873 in the *Treatise* (Maxwell 1873b) will provide a deeper appreciation of the role of generic modeling in his analysis.

The fact that Maxwell submitted the part of the analysis pertaining to electrostatics in the 1861–62 paper eight months after the work on magnetism and electromagnetic induction was published indicates that the initial representation of static electricity was difficult for him to work out on the basis of the idle wheel–vortex model. It would seem quite natural to identify charge with the accumulation of idle-wheel particles at the boundary of dielectric and conducting media. Thus, it is puzzling why Maxwell did not immediately proceed to the type of analysis he ultimately presented and he provides no clues of his path to that solution. Siegel (1991) presents a detailed and plausible analysis of the nature of the problems Maxwell would have encountered in trying to construct an account of the interface between conducting and dielectric media given the specific mechanism of the model.[8] The solution to these problems was to make the vortices elastic. So, the idle wheel–vortex model was modified in part 3 by again considering its plausibility as a mechanical system. In part 2, the system contains vortex cells of rotating fluid separated by particles very small in comparison to them. To simplify the calculations for the transmission of rotation from one cell to another via the tangential action between the surface of the vortices and the particles, Maxwell had assumed the vortices to be rigid. But he now noted that in order for the rotation to be transmitted from the exterior to the interior parts of the cells, the cell material needs to be elastic. And, although he does not comment on it, conceiving of the molecular vortices as pseudospherical blobs of elastic material would also give them the right configuration on rotation (figure 5), and thus eliminate the inconsistency of the rigid vortices with the geometrical constraints of part 1 for magnetism.

He began by noting the constraint that "electric tension" associated with a charged body is the same, experimentally, whether produced from static or current electricity. If there is a difference in tension in a body, it will produce either current or static charge, depending on whether the substance is a conductor or insulator. He likened a conductor to a "porous membrane which opposes more or less resistance to the passage of a fluid" (490) and a dielectric to an elastic membrane which does not allow passage of a fluid, but "transmits the pressure of the fluid on one side to that on the other" (491). In the process of electrostatic induction, electricity can be viewed as "displaced" within a molecule of a dielectric, so that one side becomes positive and the other negative, but does not pass from molecule to molecule. Although Maxwell did not immediately link his discussion of the different manifestations of electric tension to the hybrid model of part 2, it is clear that it figures throughout the discussion. This is made explicit in the calculations immediately following the general discussion. I note this because the notion of "displacement current" introduced before these calculations cannot properly be understood without the model. Maxwell claimed that the displacement of electricity in electrostatic induction can be likened to a current in that *change* in displacement is similar to "the commencement of a current" (491). That is, given the model, an electrostatic force produces a slight elastic distortion in the vortices causing a slight translational motion of the idle-wheel particles, which is propagated throughout the dielectric medium.

The mathematical expression relating the electromotive force and the displacement that Maxwell established is: $\mathbf{E} = -4\pi k^2\mathbf{D}$, where '**E**' is the electromotive force (electric field), 'k' the coefficient for the specific dielectric, and '**D**' is the displacement (491). The amount of current due to displacement is $\mathbf{j}_{disp} = \partial\mathbf{D}/\partial t$. The equation relating the electromotive force and the displacement has the displacement in the direction opposite from that which is customary now and in Maxwell's later work. The orientation given here can be accounted for if we keep in mind that on the model an elastic resorting force is opposite in orientation to the impressed force. Although Maxwell stressed that the relations expressed by the above formula are independent of a specific *theory* about the actual internal mechanisms of a dielectric, they are not independent of the *model*. Without the mechanism of the model, there is no basis on which to call the motion a "current." It is translational motion of the particles which constitutes current. Thus, in its initial derivation, the "displacement current" is modeled on a specific mechanical process. We can see this in the following way.

Recall the difference Maxwell specified between conductors and dielectrics when he first introduced the idle-wheel particles. In a conductor, they are free to move from vortex to vortex. In a dielectric, they can only rotate in place. In electrostatic induction, then, the particles can only be urged forward by the elastic distortion of the vortices, but cannot move out of place. This motion is similar to that of the "commencement of a current." But, their motion "does not amount to a current, because when it has attained a certain value it remains constant" (491). That is, the particles do not actually move out of place by translational motion as in conduction, instead they accumulate, creating regions of stress. Since they are not free to flow, they must react back on the vortices with a force to restore their position. The system reaches a certain level of stress and remains there. 'Charge', then, is interpreted as the excess of tension in the dielectric medium created by the accumulation of displaced particles. Without the model, "displacement current" loses its physical meaning, which is what bothered so many of the readers of the *Treatise*, where the mechanical model is no longer employed. As we will see below, it also created problems for Maxwell in his 1864 analysis.

Since the vortices are now elastic and since in a conductor the particles are free to move, the current produced by the medium (that is, net flow of particle per unit area) must include a factor for their motion due to the elasticity. So Maxwell corrected the equation for Ampère's law (equation 9) to include the total current, $\mathbf{j} = 1/4\pi\,(\mathbf{curl\ H}) - \partial\mathbf{E}/\partial t$ (equation 112, p. 496). Since the emf has rotation opposite to the rotation of the vortices, the "displacement current" actually opens the closed current of equation 9, creating a noncircuital current.[9] He coupled this equation with the equation of continuity for charge, which links current and charge, to derive an expression linking charge and the electric field, $e = 1/4\pi\ (k^2 \text{ div } \mathbf{E})$ (equation 115, p. 497), which is equivalent to $\rho = -\text{ div } \mathbf{D}$. This latter expression looks similar to what we now call Coulomb's law except for two features that turn out to be highly salient for understanding Maxwell's reasoning. First, the form of this equation and the modified equation for current (equation 112) again demonstrates Maxwell's field conception of current and charge: interpreted left to right, charge and current arise from the field. Second, the minus sign is not part of the contemporary equation, but arises out of the model because the elastic restoring force exerted on the vortices by the particles and the electromotive force have opposite orientation. Through what can be interpreted simply as a substitution error in equation 104 the equations in this paper are consistent.

Turning to the 1864 paper, we can see how the specific interpretation of the displacement current created problems. In this paper, Maxwell rederived the field equations without explicit reference to the mechanical model. Once he had abstracted the electromagnetic dynamical properties and relations it was possible to derive the electromagnetic equations using generalized dynamics and assuming only that the electromagnetic aether is a "connected system," possessing elasticity and thus having energy. Here Maxwell made the identification of the electromagnetic and light aethers that he failed to do at the end of the 1861–62 analysis, where the close agreement of the velocity of transverse vibrations in these two hypothetical media led him to state "we can scarcely avoid the inference that *light consists in transverse undulations of the same medium which is the cause of electric and magnetic phenomena*" (italics in the original, 500). We can interpret Maxwell's reticence to draw the inference in the earlier analysis as due to the value of the transverse velocity in the electromagnetic medium being derived from specific features of the idle wheel–vortex model. There were no grounds on which to assume vortex motion in the light aether. Note also that Maxwell was not avoiding the inference that light is an electromagnetic phenomenon but only the possible identity of the two media. On the then prevailing view light is a transverse wave in an elastic medium and this is not the same kind of mechanism as that provided by the model for propagating electromagnetic actions. In the 1864 paper Maxwell treated the aether as a generic elastic medium whose constraints could be satisfied by many specific mechanical instantiations (in the *Treatise*, Maxwell says an "infinite" number) and thus saw no reason for multiplying aethers. Elastic systems can receive and store energy in two forms, what Maxwell called the "energy of motion," or kinetic energy, and the "energy of tension," or potential energy. He identified kinetic energy with magnetic polarization and potential energy with electric polarization. Figure 8 illustrates my interpretation of how the 1861–62 analysis enabled him to do this through generic abstraction from the model. Although in 1864

Maxwell is thinking of the aetherial medium in more abstract, general dynamical terms, vestiges of the earlier specific mechanical model can be shown to have remained in his thinking and this created a problem with the current and charge equations.

The infamous inconsistency arises in the 1864 paper because in the absence of the mechanical model there is no basis on which to distinguish conduction current and displacement current. Thus current is treated generically in terms of the stresses in the medium created by the flow of electricity, so $\mathbf{E} = k\mathbf{D}$ and coupling the equation for the total current $\mathbf{j} = 1/4\pi\ (\mathbf{curl\ H}) - \partial\mathbf{D}/\partial t$ with the equation of continuity $\partial\rho/\partial t + \text{div } \mathbf{j} = 0$, yields $\rho - \text{div } \mathbf{D}$. However, Maxwell wrote the equivalent of $\rho = -\text{ div } \mathbf{D}$ as the "equation of free electricity" (charge)—that is, the equation in the form of the 1861–62 analysis. So the complete set of equations which he gathers together in part 3 of the 1864 paper is formally inconsistent. My interpretation is that Maxwell continued to think of charge as associated

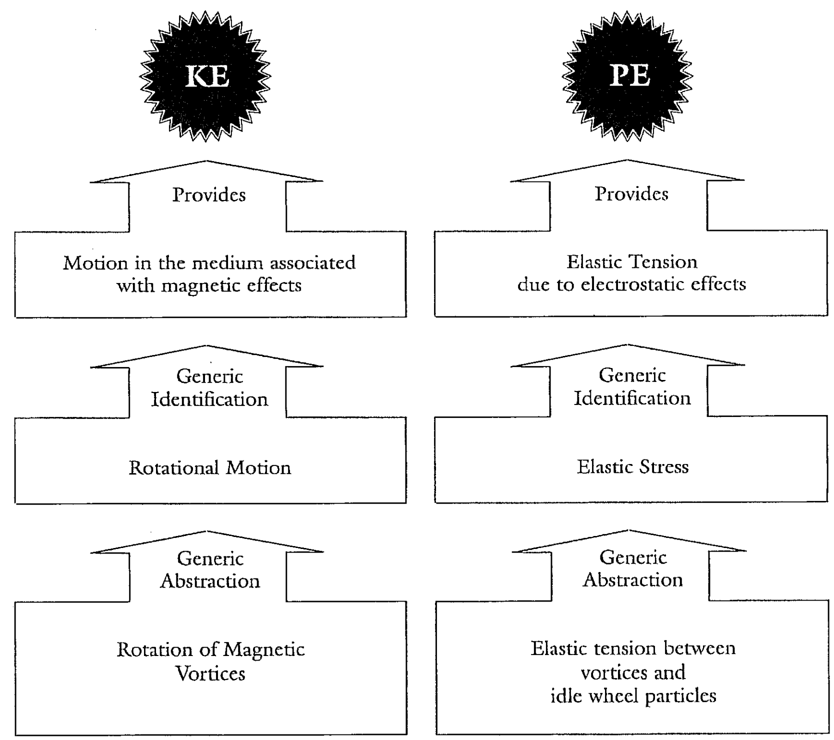

FIGURE 8
Identifying energy components via generic modeling

with the accumulation of the idle-wheel particles through the specific mechanism of "displacement" and thus with the reactive force that is oriented away from the accumulation point. However, the mathematical equations of the generic medium require that it be pointing toward the stress point and so clearly require $\rho = \text{div}\,\mathbf{D}$.

There are few existing drafts of Maxwell's published papers, but fortunately there are a couple of pages of a draft pertaining to this derivation.[10] These reveal that Maxwell had some confusion about how to think of current and charge without the medium. In the draft equation for current, Maxwell wrote the components of "electric resistance," in other words, the electromotive force required to keep the current flowing through a conductor, as pointing in the opposite direction, as would have been the case in the mechanical medium, but in the published paper, these are written correctly. In the draft equation for "statical electricity" the components are written with the negative sign as above. Maxwell's handwriting can be interpreted as indicating that he struggled with this result.[11] In writing the first two components Maxwell made the equals sign three times the length used in other samples of his handwriting and he actually blends the equals sign into the minus sign. Only for the final component is the equals sign of regular length and clearly separated from the minus sign. As noted above, it is also written with the minus sign in the published paper.[12]

Maxwell never discussed this inconsistency and then in the *Treatise*, again without discussion, the inconsistency is gone. Although it is just speculation on my part, given how Maxwell collects all of his equations in part 3 of the published paper, it is hard to imagine that he did not notice the inconsistency. I believe he kept the Coulomb equation $\rho = -\text{div}\,\mathbf{D}$ (equation G; p. 561) in 1864 because he had not figured out how to conceive of charge generically, that is to abstract it from the specific mechanism through which he had been able initially to represent it. In the *Treatise* charge is abstracted from the notion of stress and reactive force due to accumulating particles and treated generically as elastic stress from the flow of electricity through a medium, with orientation in the direction of flow. In conduction the current flow is unimpeded and in electrostatics, stress is created at points of discontinuity, such as where a charging capacitor and a dielectric meet. The generic notion of charge as associated with elastic stress is compatible with Faraday's field notion of charge, but was to cause difficulties in comprehending Maxwell's for those who held the customary action-at-a-distance notion that charge is associated with a particle. As H. A. Lorentz noted in a discussion of the need for a clear separation of field and charge, "Poincaré mentions a physicist who declared that he understood the whole of Maxwell's theory, but that he still had not grasped what an electrified sphere was !" (Lorentz 1891, 95, my translation from Dutch).

5. Further Reflections: Model-based Reasoning and Conceptual Change

My analysis has shown how Maxwell was able to formulate the laws of the electromagnetic field by abstracting from specific mechanical models the dynamical properties and relations that continuum-mechanical systems, certain machine mechanisms, and electromagnetic systems have in common. In their mathematical treatment these common dynamical properties and relations were separated from the specific instantiations provided in the models through which they had been rendered concrete. Thus the underlying inconsistencies in the models could be ignored. The generic mechanical relationships represented by the imaginary systems of the models served as the basis from which he abstracted a mathematical structure of sufficient generality that it represented causal processes in the electromagnetic medium without requiring knowledge of specific causal mechanisms (see also Stein 1970; Stein 1976; Stein 1981). In the final analysis in the *Treatise* even the "theory of molecular vortices" is presented in generic form, with vortex motion in the medium becoming "something belonging to the same mathematical class as an angular velocity, whose axis is in the direction of the magnetic force" (Maxwell 1873b, 459). However, we have also seen that at an intermediate stage of development specific, features of the model seemed to figure so strongly in his thinking that he introduced a formal inconsistency in the set of equations that was only eliminated several years later—again through generic abstraction from a mechanism of the idle wheel–vortex model.

With hindsight, we know, as Maxwell did not, that Maxwellian electrodynamics cannot be given a Newtonian formulation. Thus, the Maxwell case presents an interesting problem for those wanting to understand the nature of the creative reasoning employed by scientists in the processes of representational change. If Maxwell really did derive the mathematical laws of the electromagnetic field in the way he presents the analysis, how is it possible that employing analogies drawn from Newtonian mechanical domains, he constructed the laws of a non-Newtonian system, electrodynamics? My answer is that employing the process of generic abstraction in model-based reasoning enabled Maxwell to abstract a system of dynamical relations of greater generality than Newtonian mechanics. In this case, generic abstraction from mechanical models enabled Maxwell to construct a system of abstract laws that when applied to the class of electromagnetic systems yield the laws of a dynamical system that is nonmechanical, that is, one that is inconsistent with the laws of the mechanical domains from which its mathematical structure was abstracted. For Maxwell, Newtonian mechanics and general (Lagrangian) dynamics were thought to be coextensive. In arriving at the general dynamical form of the field equations starting from considerations of the specific mechanisms of the models, he could think that he had demonstrated the possibility of a "mechanical explanation" for the electromagnetic phenomena. Bu the Lagrangian formalism provides no information about the underlying system. Maxwell did not, however, "turn away from mechanical models" (Siegel 1991, 54). A complete explanation would need to provide an account of the actual mechanisms in the aether that create the phenomena, and Maxwell fully expected such an account to be forthcoming. The laws he had formulated would act as a constraint on any acceptable mechanism. Maxwell saw himself as following the same strategy Newton had in formulating the universal law of gravitation: "investigat[ing] the forces with which bodies act on each other in the first place, before attempting to explain *how* that force is transmitted" [italics in original] (Maxwell 1890a). Just what kind of mechanical system remained a problem for Maxwell and he saw his theory as incomplete without that specification. With hindsight, we can see Maxwell as having abstracted away the notion of mechanism, creating a representation of a nonmechanical, dynamical system.

I do not know if Howard will feel that I have provided a satisfactory assessment of the role Maxwell's analogies played in the development of his electromagnetic theory but I thank him for inspiring me to try.

NOTES

1. See also Larmor 1937; Maxwell 1855–56; Maxwell 1856.

2. See also Chalmers 1986 which argues in response to my1984a and 1984b analyses to the contrary that the analogy was an "unproductive digression."

3. Berkson also criticizes the symmetry argument as presented by Jackson (Berkson 1974, 338–39).

4. A few commentators at that time did take seriously at least parts of the analogy, most significantly Bromberg (1968) whose analysis of the displacement current I found useful; and Berkson (1974), whose work I did not come across until much later. Later, Seigel (1986 and 1991) presented arguments in favor of the centrality of the analogical model in the 1861–62 paper. Hesse's position is more difficult to characterize because she does see analogy as playing a significant role in hypothesis formation and theory interpretation in general. However, her analysis of Maxwell's theory passes over the analysis in the 1861–62 paper—which I am arguing contains the main generative work—entirely (Hesse 1973). Although I cannot elaborate here, I believe the problem is that her theory of analogy does not allow for the kind of abstraction I will be discussing in sections 3 and 4.

5. "If you want to find out anything from the theoretical physicists about the methods they use, I advise you to stick closely to one principle: don't listen to their words, fix your attention on their deeds. To him who is a discoverer in this field, the products of his imagination appear so necessary and natural that he regards them, and would like to have them regarded by others, not as creations of thought but as given realities" (Einstein 1973, 264).

6. See especially letters of November 13, 1854; May 18, 1855, pp. 8 and 11; and September 13, 1855, pp. 1718.

7. I have written this equation, which Maxwell wrote in component form, in modern vector notation and will do so throughout. The vector calculus was just being developed around the time of Maxwell's analysis. Note the actual physical meaning of the vector operators here: the gradient is a slope along a vortex, the curl is the rotation of the fluid vortex, and the divergence is the flowing of fluid in the medium. Maxwell reformulated his electromagnetic theory in quaternions, a form of vector calculus developed by Hamilton, in the *Treatise* (1873b).

8. I disagree, however, with his claim (Siegel 1991, 75) that Maxwell was prevented initially from making the vortices elastic since "Thomson saw elasticity as resulting from motion in a mechanical substratum." The vortices *are* a form of motion in a mechanical substratum and, as noted above, elasticity is consistent with the analysis in part 1. A more likely reason is that Maxwell had considered and rejected this alternative in the calculations for part 2 and was reluctant to modify this specific feature of the idle wheel–vortex version of the model.

9. See Nersessian 1984, 82 and Siegel 1991, 112.

10. Add MS 7655, V, c/8. University Library, Cambridge University.

11. See Siegel 1991, 174–75 for a similar point.

12. Harman mistakenly says this equation appears without the minus sign in the 1864 paper (Harman [Heimann] 1995, 161, n. 6).

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